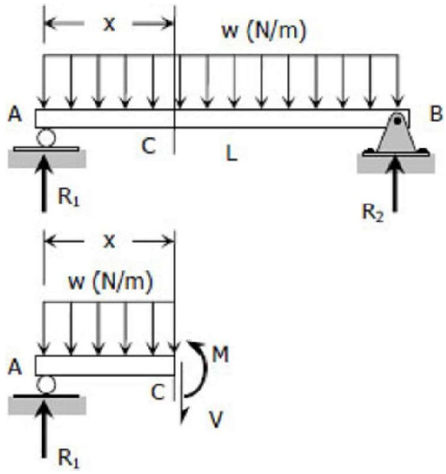
Consider a simple beam shown of length L that carries a uniform load of w (N/m) throughout its length and is held in equilibrium by reactions $R_1$ and $R_2$. Assume that the beam is cut at point C a distance of x from he left support and the portion of the beam to the right of C be removed. The portion removed must then be replaced by vertical shearing force V together with a couple M to hold the left portion of the bar in equilibrium under the action of $R_1$ and wx.




The couple M is called the resisting moment or moment and the force V is called the resisting shear or shear. The sign of V and M are taken to be positive if they have the senses indicated above.

**INSTRUCTION:**

Write shear and moment equations for the beams in the following problems. In each problem, let x be the distance measured from left end of the beam. Also, draw shear and moment diagrams, specifying values at all change of loading positions and at points of zero shear. Neglect the mass of the beam in each problem.

# Solution to Problem 403 | Shear and Moment Diagrams

## Problem 403

Beam loaded as shown in Fig. P-403. See the instruction.

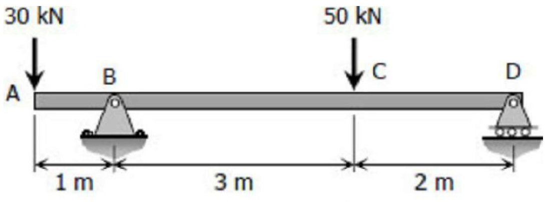


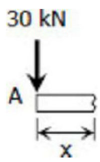

Figure P-403

# Solution 403

**From the load diagram:**

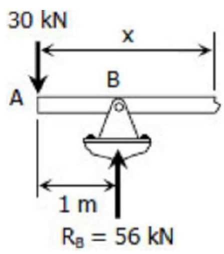
$\Sigma M_B = 0$
$5R_D + 1(30) = 3(50)$
$R_D = 24\ \text{kN}$

$\Sigma M_D = 0$
$5R_B = 2(50) + 6(30)$
$R_B = 56\ \text{kN}$




**Segment AB:**

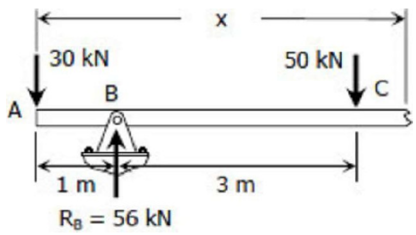
$V_{AB} = -30\ \text{kN}$
$M_{AB} = -30x\ \text{kN} \cdot \text{m}$




**Segment BC:**

$V_{BC} = -30 + 56$
$V_{BC} = 26\ \text{kN}$

$M_{BC} = -30x + 56(x-1)$
$M_{BC} = 26x - 56\ \text{kN} \cdot \text{m}$




**Segment CD:**

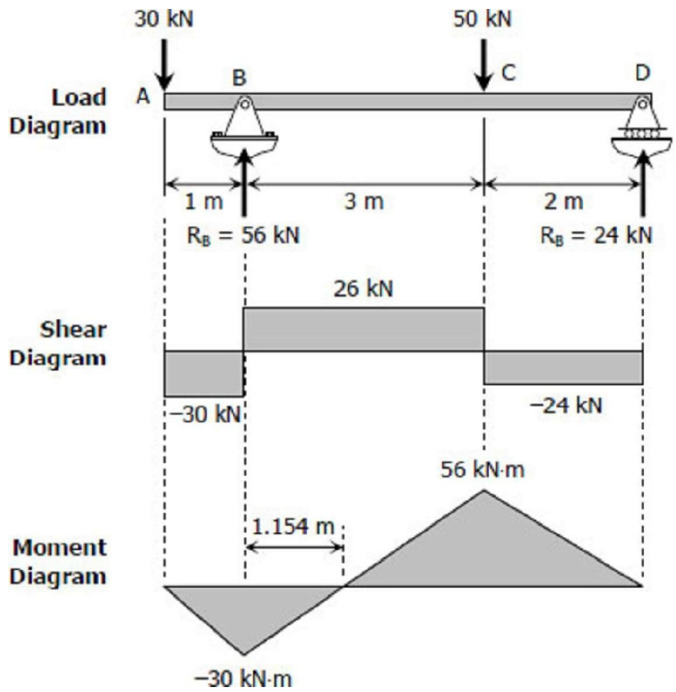
$V_{CD} = -30 + 56 - 50$
$V_{CD} = -24\ \text{kN}$
$M_{CD} = -30x + 56(x-1) - 50(x-4)$
$M_{CD} = -30x + 56x - 56 - 50x + 200$
$M_{CD} = -24x + 144\ \text{kN} \cdot \text{m}$

### To draw the Shear Diagram:

1. In segment AB, the shear is uniformly distributed over the segment at a magnitude of –30 kN.
2. In segment BC, the shear is uniformly distributed at a magnitude of 26 kN.
3. In segment CD, the shear is uniformly distributed at a magnitude of –24 kN.

### To draw the Moment Diagram:

1. The equation $M_{AB} = -30x$ is linear, at x = 0, $M_{AB} = 0$ and at x = 1 m, $M_{AB} = -30$ kN·m.
2. $M_{BC} = 26x - 56$ is also linear. At x = 1 m, $M_{BC} = -30$ kN·m; at x = 4 m, $M_{BC} = 48$ kN·m. When $M_{BC} = 0$, x = 2.154 m, thus the moment is zero at 1.154 m from B.
3. $M_{CD} = -24x + 144$ is again linear. At x = 4 m, $M_{CD} = 48$ kN·m; at x = 6 m, $M_{CD} = 0$.




# Solution to Problem 404 | Shear and Moment Diagrams

Beam loaded as shown in Fig. P-404. See the instruction.

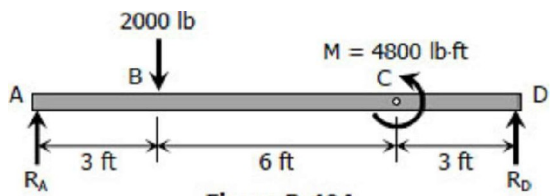
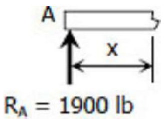



Figure P-404

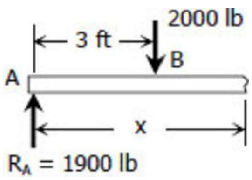
## Solution 404

$$\Sigma M_A = 0$$
$$12R_D + 4800 = 3(2000)$$
$$R_D = 100 \text{ lb}$$

$\Sigma M_D = 0$
$12R_A = 9(2000) + 4800$
$R_A = 1900\text{ lb}$

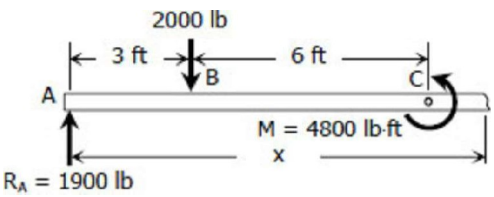



**Segment AB:**
$V_{AB} = 1900\text{ lb}$
$M_{AB} = 1900x\text{ lb}\cdot\text{ft}$




**Segment BC:**
$V_{BC} = 1900 – 2000$
$V_{BC} = –100\text{ lb}$

$M_{BC} = 1900x – 2000(x – 3)$
$M_{BC} = 1900x – 2000x + 6000$
$M_{BC} = –100x + 6000\text{ lb}\cdot\text{ft}$



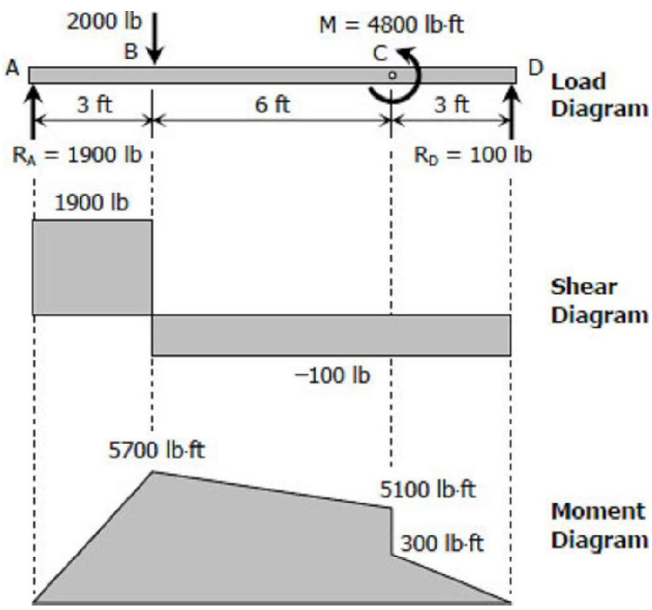

**Segment CD:**
$V_{CD} = 1900 – 2000$
$V_{CD} = –100\text{ lb}$

$M_{CD} = 1900x – 2000(x – 3) – 4800$
$M_{CD} = 1900x – 2000x + 6000 – 4800$
$M_{CD} = –100x + 1200\text{ lb}\cdot\text{ft}$

## To draw the Shear Diagram:

1. At segment AB, the shear is uniformly distributed at 1900 lb.
2. A shear of –100 lb is uniformly distributed over segments BC and CD.

## To draw the Moment Diagram:



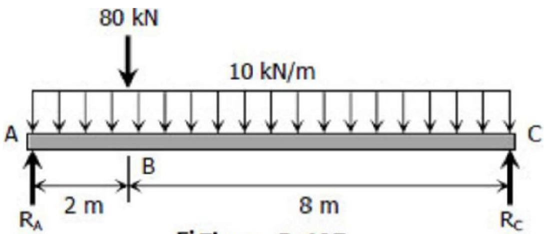

1. $M_{AB} = 1900x$ is linear; at x = 0,

$M_{AB} = 0$; at x = 3 ft, $M_{AB} = 5700$ lb·ft.

2. For segment BC, $M_{BC} = –100x + 6000$ is linear; at x = 3 ft, $M_{BC} = 5700$ lb·ft; at x = 9 ft, $M_{BC} = 5100$ lb·ft.
3. $M_{CD} = –100x + 1200$ is again linear; at x = 9 ft, $M_{CD} = 300$ lb·ft; at x = 12 ft, $M_{CD} = 0$.

# Solution to Problem 405 | Shear and Moment Diagrams

Beam loaded as shown in Fig. P-405. See the instruction.

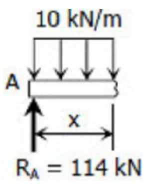


FiFigure P-405

## Solution 405

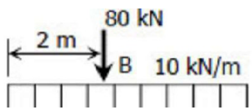
$$\Sigma M_A = 0$$
$$10R_C = 2(80) + 5[10(10)]$$
$$R_C = 66 \text{ kN}$$

$$\Sigma M_C = 0$$
$$10R_A = 8(80) + 5[10(10)]$$
$$R_A = 114 \text{ kN}$$



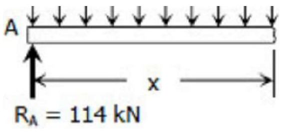

**Segment AB:**

$$V_{AB} = 114 – 10x \text{ kN}$$
$$M_{AB} = 114x – 10x(x/2)$$
$$M_{AB} = 114x – 5x^2 \text{ kN} \cdot \text{m}$$

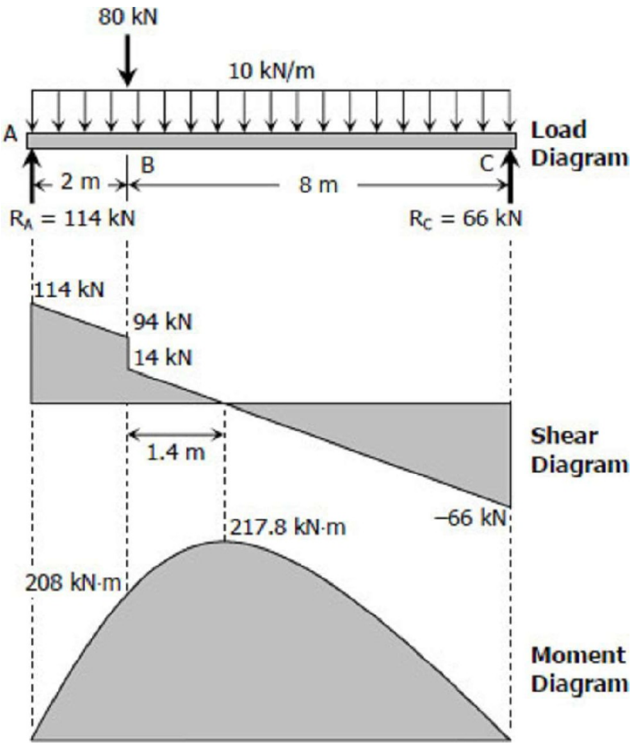



**Segment BC:**

$$V_{BC} = 114 – 80 – 10x$$




$$V_{BC} = 34 – 10x \text{ kN}$$
$$M_{BC} = 114x – 80(x – 2) – 10x(x/2)$$
$$M_{BC} = 160 + 34x – 5x^2 \text{ kN} \cdot \text{m}$$




**To draw the Shear Diagram:**

1. For segment AB, $V_{AB} = 114 – 10x$ is linear; at x = 0, $V_{AB}$ = 14 kN; at x = 2 m, $V_{AB}$ = 94 kN.
2. $V_{BC} = 34 – 10x$ for segment BC is linear; at x = 2 m, $V_{BC}$ = 14 kN; at x = 10 m, $V_{BC}$ = –66 kN. When $V_{BC}$ = 0, x = 3.4 m thus $V_{BC}$ = 0 at 1.4 m from B.
3.

**To draw the Moment Diagram:**

1. $M_{AB} = 114x – 5x^2$ is a second degree curve for segment AB; at x = 0, $M_{AB}$ = 0; at x = 2 m, $M_{AB}$ = 208 kN·m.
2. The moment diagram is also a second degree curve for segment BC given by $M_{BC} = 160 + 34x – 5x^2$; at x = 2 m, $M_{BC}$ = 208 kN·m; at x = 10 m, $M_{BC}$ = 0.
3. Note that the maximum moment occurs at point of zero shear. Thus, at x = 3.4 m, $M_{BC}$ = 217.8 kN·m.

# Solution to Problem 406 | Shear and Moment Diagrams

Beam loaded as shown in Fig. P-406. See the instruction.

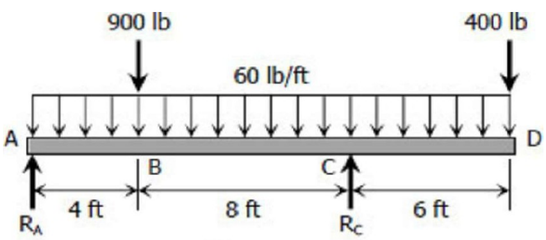


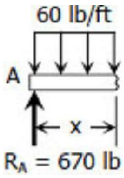

**Figure P-406**

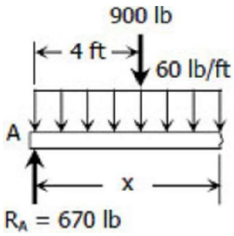
# Solution 406

$\Sigma M_A = 0$
$12R_C = 4(900) + 18(400) + 9[(60)(18)]$
$R_C = 1710\,\text{lb}$

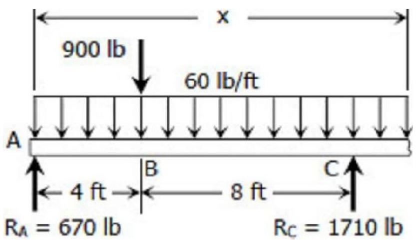
$\Sigma M_C = 0$
$12R_A + 6(400) = 8(900) + 3[60(18)]$
$R_A = 670\,\text{lb}$




**Segment AB:**
$V_{AB} = 670 – 60x\,\text{lb}$
$M_{AB} = 670x – 60x(x/2)$
$M_{AB} = 670x – 30x^2\ \text{lb}\cdot\text{ft}$



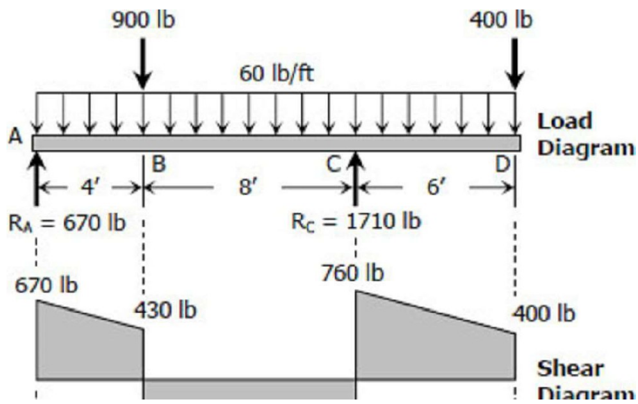

**Segment BC:**
$V_{BC} = 670 – 900 – 60x$
$V_{BC} = –230 – 60x\,\text{lb}$
$M_{BC} = 670x – 900(x–4) – 60x(x/2)$
$M_{BC} = 3600 – 230x – 30x^2\ \text{lb}\cdot\text{ft}$



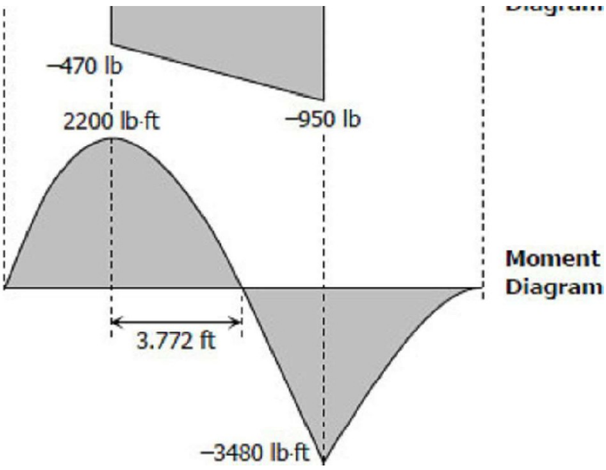

**Segment CD:**
$V_{CD} = 670 + 1710 – 900 – 60x$
$V_{CD} = 1480 – 60x\,\text{lb}$
$M_{CD} = 670x + 1710(x–12) – 900(x–4) – 60x(x/2)$
$M_{CD} = –16920 + 1480x – 30x^2\ \text{lb}\cdot\text{ft}$

**To draw the Shear Diagram:**

1. $V_{AB} = 670 – 60x$ for segment AB is linear; at x = 0, $V_{AB}$= 670 lb; at x = 4 ft, $V_{AB}$ = 430 lb.
2. For segment BC, $V_{BC} = –230 – 60x$ is also linear; at x= 4 ft, $V_{BC}$ = –470 lb, at x = 12 ft, $V_{BC}$ = –950 lb.
3. $V_{CD} = 1480 – 60x$ for segment CD is



again linear; at x = 12, $V_{CD}$ = 760 lb; at x = 18 ft, $V_{CD}$ = 400 lb.

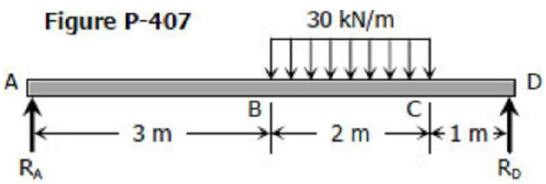



**To draw the Moment Diagram:**

1. $M_{AB} = 670x - 30x^2$ for segment AB is a second degree curve; at x = 0, $M_{AB}$ = 0; at x = 4 ft, $M_{AB}$ = 2200 lb·ft.
2. For BC, $M_{BC} = 3600 - 230x - 30x^2$, is a second degree curve; at x = 4 ft, $M_{BC}$ = 2200 lb·ft, at x = 12 ft, $M_{BC}$ = –3480 lb·ft; When $M_{BC}$ = 0, $3600 - 230x - 30x^2 = 0$, x = –15.439 ft and 7.772 ft. Take x = 7.772 ft, thus, the moment is zero at 3.772 ft from B.
3. For segment CD, $M_{CD} = -16920 + 1480x - 30x^2$ is a second degree curve; at x = 12 ft, $M_{CD}$ = –3480 lb·ft; at x = 18 ft, $M_{CD}$ = 0.

# Solution to Problem 407 | Shear and Moment Diagrams

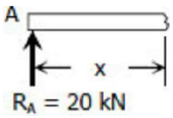
Beam loaded as shown in Fig. P-407. See the instruction.




## Solution 407

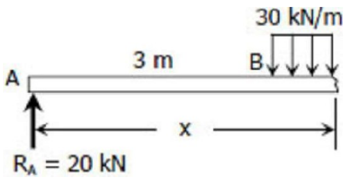
$\Sigma M_A = 0$
$6R_D = 4[2(30)]$
$R_D = 40\text{ kN}$

$\Sigma M_D = 0$
$6R_A = 2[2(30)]$
$R_A = 20\text{kN}$



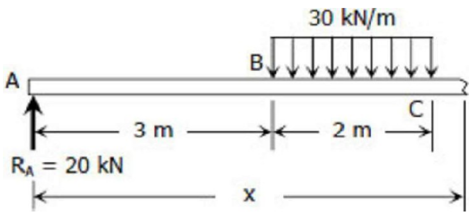

**Segment AB:**

$$V_{AB} = 20\,\text{kN}$$
$$M_{AB} = 20x\,\text{kN}\cdot\text{m}$$




**Segment BC:**

$$V_{BC} = 20 – 30(x – 3)$$
$$V_{BC} = 110 – 30x\,\text{kN}$$
$$M_{BC} = 20x – 30(x – 3)(x – 3)/2$$
$$M_{BC} = 20x – 15(x – 3)^2\,\text{kN}\cdot\text{m}$$

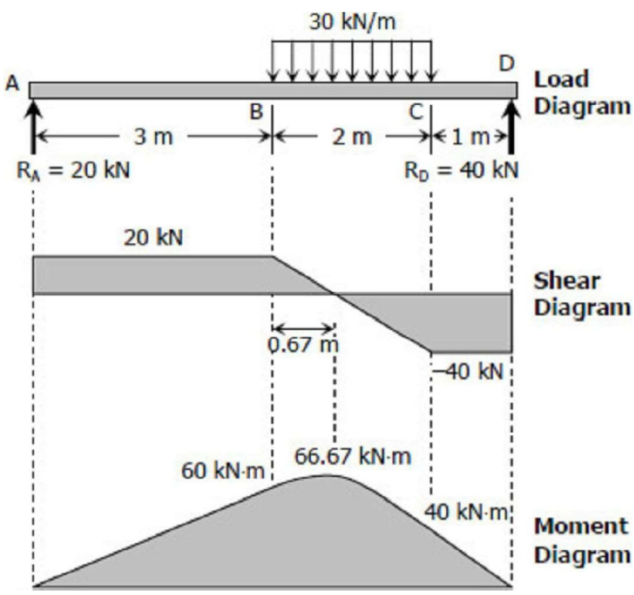



**Segment CD:**

$$V_{CD} = 20 – 30(2)$$
$$V_{CD} = –40\,\text{kN}$$
$$M_{CD} = 20x – 30(2)(x – 4)$$
$$M_{CD} = 20x – 60(x – 4)\,\text{kN}\cdot\text{m}$$

## To draw the Shear Diagram:

1. For segment AB, the shear is uniformly distributed at 20 kN.
2. $V_{BC}$ = 110 – 30x for segment BC; at x = 3 m, $V_{BC}$ = 20 kN; at x = 5 m, $V_{BC}$ = –40 kN. For $V_{BC}$ = 0, x = 3.67 m or 0.67 m from B.
3. The shear for segment CD is uniformly distributed at –40 kN.




## To draw the Moment Diagram:

1. For AB, $M_{AB}$ = 20x; at x = 0, $M_{AB}$ = 0; at x = 3 m, $M_{AB}$ = 60 kN·m.
2. $M_{BC} = 20x – 15(x – 3)^2$ for segment BC is second degree curve; at x = 3 m, $M_{BC}$ = 60 kN·m; at x = 5 m, $M_{BC}$ = 40 kN·m. **Note:** that maximum moment occurred at zero shear; at x = 3.67 m, $M_{BC}$ = 66.67 kN·m.
3. $M_{CD}$ = 20x – 60(x – 4) for segment BC is linear; at x = 5 m, $M_{CD}$ = 40 kN·m; at x = 6 m, $M_{CD}$ = 0.

# Solution to Problem 408 | Shear and Moment Diagrams

Beam loaded as shown in Fig. P-408. See the instruction.

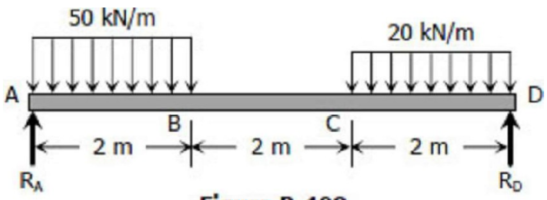
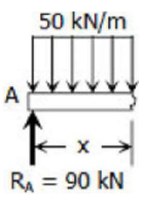


**Figure P-408**

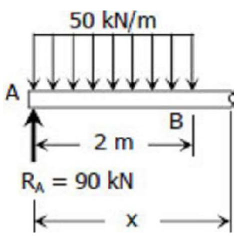
## Solution 408

$\Sigma M_A = 0$
$6R_D = 1[2(50)] + 5[2(20)]$
$R_D = 50 \text{ kN}$

$\Sigma M_D = 0$
$6R_A = 5[2(50)] + 1[2(20)]$
$R_A = 90 \text{ kN}$

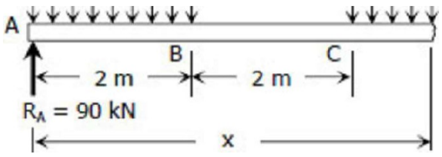



**Segment AB:**
$V_{AB} = 90 – 50x \text{ kN}$
$M_{AB} = 90x – 50x(x/2)$
$M_{AB} = 90x – 25x^2 \text{ kN} \cdot \text{m}$



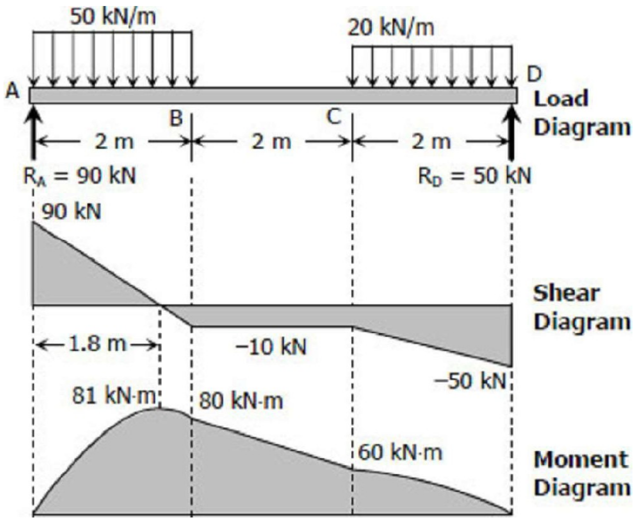

**Segment BC:**
$V_{BC} = 90 – 50(2)$
$V_{BC} = –10 \text{ kN}$
$M_{BC} = 90x – 2(50)(x – 1)$
$M_{BC} = –10x + 100 \text{ kN} \cdot \text{m}$


**Segment CD:**




$$V_{CD} = 90–2(50)–20(x–4)$$
$$V_{CD} = –20x + 70 \text{ kN}$$
$$M_{CD} = 90x–2(50)(x–1)–20(x–4)(x–4)/2$$
$$M_{CD} = 90x–100(x–1)–10(x–4)^2$$

$$M_{CD} = –10x^2 + 70x–60 \text{ kN} \cdot \text{m}$$




## To draw the Shear Diagram:

1. $V_{AB} = 90 – 50x$ is linear; at x = 0, $V_{BC}$ = 90 kN; at x = 2 m, $V_{BC}$ = –10 kN. When $V_{AB}$ = 0, x = 1.8 m.
2. $V_{BC}$ = –10 kN along segment BC.
3. $V_{CD} = –20x + 70$ is linear; at x = 4 m, $V_{CD}$ = –10 kN; at x = 6 m, $V_{CD}$ = –50 kN.

## To draw the Moment Diagram:

1. $M_{AB} = 90x – 25x^2$ is second degree; at x = 0, $M_{AB}$ = 0; at x = 1.8 m, $M_{AB}$ = 81 kN·m; at x = 2 m, MAB = 80 kN·m.
2. $M_{BC} = –10x + 100$ is linear; at x = 2 m, $M_{BC}$ = 80 kN·m; at x = 4 m, $M_{BC}$ = 60 kN·m.
3. $M_{CD} = –10x^2 + 70x – 60$; at x = 4 m, $M_{CD}$ = 60 kN·m; at x = 6 m, $M_{CD}$ = 0.

# Solution to Problem 409 | Shear and Moment Diagrams

Cantilever beam loaded as shown in Fig. P-409. See the instruction.

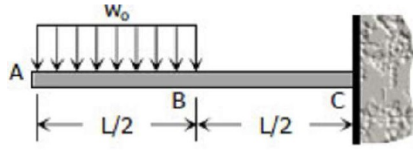
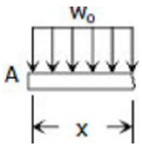


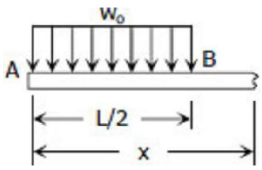

Figure P-409

## Solution 409



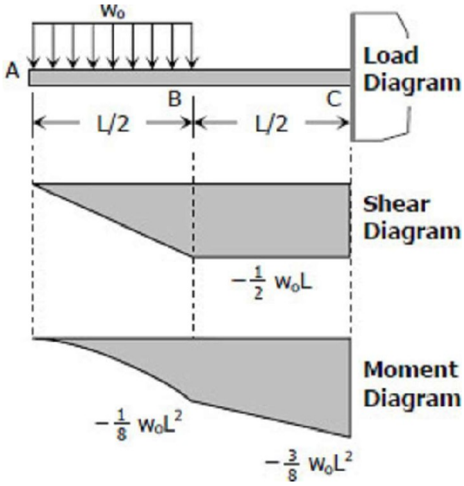

**Segment AB:**

$$V_{AB} = -w_o x$$
$$M_{AB} = -w_o x(x/2)$$
$$MAB = -\frac{1}{2}w_o x^2$$




**Segment BC:**

$$V_{BC} = -w_o(L/2)$$
$$V_{BC} = -\frac{1}{2}w_o L$$
$$M_{BC} = -w_o(L/2)(x-L/4)$$
$$M_{BC} = -\frac{1}{2}w_o Lx + \frac{1}{8}w_o L^2$$




## To draw the Shear Diagram:

1. $V_{AB} = -w_o x$ for segment AB is linear; at x = 0, $V_{AB} = 0$; at x = L/2, $V_{AB} = -½w_oL$.
2. At BC, the shear is uniformly distributed by $-½w_oL$.

## To draw the Moment Diagram:

1. $M_{AB} = -½w_o x^2$ is a second degree curve; at x = 0, $M_{AB} = 0$; at x = L/2, $M_{AB} = -1/8\ w_oL^2$.
2. $M_{BC} = -½w_oLx + 1/8\ w_oL^2$ is a second degree; at x = L/2, $M_{BC} = -1/8\ w_oL^2$; at x = L, $M_{BC} = -3/8\ w_oL^2$.

# Solution to Problem 410 | Shear and Moment Diagrams

Cantilever beam carrying the uniformly varying load shown in Fig. P-410. See the instruction.

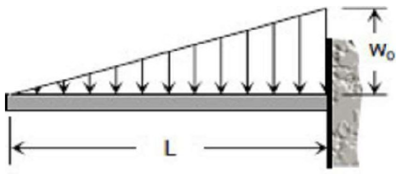
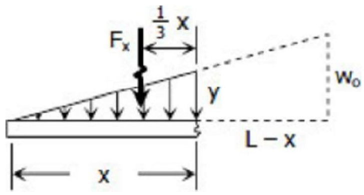



Figure P-410

# Solution 410

$$\frac{y}{x} = \frac{w_o}{L}$$
$$y = \frac{w_o}{L}\,x$$




$$F_x = \tfrac{1}{2}xy$$
$$F_x = \frac{1}{2}\,x\left(\frac{w_o}{L}\,x\right)$$
$$F_x = \frac{w_o}{2L}\,x^2$$

**Shear equation:**

$$V = -\frac{w_o}{2L}\,x^2$$

**Moment equation:**

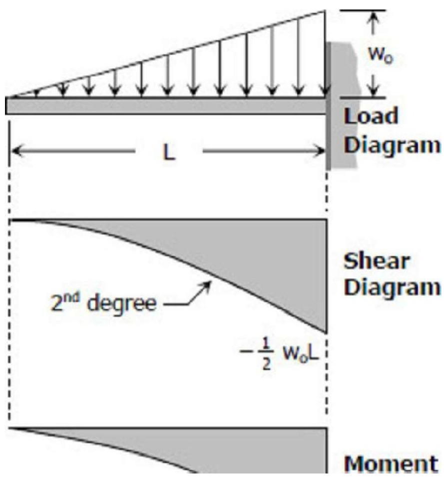
$$M = -\tfrac{1}{3}xF_x = -\frac{1}{3}\,x\left(\frac{w_o}{2L}\,x^2\right)$$
$$M = -\frac{w_o}{6L}\,x^3$$

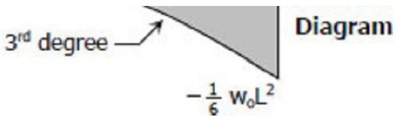
## To draw the Shear Diagram:

1. $V = -w_o x^2 / 2L$ is a second degree curve; at $x = 0$, $V = 0$; at $x = L$, $V = -\frac{1}{2} w_o L$.

## To draw the Moment Diagram:

1. $M = -w_o x^3 / 6L$ is a third degree curve; at $x = 0$, $M = 0$; at $x = L$, $M = -1/6\ w_o L^2$.






# Solution to Problem 411 | Shear and Moment Diagrams

Cantilever beam carrying a distributed load with intensity varying from wo at the free end to zero at the wall, as shown in Fig. P-411. See the instruction.

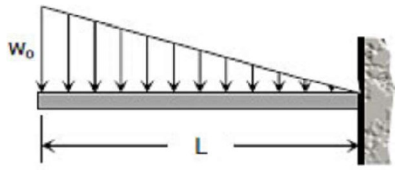
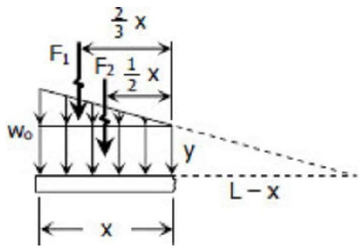



Figure P-411

## Solution 411

$$\frac{y}{L-x} = \frac{w_o}{L}$$
$$y = \frac{w_o}{L}(L-x)$$




$$F_1 \tfrac{1}{2}x(w_o - y)$$
$$F_1 = \frac{1}{2}x\left[w_o - \frac{w_o}{L}(L-x)\right]$$
$$F_1 = \frac{1}{2}x\left[w_o - w_oL - \frac{w_o}{L}x\right]$$
$$F_1 = \frac{w_o}{2L}x^2$$

$$F_2 = xy = x\left[\frac{w_o}{L}(L-x)\right]$$
$$F_2 = \frac{w_o}{L}(Lx - x^2)$$

**Shear equation:**

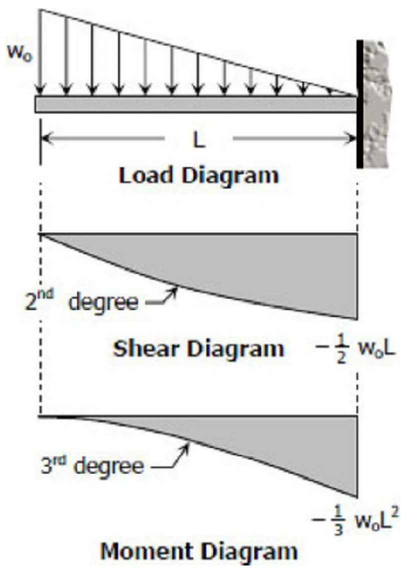
$$V = -F_1 - F_2 = -\frac{w_o}{2L}x^2 - \frac{w_o}{L}(Lx - x^2)$$
$$V = -\frac{w_o}{2L}x^2 - w_o x + \frac{w_o}{L}x^2$$
$$V = \frac{w_o}{2L}x^2 - w_o x$$

**Moment equation:**

$$M = -\tfrac{2}{3}xF_1 - \tfrac{1}{2}xF_2$$
$$M = -\frac{2}{3}x\left(\frac{w_o}{2L}x^2\right) - \tfrac{1}{2}x\left[\frac{w_o}{L}(Lx - x^2)\right]$$
$$M = -\frac{w_o}{3L}x^3 - \frac{w_o}{2}x^2 + \frac{w_o}{2L}x^3$$
$$M = -\frac{w_o}{2}x^2 + \frac{w_o}{6L}x^3$$

## To draw the Shear Diagram:

1. $V = w_o x^2/2L - w_o x$ is a concave upward second degree curve; at x = 0, V = 0; at x = L, V = –1/2 $w_o L$.




## To draw the Moment diagram:

1. $M = -w_o x^2/2 + w_o x^3/6L$ is in third degree; at x = 0, M = 0; at x = L, M = –1/3 $w_o L^2$.

# Solution to Problem 412 | Shear and Moment Diagrams

Beam loaded as shown in [Fig. P-412](). See the [instruction]().