EXAMPLE 4–3 **Isothermal Compression of an Ideal Gas**

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

SOLUTION Air in a piston–cylinder device is compressed isothermally. The boundary work done is to be determined.
Analysis A sketch of the system and the *P-V* diagram of the process are shown in Fig. 4–8.
Assumptions **1** The compression process is quasi-equilibrium. **2** At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.
Analysis For an ideal gas at constant temperature T_0,

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

where C is a constant. Substituting this into Eq. 4–2, we have

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}\,dV = C\int_1^2 \frac{dV}{V} = C\ln\frac{V_2}{V_1} = P_1V_1\ln\frac{V_2}{V_1} \qquad \textbf{(4–7)}$$

In Eq. 4–7, P_1V_1 can be replaced by P_2V_2 or mRT_0. Also, V_2/V_1 can be replaced by P_1/P_2 for this case since $P_1V_1 = P_2V_2$.

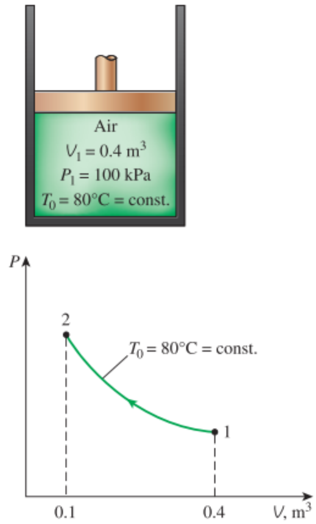


FIGURE 4–8
Schematic and *P-V* diagram for Example 4–3.

Substituting the numerical values into Eq. 4–7 yields

$$W_b = (100\text{ kPa})(0.4\text{ m}^3)\left(\ln\frac{0.1}{0.4}\right)\left(\frac{1\text{ kJ}}{1\text{ kPa·m}^3}\right)$$
$$= \mathbf{-55.5\ kJ}$$

Discussion The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.

EXAMPLE 4–1 **Boundary Work for a Constant-Volume Process**

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

SOLUTION Air in a rigid tank is cooled, and both the pressure and temperature drop. The boundary work done is to be determined.
Analysis A sketch of the system and the *P-V* diagram of the process are shown in Fig. 4–6. The boundary work can be determined from Eq. 4–2 to be

$$W_b = \int_1^2 P\,\cancel{dV}^{\,0} = \mathbf{0}$$

Discussion This is expected since a rigid tank has a constant volume and $dV = 0$ in this equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant-volume process is always zero. This is also evident from the *P-V* diagram of the process (the area under the process curve is zero).

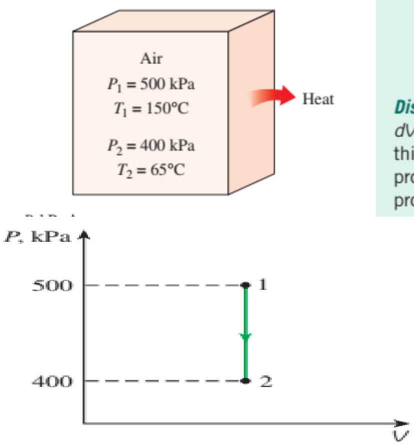


FIGURE 4–6
Schematic and *P-V* diagram for Example 4–1.

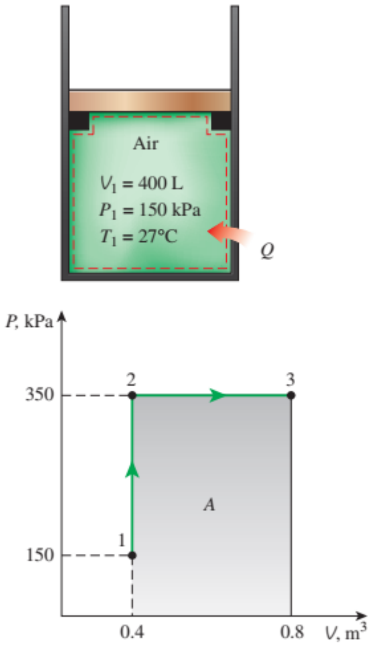


FIGURE 4–32

Schematic and *P-V* diagram for Example 4–10.

EXAMPLE 4–10 Heating of a Gas at Constant Pressure

A piston–cylinder device initially contains air at 150 kPa and 27°C. At this state, the piston is resting on a pair of stops, as shown in Fig. 4–32, and the enclosed volume is 400 L. The mass of the piston is such that a 350-kPa pressure is required to move it. The air is now heated until its volume has doubled. Determine (*a*) the final temperature, (*b*) the work done by the air, and (*c*) the total heat transferred to the air.

SOLUTION Air in a piston–cylinder device with a set of stops is heated until its volume is doubled. The final temperature, work done, and the total heat transfer are to be determined.

Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **2** The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. **3** The volume remains constant until the piston starts moving, and the pressure remains constant afterwards. **4** There are no electrical, shaft, or other forms of work involved.

Analysis We take the contents of the cylinder as the *system* (Fig. 4–32). This is a *closed system* since no mass crosses the system boundary during the process. We observe that a piston-cylinder device typically involves a moving boundary and thus boundary work, W_b. Also, the boundary work is done by the system, and heat is transferred to the system.

(*a*) The final temperature can be determined easily by using the ideal-gas relation between states 1 and 3 in the following form:

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \longrightarrow \frac{(150\text{ kPa})(V_1)}{300\text{ K}} = \frac{(350\text{ kPa})(2V_1)}{T_3}$$

$$T_3 = \mathbf{1400\ K}$$

(*b*) The work done could be determined by integration, but for this case it is much easier to find it from the area under the process curve on a *P-V* diagram, shown in Fig. 4–32:

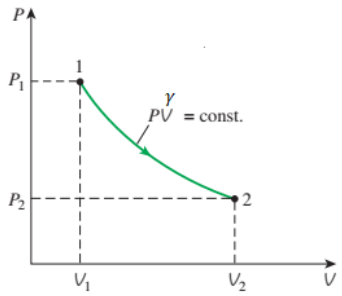
$$A = (V_2 - V_1)P_2 = (0.4\text{ m}^3)(350\text{ kPa}) = 140\text{ m}^3\cdot\text{kPa}$$

Therefore,

$$W_{13} = \mathbf{140\ kJ}$$

The work is done by the system (to raise the piston and to push the atmospheric air out of the way), and thus it is work output.

Q1 . A gas expand adiabatically from a pressure , Temperature and volume of 700 Kpa 320 ^{0}C and 0.015 m^3 respectively to pressure of 140 kpa. Determine the final volume , temperature and the work done by the gas, what is the change of internal energy in this case ? take C_p= 1.046 KJ/Kg.K , C_v= 0.752 KJ/Kg.K



Solution

$$\gamma = \frac{C_p}{C_v} = \frac{1.046}{0.752} = 1.39$$

$$\frac{P_2}{P_1} = (\frac{V_1}{V_2})^{\gamma}$$

$$\frac{V_1}{V_2} = (\frac{P_2}{P_1})^{\frac{1}{\gamma}} \qquad V_2 = V_1(\frac{P_1}{P_2})^{\frac{1}{\gamma}} \qquad V_2 = 0.015(\frac{700}{140})^{\frac{1}{1.39}} = 0.048 \text{ m}^3$$

$$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{\gamma-1} \qquad T_2 = T_1(\frac{V_1}{V_2})^{\gamma-1} \qquad T_2 = 320(\frac{0.015}{0.048})^{1.39-1} = 203.3\ ^0\text{C}$$

$$W = \frac{P_1V_1 - P_2V_2}{\gamma-1} = \frac{700\times10^3\times0.015 - 140\times10^3\times0.048}{1.39-1} = 9.69\ KJ$$

Q-W= ΔU

0- 9.69= ΔU

ΔU= -9.69 KJ/Kg

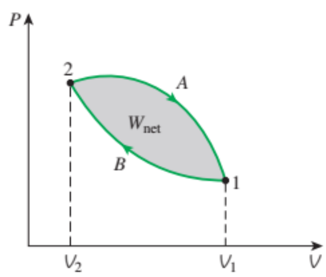
Q2 Quantity of gas occupies a volume of 0.3 m^3 at pressure of 100 Kpa and temperature of 20 ^{0}C. The gas is compressed isothermal to pressure of 500 Kpa and then expanded a adiabatically to initial volume. determine for this quantity of gas a) the heat received or rejected (state which) during the compression b) the change of internal energy during the expansion c) the mas of gas take C_p= 1.046 KJ/Kg.K , C_v= 0.752 KJ/Kg.K

Solution

P_1= 100 Kpa , T_1= 20 ^{0}C, V_1=0.3 m^3

P_2= 500 Kpa

$P_1 V_1$=mR T_1



$$m = \frac{P_1V_1}{\text{R T1}} = \frac{100\times0.3}{0.287\times(20+273)} = 0.3567\ Kg$$

1-2 process isothermal T_1= T_2

$$P_1V_1 = P_2V_2$$

$$V_2 = \frac{P_1V_1}{P_2} = \frac{100\times10^3\times0.3}{500\times10^3} = 0.1\ m^3$$

Q=W= $P_1V_1\ln\frac{P_1}{P_2} = 100 \times\ 0.3\ln\frac{100}{500} =$

ΔU=0

2-1 process adiabatically

$\gamma = \frac{C_p}{C_v} = \frac{1.046}{0.752} = 1.39$

W=$\frac{P_2V_2 - P_1V_1}{\gamma - 1} = \frac{500\times10^3\times0.1 - 100\times10^3\times0.3}{1.39-1} = 51.28\ K\ J$

Q=0

Q-W= ΔU

W= ΔU

Q3, 0.05 m³ of a gas at 6.9 bar expands reversibly in a cylinder behind a piston according to the law $PV^{1.2}$ =constant until volume is 0.08 m³. Calculate the work done by the gas and sketch the process on p-v diagram.

SOLUTION

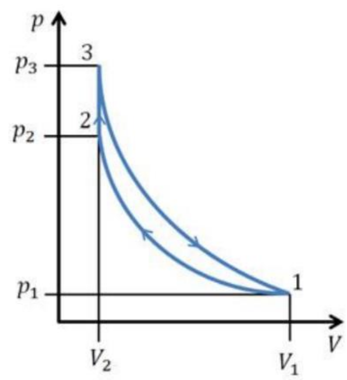
$\frac{P_2}{P_1} = (\frac{V_1}{V_2})^n$

$P_2 = P_1(\frac{V_1}{V_2})^n = 6.9\ (\frac{0.05}{0,08})^{1.2} = 3.9\ bar$

W=$\frac{P_1V_1 - P_2V_2}{n-1} = \frac{6.9\times10^5\times0.05 - 3.9\ \times10^5\times0.08}{1.3-1} = 15300\ J$

Q4/ A fluid at 0.7 bar occupying 0.09m³ is compressed reversibly to a pressure of 3.5 bar according to law $pv^{1.3}$ = constant. The fluid is then heated reversibly at constant volume until the pressure is 4 bar; the specific volume is then 0.5 m³ / kg. A reversible expansion according to a law pv^2 = constant restores the fluid to its initial state. Sketch the cycle to a p-v diagram and calculate: iii) the network of the cycle

SOLUTION



1) For 1-2

$\frac{P_2}{P_1} = (\frac{V_1}{V_2})^n$

$\frac{V_1}{V_2} = (\frac{P_2}{P_1})^{\frac{1}{n}}$ $V_2 = V_1(\frac{P_1}{P_2})^{\frac{1}{n}}$ $V_2 = 0.09\ (\frac{0.7}{3.5})^{\frac{1}{1.3}} = 0.584$ m3

$$W=\frac{P_1V_1-P_2V_2}{n-1}=\frac{0.7\times10^5\times0.09-3.5\times10^5\times0.584}{1.3-1}=-660\ KJ/Kg$$

2) for 2-3 W =0 constant volume

3) for 3-1 polytrophic process

$V_2=V_3$ constant volume

$P_3=4\ bar$

$$W=\frac{P_3V_3-P_1V_1}{n-1}=\frac{4\times10^5\times0.584-0.7\times10^5\times0.09}{2-1}=227.3\ KJ/Kg$$

Q5/ In the compression stroke of the gas engine the work done on the gas by the piston is 70 kJ/kg and the heat rejected to the cooling water is 42 kJ/kg calculate the change of specific internal energy stating whether it is gain or loss.

Solution:

Heat given to the cooling water Q = – 42 kJ/kg , Work input W = – 70 kJ/kg

Q-W= ΔU

$Q=(U2-U1)+W$

$-42=(U2-U1)-70$

So, the change of specific internal energy $\Delta U=-42+70=28\ kJ/kg$ Hence gain in internal energy = $28\ kJ/kg$ Answer. $28\ kJ/kg$ (gain).

Q6/ A mass of gas at an initial pressure of 28 bar and with an internal energy of 1500 kJ, is contained in a well-insulated cylinder of volume 0.06 m^3. The gas is allowed to expand behind a piston until its internal energy is 1400 kJ; the law of expansion is PV^3=Constant. Calculate 1. Work done; 2. The final volume; 3. The final pressure; 4. Draw P-V diagram.

Solution:

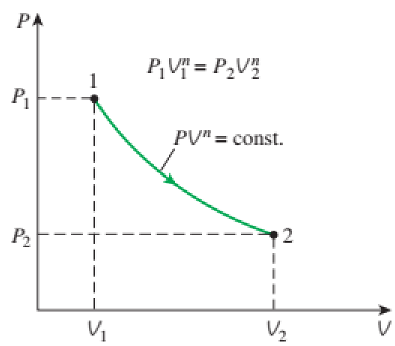
1) As there is no heat transfer in the system:

$A=\Delta U=1400-1500=-100kJ$

Q=0 due to insulated cylinder

Q-W= ΔU

W=- ΔU= $100\ kJ$



Q7/ A certain fluid at 10 bar is contained in a cylinder behind a piston, the initial volume being 0.05 m^3. Calculate the work done by the fluid when it expands reversibly:

i. at constant pressure to a final volume of 0.2 m^3

iv. according to a law pV^3 = constant to a final volume 0f 0.06 m^3

Sol

1.P=C

$W=P(V_2 - V_1) = 10 \times 10^5 (0.2 - 0.05) = 150\ KJ$

2.

$\frac{P_2}{P_1} = (\frac{V_1}{V_2})^n$

$P_2 = P_1(\frac{V_1}{V_2})^n = 10\ (\frac{0.05}{0{,}06})^3 = 5.7\ bar$

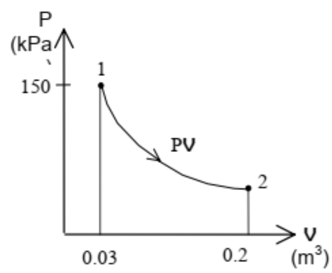
$W=\frac{P_1V_1 - P_2V_2}{n-1} = \frac{10\times10^5\times0.05-5{,}7\ \times10^5\times0.06}{3-1} = 7.9\ KJ$

Q8, During some actual expansion and compression processes in piston–cylinder devices, the gases have been observed to satisfy the relationship $PV^n = C$, where n and C are constants. Calculate the work done when a gas expands from 150 kPa and 0.03 m^3 to a final volume of 0.2 m^3 for the case of n = 1.3.

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^n = (150\ \text{kPa})\left(\frac{0.03\ \text{m}^3}{0.2\ \text{m}^3}\right)^{1.3} = 12.74\ \text{kPa}$$

and,

$$W_{b,\text{out}} = \int_1^2 P\,dV = \frac{P_2V_2 - P_1V_1}{1-n}$$
$$= \frac{(12.74\times0.2-150\times0.03)\ \text{kPa}\cdot\text{m}^3}{1-1.3}\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right)$$
$$= \mathbf{6.51\ kJ}$$



Q9/ frictionless piston–cylinder device contains 5 kg of nitrogen at 100 kPa and 250 K. Nitrogen is now compressed slowly according to the relation $PV^{1.4}$ = constant until it reaches a final temperature of 360 K. Calculate the work input during this process. Answer: *408 kJ*

Q10/ A gas is compressed hyperbolically $PV^{1.5} = C$ from a pressure and volume of 100 kN/m^2 and 0.056 m^3, respectively, to a volume of 0.007 m^3. Determine the final pressure and the work done on the gas.

Q11 Nitrogen at an initial state of 300 K, 150 kPa, and 0.2 m^3 is compressed slowly in an isothermal process to a final pressure of 800 kPa. Determine the work done during this process

Q12 A piston–cylinder device initially contains 0.07 m^3 of nitrogen gas at 130 kPa and 180 ^0C. The nitrogen is now expanded to a pressure of 80 kPa polytropically with a polytropic exponent whose value is equal to the specific heat ratio (called isentropic expansion). Determine the final temperature and the boundary work done during this process.

$$n = \frac{\ln\left(\frac{p_1}{p_2}\right)}{\ln\frac{V_2}{V_1}}$$

Q13/ one kg of a fluid expands reversibly according to a linear law from 4.2 bar to 1.4 bar; the initial] and final volumes are 0.004 m) and 0.02 m). The fluid is then cooled reversibly at constant pressure, and finally compressed reversibly according to a law po = constant back to the initial conditions of 4.2 bar and 0.004 m '. Calculate the work done in each process and the network of the cycle. Sketch the cycle on a. p-v diagram

Q14/ 1Kg of a fluid is compressed reversibly according to a law pv=0.25 where p is in bar v is in m^3/Kg. The final volume is 1/4 of the initial volume. Calculate the work done on the fluid and sketch the P-V diagram

Q15/ Air at 200 kPa, 30°C is contained in a cylinder/piston arrangement with initial volume .0.1 m^3. The inside pressure balances ambient pressure of 100 kPa plus an externally imposed force that is proportional to $P\ V^{0.5}$.=C Now heat is transferred to the system to a final pressure of 225 kPa. Find the final temperature and the work done in the process.