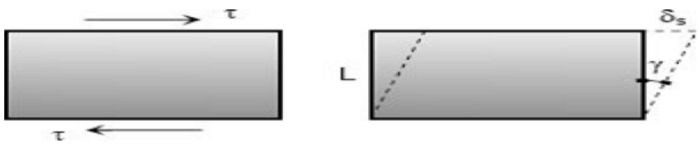
Shearing Deformation

Shearing Deformation

Shearing forces cause shearing deformation. An element subject to shear does not change in length but undergoes a change in shape.



The change in angle at the corner of an original rectangular element is called the **shear strain** and is expressed as

$$\gamma = \frac{\delta_s}{L}$$

The ratio of the shear stress τ and the shear strain γ is called the *modulus of elasticity in shear* or **modulus of rigidity** and is denoted as G, in MPa.

$$G = \frac{\tau}{\gamma}$$

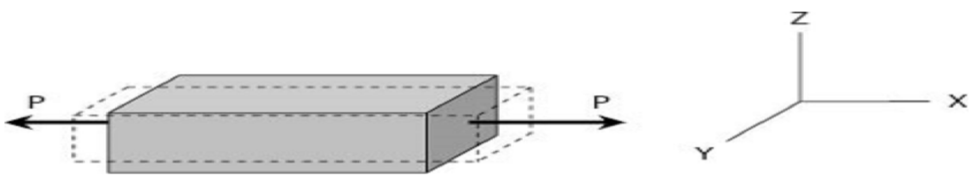
The relationship between the shearing deformation and the applied shearing force is

$$\delta_s = \frac{VL}{A_s G} = \frac{\tau L}{G}$$

where V is the shearing force acting over an area A_s.

Poisson's Ratio

When a bar is subjected to a tensile loading there is an increase in length of the bar in the direction of the applied load, but there is also a decrease in a lateral dimension perpendicular to the load. The ratio of the sidewise deformation (or strain) to the longitudinal deformation (or strain) is called the Poisson's ratio and is denoted by ν. For most steel, it lies in the range of 0.25 to 0.3, and 0.20 for concrete.



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

where ε_x is strain in the x-direction and ε_y and ε_z are the strains in the perpendicular direction. The negative sign indicates a decrease in the transverse dimension when ε_x is positive.

Biaxial Deformation

If an element is subjected simultaneously by ensile stresses, σ_x and σ_y, in the x and y directions, the strain in the x direction is σ_x / E and the strain in the y direction is σ_y / E. Simultaneously, the stress in the y direction will produce a lateral contraction on the x direction of the amount $-\nu\ \varepsilon_y$ or $-\nu\ \sigma_y / E$. The resulting strain in the x direction will be

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\,\frac{\sigma_y}{E} \quad \text{or} \quad \sigma_x = \frac{(\varepsilon_x + \nu\varepsilon_y)E}{1 - \nu^2}$$

and

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\,\frac{\sigma_x}{E} \quad \text{or} \quad \sigma_y = \frac{(\varepsilon_y + \nu\varepsilon_x)E}{1 - \nu^2}$$

Triaxial Deformation

If an element is subjected simultaneously by three mutually perpendicular normal stresses σ_x, σ_y, and σ_z, which are accompanied by strains ε_x, ε_y, and ε_z, respectively,

$$\varepsilon_x = \frac{1}{E}\,[\,\sigma_x - \nu(\sigma_y + \sigma_z)\,]$$

$$\varepsilon_y = \frac{1}{E}\,[\,\sigma_y - \nu(\sigma_x + \sigma_z)\,]$$

$$\varepsilon_z = \frac{1}{E}\,[\,\sigma_z - \nu(\sigma_x + \sigma_y)\,]$$

Tensile stresses and elongation are taken as positive. Compressive stresses and contraction are taken as negative.

Relationship Between E, G, and ν

The relationship between modulus of elasticity E, shear modulus G and Poisson's ratio ν is:

$$G = \frac{E}{2(1 + \nu)}$$

Bulk Modulus of Elasticity or Modulus of Volume Expansion, K

The bulk modulus of elasticity K is a measure of a resistance of a material to change in volume without change in shape or form. It is given as

$$K = \frac{E}{3(1 - 2\nu)} = \frac{\sigma}{\Delta V/V}$$

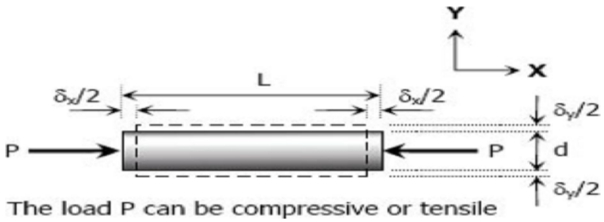
where V is the volume and ΔV is change in volume. The ratio $\Delta V/V$ is called volumetric strain and can be expressed as

$$\frac{\Delta V}{V} = \frac{\sigma}{K} = \frac{3(1 - 2\nu)}{E}$$

Solution to Problem 222 Poisson's Ratio

A solid cylinder of diameter d carries an axial load P. Show that its change in diameter is $4P\nu / \pi E d$.

Solution 222



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\nu\,\varepsilon_x$$

$$\varepsilon_y = -\nu\,\frac{\sigma_x}{E}$$

$$\frac{\delta_y}{d} = -\nu\,\frac{-P}{AE}$$

$$\delta_y = \frac{Pd}{\frac{1}{4}\pi d^2\,E}$$

$$\delta_y = \frac{4P\nu}{\pi E d} \longrightarrow \textbf{\textit{ok}}$$

Solution to Problem 223 Triaxial Deformation

Strength of Materials 4th Edition by Pytel and Singer

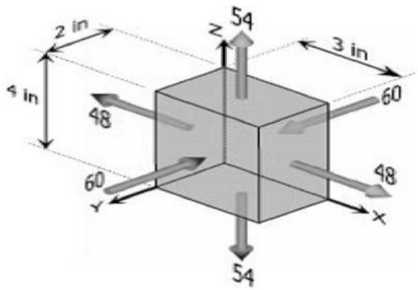
Given:
Dimensions of the block:
- x direction = 3 inches
- y direction = 2 inches
- z direction = 4 inches

Triaxial loads in the block
- x direction = 48 kips tension
- y direction = 60 kips compression
- z direction = 54 kips tension

Poisson's ratio, $\nu = 0.30$
Modulus of elasticity, $E = 29 \times 10^6$ psi

Required:
Single uniformly distributed load in the x direction that would produce the same deformation in the y direction as the original loading.

Solution 223



For triaxial deformation (tensile triaxial stresses):
(compressive stresses are negative stresses)

$$\varepsilon_y = \frac{1}{E}\left[\,\sigma_y - \nu(\sigma_x + \sigma_z)\,\right]$$

$$\sigma_x = \frac{P_x}{A_{yz}} = \frac{48}{4(2)} = 6.0\ \text{ksi}\ \ \text{(tension)}$$

$$\sigma_y = \frac{P_y}{A_{xz}} = \frac{60}{4(3)} = 5.0\ \text{ksi}\ \ \text{(compression)}$$

$$\sigma_z = \frac{P_z}{A_{xy}} = \frac{54}{2(3)} = 9.0\ \text{ksi}\ \ \text{(tension)}$$

$$\varepsilon_y = \frac{1}{29\times10^6}\left[\,-5000 - 0.30(6000 + 9000)\,\right]$$

$$\varepsilon_y = -3.276\times10^{-4}$$

ε_y is negative, thus tensile force is required in the x-direction to produce the same deformation in the y-direction as the original forces.

For equivalent single force in the x-direction:
(uniaxial stress)

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$-\nu\varepsilon_x = \varepsilon_y$$

$$-\nu\,\frac{\sigma_x}{E} = \varepsilon_y$$

$$-0.30\left(\frac{\sigma_x}{29\times10^6}\right) = -3.276\times10^{-4}$$

$$\sigma_x = 31\,666.67\ \text{psi}$$

$$\sigma_x = \frac{P_x}{4(2)} = 31\,666.67$$

$$P_x = 253\,333.33\ \text{lb}\ \ \text{(tension)}$$

$$P_x = 253.33\ \text{kips}\ \ \text{(tension)} \longrightarrow \textit{answer}$$

Problem 224

For the block loaded triaxially as described in Prob. 223, find the uniformly distributed load that must be added in the x direction to produce no deformation in the z direction.

Solution 224

$$\varepsilon_z = \frac{1}{E}\left[\ \sigma_z - \nu(\sigma_x + \sigma_y)\ \right]$$

Where
σ_x = 6.0 ksi (tension)
σ_y = 5.0 ksi (compression)
σ_z = 9.0 ksi (tension)

$$\varepsilon_z = \frac{1}{29 \times 10^6}\left[\ 9000 - 0.3(6000 - 5000)\ \right]$$
$$\varepsilon_z = 2.07 \times 10^{-5}$$
ε_z is positive, thus positive stress is needed in the x-direction to eliminate deformation in z-direction.

The application of loads is still simultaneous:
(*No deformation means zero strain*)

$$\varepsilon_z = \frac{1}{E}\left[\ \sigma_z - \nu(\sigma_x + \sigma_y)\ \right] = 0$$
$$\sigma_z = \nu(\sigma_x + \sigma_y)$$
σ_y = 5.0 ksi (compression)
σ_z = 9.0 ksi (tension)
$$9000 = 0.30(\sigma_x - 5000)$$
$$\sigma_x = 35\,000 \text{ psi}$$

$$\sigma_{added} + 6000 = 35\,000$$
$$\sigma_{added} = 29\,000 \text{ psi}$$
$$\frac{P_{added}}{2(4)} = 29\,000$$
$$P_{added} = 232\,000 \text{ lb}$$
$$P_{added} = 232 \text{ kips} \longrightarrow \textbf{\textit{answer}}$$

Solution to Problem 225 Biaxial Deformation

A welded steel cylindrical drum made of a 10-mm plate has an internal diameter of 1.20 m. Compute the change in diameter that would be caused by an internal pressure of 1.5 MPa. Assume that Poisson's ratio is 0.30 and E = 200 GPa.

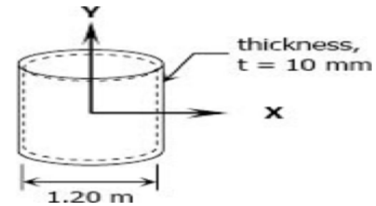
Solution 225

σ_y = longitudinal stress

$$\sigma_y = \frac{pD}{4t} = \frac{1.5(1200)}{4(10)}$$
$$\sigma_y = 45\ \text{MPa}$$

σ_x = tangential stress

$$\sigma_y = \frac{pD}{2t} = \frac{1.5(1200)}{2(10)}$$
$$\sigma_y = 90\ \text{MPa}$$

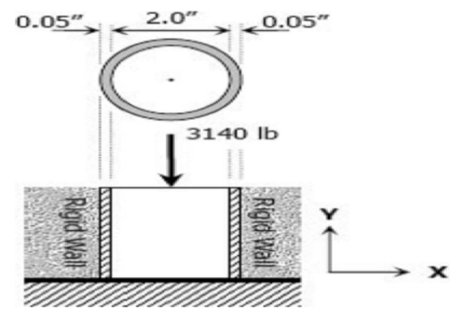


$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\,\frac{\sigma_y}{E}$$
$$\varepsilon_x = \frac{90}{200\,000} - 0.3\left(\frac{45}{200\,000}\right)$$
$$\varepsilon_x = 3.825 \times 10^{-4}$$

$$\varepsilon_x = \frac{\Delta D}{D}$$
$$\Delta D = \varepsilon_x\, D == (3.825 \times 10^{-4})(1200)$$
$$\Delta D = 0.459\ \text{mm} \longrightarrow \textit{answer}$$

Solution to Problem 226 Biaxial Deformation

A 2-in.-diameter steel tube with a wall thickness of 0.05 inch just fits in a rigid hole. Find the tangential stress if an axial compressive load of 3140 lb is applied. Assume $\nu = 0.30$ and neglect the possibility of buckling.



Solution 226

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\,\frac{\sigma_y}{E} = 0$$

where
σ_x = tangential stress
σ_y = longitudinal stress
$\sigma_y = P_y / A = 3140 / (\pi \times 2 \times 0.05)$
$\sigma_y = 31{,}400/\pi$ psi

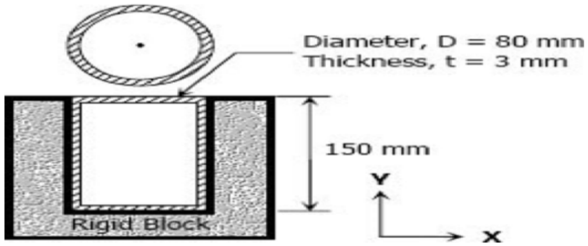
$$\sigma_x = 0.30(31400/\pi)$$
$$\sigma_x = 9430/\pi\ \text{psi}$$
$$\sigma_x = 2298.5\ \text{psi}$$

Solution to Problem 227 Biaxial Deformation

Problem 227

A 150-mm-long bronze tube, closed at its ends, is 80 mm in diameter and has a wall thickness of 3 mm. It fits without clearance in an 80-mm hole in a rigid block. The tube is then subjected to an internal pressure of 4.00 MPa. Assuming ν = 1/3 and E = 83 GPa, determine the tangential stress in the tube.

Solution 227



Longitudinal stress:

$$\sigma_y = \frac{pD}{4t} = \frac{4(80)}{4(3)}$$

$$\sigma_y = \frac{80}{3}\ \text{MPa}$$

The **strain in the x-direction** is:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\,\frac{\sigma_y}{E} = 0$$

$$\sigma_x = \nu\,\sigma_y = \text{tangential stress}$$

$$\sigma_x = \tfrac{1}{3}(\tfrac{80}{3})$$

$$\sigma_x = 8.89\ \text{MPa} \ \longrightarrow\ \textit{answer}$$

Solution to Problem 228 Biaxial Deformation

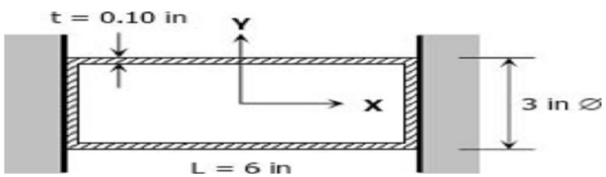
A 6-in.-long bronze tube, with closed ends, is 3 in. in diameter with a wall thickness of 0.10 in. With no internal pressure, the tube just fits between two rigid end walls. Calculate the longitudinal and tangential stresses for an internal pressure of 6000 psi. Assume ν = 1/3 and E = 12 × 10^6 psi.

Solution 228

$$\varepsilon = \frac{\sigma_x}{E} - \nu\,\frac{\sigma_y}{E} = 0$$

$$\sigma_x = \nu\,\sigma_y = \sigma_l \ \longrightarrow\ \text{longitudinal stress}$$

$$\sigma_t = \sigma_y \ \longrightarrow\ \text{tangential stress}$$



$$\sigma_t = \frac{pD}{2t} = \frac{6000(3)}{2(0.10)}$$

$$\sigma_t = 90\,000\ \text{psi} \ \longrightarrow\ \textit{answer}$$

$$\sigma_l = \nu\,\sigma_y = \tfrac{1}{3}(90\,000)$$

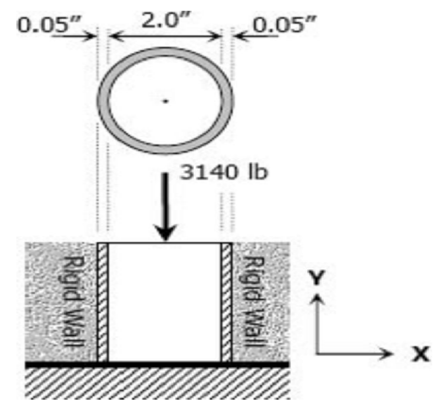
$$\sigma_l = 30\,000\ \text{psi} \ \longrightarrow\ \textit{answer}$$

Statically Indeterminate Members

When the reactive forces or the internal resisting forces over a cross section exceed the number of independent equations of equilibrium, the structure is called statically indeterminate. These cases require the use of additional relations that depend on the elastic deformations in the members.

Solution to Problem 226 Biaxial Deformation

A 2-in.-diameter steel tube with a wall thickness of 0.05 inch just fits in a rigid hole. Find the tangential stress if an axial compressive load of 3140 lb is applied. Assume $\nu = 0.30$ and neglect the possibility of buckling.



Solution 226

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} = 0$$

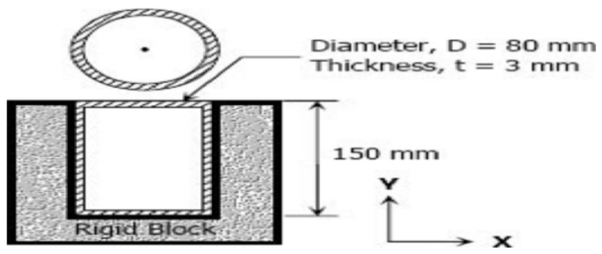
where
σ_x = tangential stress
σ_y = longitudinal stress
$\sigma_y = P_y / A = 3140 / (\pi \times 2 \times 0.05)$
$\sigma_y = 31{,}400/\pi$ psi

$$\sigma_x = 0.30(31400/\pi)$$
$$\sigma_x = 9430/\pi \text{ psi}$$
$$\sigma_x = 2298.5 \text{ psi}$$

Problem 227

A 150-mm-long bronze tube, closed at its ends, is 80 mm in diameter and has a wall thickness of 3 mm. It fits without clearance in an 80-mm hole in a rigid block. The tube is then subjected to an internal pressure of 4.00 MPa. Assuming ν = 1/3 and E = 83 GPa, determine the tangential stress in the tube.

Solution 227



Longitudinal stress:

$$\sigma_y = \frac{pD}{4t} = \frac{4(80)}{4(3)}$$

$$\sigma_y = \frac{80}{3}\ \text{MPa}$$

The **strain in the x-direction** is:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\,\frac{\sigma_y}{E} = 0$$

$$\sigma_x = \nu\,\sigma_y = \text{tangential stress}$$

$$\sigma_x = \tfrac{1}{3}(\tfrac{80}{3})$$

$$\sigma_x = 8.89\ \text{MPa} \quad \longrightarrow \quad \textbf{\textit{answer}}$$

Solution to Problem 228 Biaxial Deformation

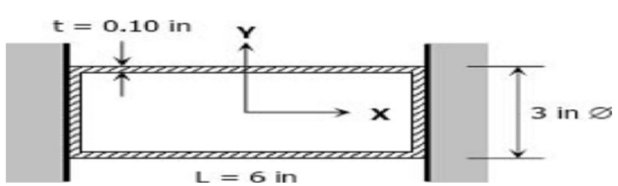
A 6-in.-long bronze tube, with closed ends, is 3 in. in diameter with a wall thickness of 0.10 in. With no internal pressure, the tube just fits between two rigid end walls. Calculate the longitudinal and tangential stresses for an internal pressure of 6000 psi. Assume ν = 1/3 and E = 12 × 10^6 psi.

Solution 228

$$\varepsilon = \frac{\sigma_x}{E} - \nu\,\frac{\sigma_y}{E} = 0$$

$$\sigma_x = \nu\,\sigma_y = \sigma_l \quad \longrightarrow \quad \text{longitudinal stress}$$

$$\sigma_t = \sigma_y \quad \longrightarrow \quad \text{tangential stress}$$



$$\sigma_t = \frac{pD}{2t} = \frac{6000(3)}{2(0.10)}$$

$$\sigma_t = 90\,000\ \text{psi} \quad \longrightarrow \quad \textbf{\textit{answer}}$$

$$\sigma_l = \nu\,\sigma_y = \tfrac{1}{3}(90\,000)$$

$$\sigma_l = 30\,000\ \text{psi} \quad \longrightarrow \quad \textbf{\textit{answer}}$$

Statically Indeterminate Members

When the reactive forces or the internal resisting forces over a cross section exceed the number of independent equations of equilibrium, the structure is called statically indeterminate. These cases require the use of additional relations that depend on the elastic deformations in the members.

Solution to Problem 233 Statically Indeterminate

A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.

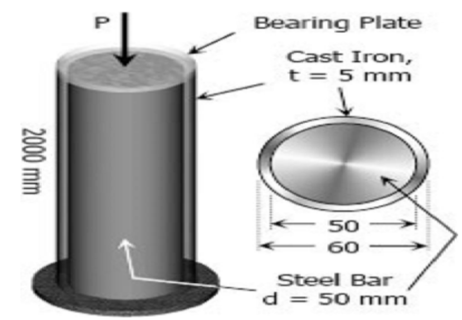
Solution 233

$$\delta = \frac{PL}{AE}$$
$$\delta = \delta_{cast\ iron} = \delta_{steel} = 0.8 \text{ mm}$$

$$\delta_{cast\ iron} = \frac{P_{cast\ iron}(2000)}{[\ \frac{1}{4}\pi(60^2 - 50^2)\ ](100\,000)} = 0.8$$
$$P_{cast\ iron} = 11\,000\pi \text{ N}$$

$$\delta_{steel} = \frac{P_{steel}(2000)}{[\ \frac{1}{4}\pi(50^2)\ ](200\,000)} = 0.8$$
$$P_{steel} = 50\,000\pi \text{ N}$$

$$\Sigma F_V = 0$$
$$P = P_{cast\ iron} + P_{steel}$$
$$P = 11\,000\pi + 50\,000\pi$$
$$P = 61\,000\pi \text{ N}$$
$$P = 191.64 \text{ kN} \longrightarrow \textit{answer}$$



Solution to Problem 234 Statically Indeterminate

Problem 234

A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use E_{co} = 14 GPa and E_{st} = 200 GPa.

Solution 234

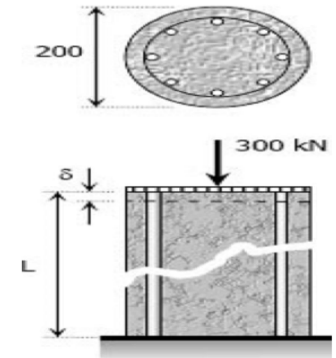
$$\delta_{co} = \delta_{st} = \delta$$
$$\left(\frac{PL}{AE}\right)_{co} = \left(\frac{PL}{AE}\right)_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{co} L}{14000} = \frac{\sigma_{st} L}{200\,000}$$

$100\sigma_{co} = 7\sigma_{st}$

When $\sigma_{st} = 120\,\text{MPa}$
$100\sigma_{co} = 7(120)$
$\sigma_{co} = 8.4\,\text{MPa} > 6\,\text{MPa}$ (not ok!)

When $\sigma_{co} = 6\,\text{MPa}$
$100(6) = 7\sigma_{st}$
$\sigma_{st} = 85.71\,\text{MPa} < 120\,\text{MPa}$ (ok!)

Use $\boldsymbol{\sigma_{co}} = \mathbf{6\ MPa}$ and $\boldsymbol{\sigma_{st}} = \mathbf{85.71\ MPa}$

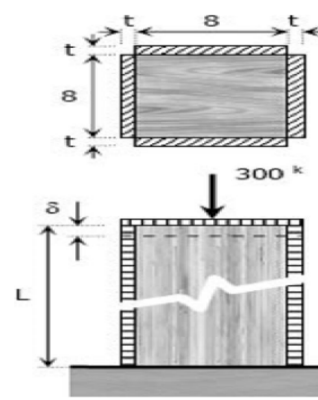


$\Sigma F_V = 0$
$P_{st} + P_{co} = 300$
$\sigma_{st} A_{st} + \sigma_{co} A_{co} = 300$
$85.71Ast + 6\left[\,\frac{1}{4}\pi(200)^2 - A_{st}\,\right] = 300(1000)$
$79.71A_{st} + 60\,000\pi = 300\,000$
$A_{st} = 1398.9\,\text{mm}^2 \;\longrightarrow$ ***answer***

Solution to Problem 235 Statically Indeterminate

A timber column, 8 in. × 8 in. in cross section, is reinforced on each side by a steel plate 8 in. wide and t in. thick. Determine the thickness t so that the column will support an axial load of 300 kips without exceeding a maximum timber stress of 1200 psi or a maximum steel stress of 20 ksi. The moduli of elasticity are 1.5×10^6 psi for timber, and 29×10^6 psi for steel.

Solution 235



$\delta_{steel} = \delta_{timber}$
$\left(\dfrac{\sigma L}{E}\right)_{steel} = \left(\dfrac{\sigma L}{E}\right)_{timber}$
$\dfrac{\sigma_{steel} L}{29 \times 10^6} = \dfrac{\sigma_{bronze} L}{1.5 \times 10^6}$
$1.5\sigma_{steel} = 29\sigma_{timber}$

When $\boldsymbol{\sigma_{timber}} = \mathbf{1200\ psi}$
$1.5\sigma_{steel} = 29(1200)$
$\sigma_{steel} = 23\,200\ \text{psi} = 23.2\ \text{ksi} > 20\ \text{ksi}$ (not ok!)

When $\boldsymbol{\sigma_{steel}} = \mathbf{20\ ksi}$
$1.5(20 \times 1000) = 29\sigma_{timber}$
$\sigma_{timber} = 1034.48\ \text{psi} < 1200\ \text{psi}$ (ok!)

Use $\boldsymbol{\sigma_{steel}} = \mathbf{20\ ksi}$ and $\boldsymbol{\sigma_{timber}} = \mathbf{1.03\ ksi}$

$$\Sigma F_V = 0$$
$$F_{steel} + F_{timber} = 300$$
$$(\sigma A)_{steel} + (\sigma A)_{timber} = 300$$
$$20\,[\,4(8t)\,] + 1.03(82) = 300$$
$$t = 0.365 \text{ in} \quad \longrightarrow \textit{answer}$$

Solution to Problem 236 Statically Indeterminate

A rigid block of mass M is supported by three symmetrically spaced rods as shown in Fig. P-236. Each copper rod has an area of 900 mm^2; E = 120 GPa; and the allowable stress is 70 MPa. The steel rod has an area of 1200 mm^2; E = 200 GPa; and the allowable stress is 140 MPa. Determine the largest mass M which can be supported.

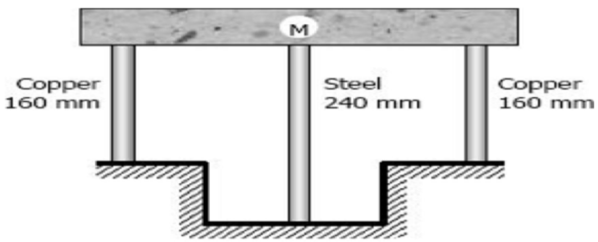
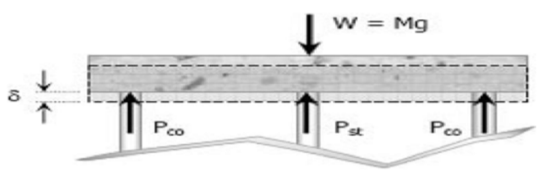


Figure P-236 and P-237

Solution 236



$$\delta_{co} = \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{co}\,L}{120\,000} = \frac{\sigma_{st}\,L}{200\,000}$$
$$10\sigma_{co} = 9\sigma_{st}$$

When σ_{st} = 140 MPa

$$\sigma_{co} = \tfrac{9}{10}(140)$$
$$\sigma_{co} = 126 \text{ MPa} > 70 \text{ MPa} \quad \textbf{(not ok!)}$$

When σ_{co} = 70 MPa

$$\sigma_{st} = \frac{10}{9}(70)$$
$$\sigma_{st} = 77.78 \text{ MPa} < 140 \text{ MPa} \textbf{ (ok!)}$$

Use **σ_{co} = 70 MPa** and **σ_{st} = 77.78 MPa**

$$\Sigma F_V = 0$$
$$2P_{co} + P_{st} = W$$
$$2(\sigma_{co} A_{co}) + \sigma_{st} A_{st} = Mg$$
$$2\,[\,70(900)\,] + 77.78(1200) = M(9.81)$$
$$M = 22358.4 \text{ kg} \quad \longrightarrow \textit{answer}$$

Solution to Problem 237 Statically Indeterminate

In Problem 236, how should the lengths of the two identical copper rods be changed so that each material will be stressed to its allowable limit?

Solution 237

Use **σ_{co} = 70 MPa** and **σ_{st} = 140 MPa**

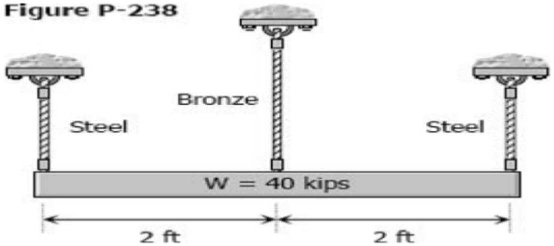
$$\delta_{co} = \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{70 L_{co}}{120\,000} = \frac{140(240)}{200\,000}$$
$$L_{co} = 288 \text{ mm} \textit{ answer}$$

Solution to Problem 238 Statically Indeterminate

Problem 238

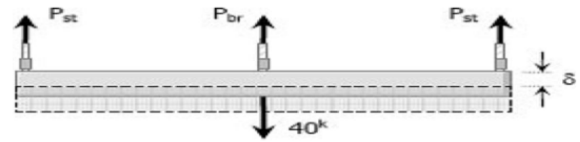
The lower ends of the three bars in Fig. P-238 are at the same level before the uniform rigid block weighing 40 kips is attached. Each steel bar has a length of 3 ft, and area of 1.0 in.^{2}, and E = 29 × 10^6 psi. For the bronze bar, the area is 1.5 in.^{2} and E = 12 × 10^6 psi. Determine (a) the length of the bronze bar so that the load on each steel bar is twice the load on the bronze bar, and (b) the length of the bronze that will make the steel stress twice the bronze stress.

Figure P-238



Solution 238

(a) Condition: $P_{st} = 2P_{br}$



$$\Sigma F_V = 0$$
$$2P_{st} + P_{br} = 40$$
$$2(2P_{br}) + P_{br} = 40$$
$$P_{br} = 8 \text{ kips}$$
$$P_{st} = 2(8) = 16 \text{ kips}$$

$$\delta_{br} = \delta_{st}$$
$$\left(\frac{PL}{AE}\right)_{br} = \left(\frac{PL}{AE}\right)_{st}$$
$$\frac{8000\,L_{br}}{1.5(12 \times 10^6)} = \frac{16000\,(3 \times 12}{1.0(29 \times 10^6)}$$
$$L_{br} = 44.69 \text{ in}$$
$$L_{br} = 3.72 \text{ ft} \;\longrightarrow\; \textbf{answer}$$

(b) Condition: $\sigma_{st} = 2\sigma_{br}$

$$\Sigma F_V = 0$$
$$2P_{st} + P_{br} = 40$$
$$2(\sigma_{st} A_{st}) + \sigma_{br} A_{br} = 40$$
$$2\,[\,(2\sigma_{br})\,A_{st}\,] + \sigma_{br} A_{br} = 40$$
$$4\sigma_{br}(1.0) + \sigma_{br}(1.5) = 40$$
$$\sigma_{br} = 7.27 \text{ ksi}$$
$$\sigma_{st} = 2(7.27) = 14.54 \text{ ksi}$$

$$\delta_{br} = \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{br} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{7.27(1000)\,L_{br}}{12 \times 10^6} = \frac{14.54(1000)(3 \times 12)}{29 \times 10^6}$$
$$L_{br} = 29.79 \text{ in}$$
$$L_{br} = 2.48 \text{ ft} \;\longrightarrow\; \textit{answer}$$

Solution to Problem 239 Statically Indeterminate

The rigid platform in Fig. P-239 has negligible mass and rests on two steel bars, each 250.00 mm long. The center bar is aluminum and 249.90 mm long. Compute the stress in the aluminum bar after the center load P = 400 kN has been applied. For each steel bar, the area is 1200 mm2 and E = 200 GPa. For the aluminum bar, the area is 2400 mm2 and E = 70 GPa.

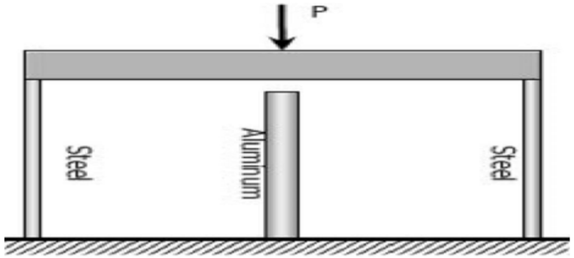
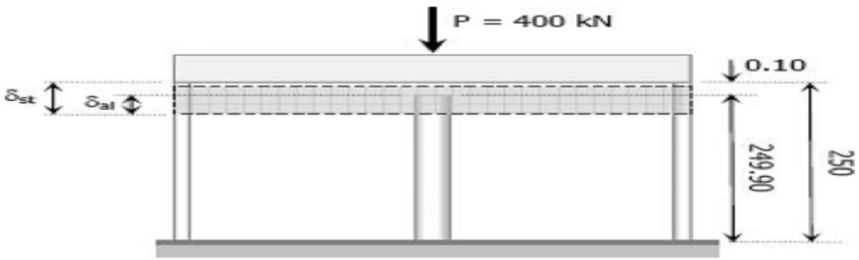
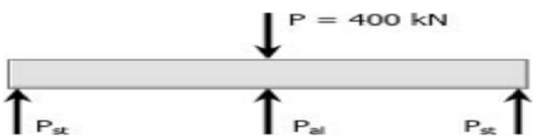


Figure P-239

Solution 239



$$\delta_{st} = \delta_{al} + 0.10$$
$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al} + 0.10$$
$$\frac{\sigma_{st}(250)}{200\,000} = \frac{\sigma_{al}(249.90)}{70\,000} + 0.10$$
$$0.00125\sigma_{st} = 0.00357\sigma_{al} + 0.10$$
$$\sigma_{st} = 2.856\sigma_{al} + 80$$



$$\Sigma F_V = 0$$
$$2P_{st} + P_{al} = 400\,000$$
$$2\sigma_{st}\,A_{st} + \sigma_{al}\,A_{al} = 400\,000$$
$$2(2.856\sigma_{al} + 80)1200 + \sigma_{al}(2400) = 400\,000$$
$$9254.4\sigma_{al} + 192\,000 = 400\,000$$
$$\sigma_{al} = 22.48\ \text{MPa} \quad \longrightarrow \textit{answer}$$

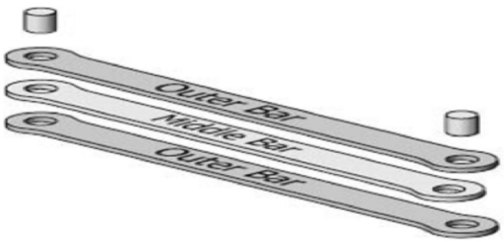
Solution to Problem 240 Statically Indeterminate

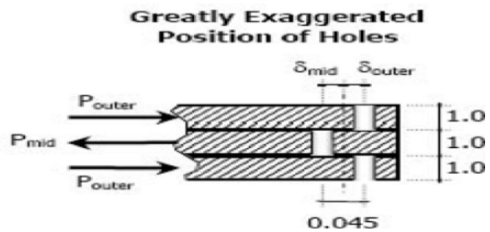
Problem 240

Three steel eye-bars, each 4 in. by 1 in. in section, are to be assembled by driving rigid 7/8-in.-diameter drift pins through holes drilled in the ends of the bars. The center-line spacing between the holes is 30 ft in the two outer bars, but 0.045 in. shorter in the middle bar. Find the shearing stress developed in the drip pins. Neglect local deformation at the holes.

Solution 240

Middle bar is 0.045 inch shorter between holes than outer bars.





$$\Sigma F_H = 0$$
$$P_{mid} = 2P_{outer}$$

$$\delta_{outer} + \delta_{mid} = 0.045$$
$$\left(\frac{PL}{AE}\right)_{outer} + \left(\frac{PL}{AE}\right)_{mid} = 0.045$$
$$\frac{P_{outer}(30 \times 12)}{[\,1.0(4.0)\,]E} + \frac{P_{mid}(30 \times 12 - 0.045)}{[\,1.0(4.0)\,]E} = 0.045$$
$$360P_{outer} + 359.955P_{mid} = 0.18E$$
$$360P_{outer} + 359.955(2P_{outer}) = 0.18E$$

(For steel: $E = 29 \times 10^6$ psi)

$$1079.91P_{outer} = 0.18(29 \times 10^6)$$
$$P_{outer} = 4833.74 \text{ lb}$$

$$P_{mid} = 2(4833.74)$$
$$P_{mid} = 9667.48 \text{ lb}$$

Use shear force $V = P_{mid}$

Shearing stress of drip pins (double shear):

$$\tau = \frac{V}{A} = \frac{9667.48}{2\,[\,\frac{1}{4}\pi\,(\frac{7}{8})^2\,]}$$
$$\tau = 8038.54 \text{ psi} \quad \longrightarrow \textit{answer}$$

Solution to Problem 241 Statically Indeterminate

As shown in Fig. P-241, three steel wires, each 0.05 in.2 in area, are used to lift a load W = 1500 lb. Their unstressed lengths are 74.98 ft, 74.99 ft, and 75.00 ft.
(a) What stress exists in the longest wire?
(b) Determine the stress in the shortest wire if W = 500 lb.

Figure P-241

Solution 241

Let L_1 = 74.98 ft; L_2 = 74.99 ft; and L_3 = 75.00 ft

Part (a)

Bring L_1 and L_2 into L_3 = 75 ft length:
(For steel: $E = 29 \times 10^6$ psi)

$$\delta = \frac{PL}{AE}$$

For L_1:

$$(75-74.98)(12) = \frac{P_1(74.98 \times 12)}{0.05(29 \times 10^6)}$$
$$P_1 = 386.77\,\text{lb}$$

For L_2:

$$(75-74.99)(12) = \frac{P_2(74.99 \times 12)}{0.05(29 \times 10^6)}$$
$$P_2 = 193.36\,\text{lb}$$

Let $P = P_3$ (Load carried by L_3)

$P + P_2$ (Total load carried by L_2)
$P + P_1$ (Total load carried by L_1)

$$\Sigma F_V = 0$$
$$(P + P_1) + (P + P_2) + P = W$$
$$3P + 386.77 + 193.36 = 1500$$
$$P = 306.62 lb = P_3$$

$$\sigma_3 = \frac{P_3}{A} = \frac{306.62}{0.05}$$
$$\sigma_3 = 6132.47\,\text{psi} \longrightarrow \textit{answer}$$

Part (b)

From the above solution:
$P_1 + P_2$ = 580.13 lb > 500 lb (L_3 carries no load)

Bring L_1 into L_2 = 74.99 ft

$$\delta = \frac{PL}{AE}$$

$$(74.99 - 74.98)(12) = \frac{P_1(74.98 \times 12)}{0.05(29 \times 10^6)}$$

$$P_1 = 193.38 \text{ lb}$$

Let P = P_2 (Load carried by L_2)

P + P_1 (Total load carried by L_1)

$$\Sigma F_V = 0$$
$$(P + P_1) + P = 500$$
$$2P + 193.38 = 500$$
$$P = 153.31 \text{ lb}$$

$$P + P_1 = 153.31 + 193.38$$
$$P + P_1 = 346.69 \text{ lb}$$

$$\sigma = \frac{P + P_1}{A} = \frac{346.69}{0.05}$$
$$\sigma = 6933.8 \text{ psi} \quad \longrightarrow \textit{answer}$$

Solution to Problem 242 Statically Indeterminate

The assembly in Fig. P-242 consists of a light rigid bar AB, pinned at O, that is attached to the steel and aluminum rods. In the position shown, bar AB is horizontal and there is a gap, Δ = 5 mm, between the lower end of the steel rod and its pin support at C. Compute the stress in the aluminum rod when the lower end of the steel rod is attached to its support.

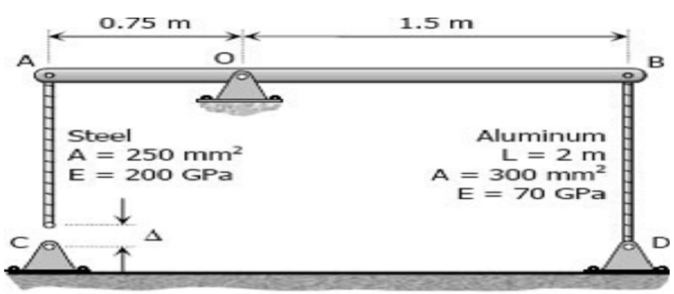
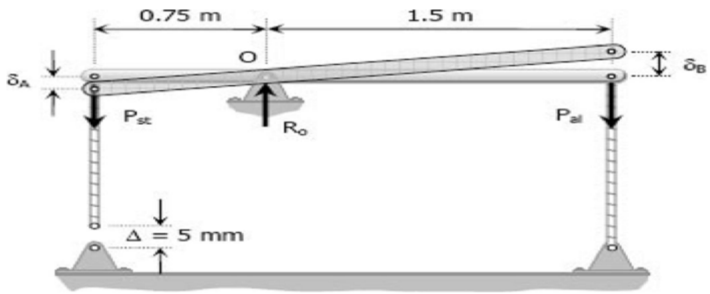


Figure P-242

Solution 242



$$\Sigma M_O = 0$$
$$0.75P_{st} = 1.5P_{al}$$
$$P_{st} = 2P_{al}$$
$$\sigma_{st} A_{st} = 2(\sigma_{al} A_{al})$$
$$\sigma_{st} = \frac{2\sigma_{al} A_{al}}{A_{st}}$$
$$\sigma_{st} = \frac{2\,[\,\sigma_{al}(3000\,]}{250}$$
$$\sigma_{st} = 2.4\sigma_{al}$$

$$\delta_{al} = \delta_B$$

By ratio and proportion:

$$\frac{\delta_A}{0.75} = \frac{\delta_B}{1.5}$$
$$\delta_A = 0.5\delta_B$$
$$\delta_A = 0.5\delta_{al}$$

$$\Delta = \delta_{st} + \delta_A$$
$$5 = \delta_{st} + 0.5\delta_{al}$$
$$5 = \frac{\sigma_{st}(2\,000{-}5)}{250(200\,000)} + 0.5\left[\frac{\sigma_{al}(2000)}{300(70\,000)}\right]$$
$$5 = (3.99 \times 10^{-5})\,\sigma_{st} + (4.76 \times 10^{-5})\,\sigma_{al}$$
$$\sigma_{al} = 105\,000{-}0.8379\,\sigma_{st}$$
$$\sigma_{al} = 105\,000{-}0.8379(2.4\,\sigma_{al})$$
$$3.01096\sigma_{al} = 105\,000$$
$$\sigma_{al} = 34\,872.6\ \text{MPa} \quad \longrightarrow \textit{answer}$$

Solution to Problem 243 Statically Indeterminate

A homogeneous rod of constant cross section is attached to unyielding supports. It carries an axial load P applied as shown in Fig. P-243. Prove that the reactions are given by $R_1 = Pb/L$ and $R_2 = Pa/L$.

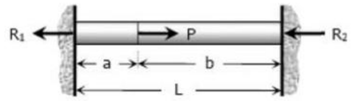
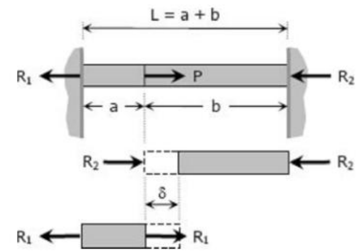


Figure P-243

Solution 243



$$\Sigma F_H = 0$$
$$R_1 + R_2 = P$$
$$R_2 = P - R_1$$

$$\delta_1 = \delta_2 = \delta$$
$$\left(\frac{PL}{AE}\right)_1 = \left(\frac{PL}{AE}\right)_2$$
$$\frac{R_1\, a}{AE} = \frac{R_2\, b}{AE}$$
$$R_1\, a = R_2\, b$$

$$R_1\, a = (P - R_1) b$$
$$R_1\, a = Pb - R_1\, b$$
$$R_1\, (a + b) = Pb$$
$$R_1\, L = Pb$$
$$R_1 = Pb/L \textbf{ ok!}$$

$$R_2 = P - Pb/L$$
$$R_2 = \frac{P\,(L - b)}{L}$$
$$R_2 = Pa/L \textbf{ ok!}$$

Solution to Problem 244 Statically Indeterminate

A homogeneous bar with a cross sectional area of 500 mm^2 is attached to rigid supports. It carries the axial loads P_1 = 25 kN and P_2 = 50 kN, applied as shown in Fig. P-244. Determine the stress in segment BC. (Hint: Use the results of Prob. 243, and compute the reactions caused by P_1 and P_2 acting separately. Then use the principle of superposition to compute the reactions when both loads are applied.)

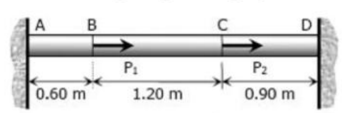
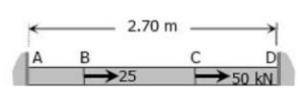
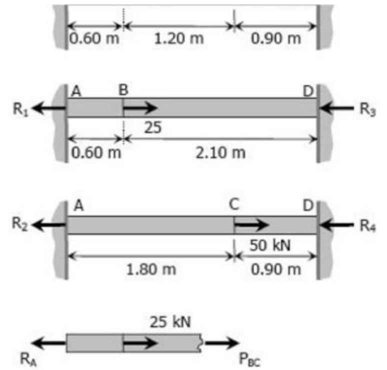


Figure P-244



Solution 244



From the results of Solution to Problem 243:

$$R_1 = 25(2.10)/2.70$$
$$R_1 = 19.44 \text{ kN}$$

$$R_2 = 50(0.90)/2.70$$
$$R_2 = 16.67 \text{ kN}$$

$$R_A = R_1 + R_2$$
$$R_A = 19.44 + 16.67$$
$$R_A = 36.11 \text{ kN}$$

For segment BC

$$P_{BC} + 25 = R_A$$
$$P_{BC} + 25 = 36.11$$
$$P_{BC} = 11.11 \text{ kN}$$

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{11.11(1000)}{500}$$
$$\sigma_{BC} = 22.22 \text{ MPa} \quad \rightarrow \textit{answer}$$

Solution to Problem 245 Statically Indeterminate

The composite bar in Fig. P-245 is firmly attached to unyielding supports. Compute the stress in each material caused by the application of the axial load P = 50 kips.

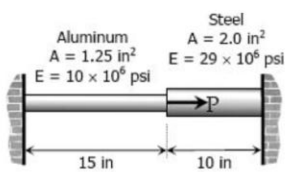
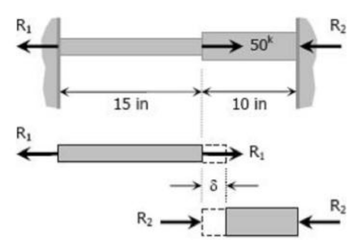


Figure P-245 and P-246

Solution 245



$$\Sigma F_H = 0$$
$$R_1 + R_2 = 50\,000$$
$$R_1 = 50\,000 - R_2$$

$$\delta_{al} = \delta_{st} = \delta$$
$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st}$$

$$\frac{R_1(15)}{1.25(10 \times 10^6)} = \frac{R_2(10)}{2.0(29 \times 10^6)}$$
$$R_2 = 6.96R_1$$
$$R_2 = 6.96(50\,000 - R_2)$$
$$7.96R_2 = 348\,000$$
$$R_2 = 43\,718.59 \text{ lb}$$

$$\sigma_{st} = \frac{R_2}{A_{st}} = \frac{43\,718.59}{2.0}$$
$$\sigma_{st} = 21\,859.30 \text{ psi} \quad \rightarrow \textit{answer}$$

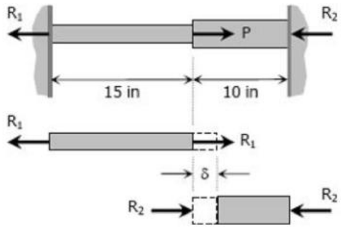
$$R_1 = 50\,000 - 43\,718.59$$
$$R_1 = 6281.41 \text{ lb}$$

$$\sigma_{al} = \frac{R_1}{A_{al}} = \frac{6281.41}{1.25}$$
$$\sigma_{al} = 5025.12 \text{ psi} \quad \rightarrow \textit{answer}$$

Solution to Problem 246 Statically Indeterminate

Referring to the composite bar in Problem 245, what maximum axial load P can be applied if the allowable stresses are 10 ksi for aluminum and 18 ksi for steel.

Solution 246



$$\delta_{st} = \delta_{al} = \delta$$
$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al}$$
$$\frac{\sigma_{st}(10)}{29 \times 10^6} = \frac{\sigma_{al}(15)}{10 \times 10^6}$$
$$\sigma_{st} = 4.35\,\sigma_{al}$$

When σ_{al} = 10 ksi

$$\sigma_{st} = 4.35(10)$$
$$\sigma_{st} = 43.5 \text{ ksi} > 18 \text{ ksi} \quad \textit{(not ok!)}$$

When σ_{st} = 18 ksi

$$18 = 4.35\,\sigma_{al}$$
$$\sigma_{al} = 4.14 \text{ ksi} < 10 \text{ ksi} \quad \textit{(ok!)}$$

Use σ_{al} = 4.14 ksi and σ_{st} = 18 ksi

$$\Sigma F_H = 0$$

$$P = R_1 + R_2$$
$$P = \sigma_{al} A_{al} + \sigma_{st} A_{st}$$
$$P = 4.14(1.25) + 18(2.0)$$
$$P = 41.17 \text{ kips} \quad \rightarrow \textit{answer}$$

Solution to Problem 247 Statically Indeterminate

The composite bar in Fig. P-247 is stress-free before the axial loads P_1 and P_2 are applied. Assuming that the walls are rigid, calculate the stress in each material if P_1 = 150 kN and P_2 = 90 kN.

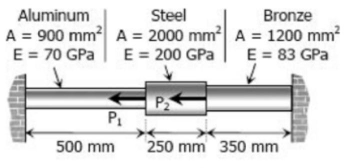
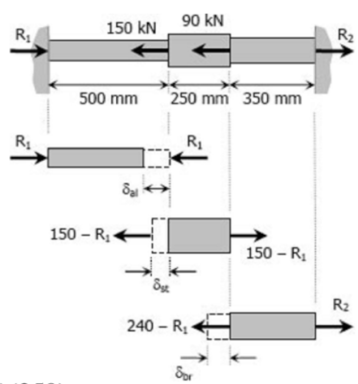


Figure P-247 and P-248

Solution 247



From the FBD of each material shown:
δ_{al} is shortening
δ_{st} and δ_{br} are lengthening

$$R_2 = 240 - R_1$$
$$P_{al} = R_1$$
$$P_{st} = 150 - R_1$$
$$P_{br} = R_2 = 240 - R_1$$

$$\delta_{al} = \delta_{st} + \delta_{br}$$
$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{br}$$

$$\frac{R_1(500)}{900(70\,000)} = \frac{(150-R_1)(250)}{2000(200\,000)} + \frac{(240-R_1)(350)}{1200(83\,000)}$$
$$\frac{R_1}{126\,000} = \frac{150-R_1}{1\,600\,000} + \frac{(240-R_1)7}{1\,992\,000}$$
$$\tfrac{1}{63}R_1 = \tfrac{1}{800}(150 - R_1) + \tfrac{7}{996}(240 - R_1)$$
$$(\tfrac{1}{63} + \tfrac{1}{800} + \tfrac{7}{996})R_1 = \tfrac{1}{800}(150) + \tfrac{7}{996}(240)$$
$$R_1 = 77.60 \text{ kN}$$

$$P_{al} = R_1 = 77.60 \text{ kN}$$
$$P_{st} = 150 - 77.60 = 72.40 \text{ kN}$$

$$P_{br} = 240 - 77.60 = 162.40 \text{ kN}$$

$$\sigma = P/A$$

$$\sigma_{al} = 77.60(1000)/900$$
$$\sigma_{al} = 86.22 \text{ MPa} \rightarrow \textbf{\textit{answer}}$$

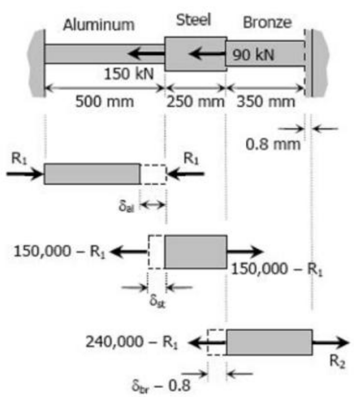
$$\sigma_{st} = 72.40(1000)/2000$$
$$\sigma_{st} = 36.20 \text{ MPa} \rightarrow \textbf{\textit{answer}}$$

$$\sigma_{br} = 162.40(1000)/1200$$
$$\sigma_{br} = 135.33 \text{ MPa} \rightarrow \textbf{\textit{answer}}$$

Solution to Problem 248 Statically Indeterminate

Solve Problem 247 if the right wall yields 0.80 mm.

Solution 248



$$\delta_{al} = \delta_{st} + (\delta_{br} + 0.8)$$
$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{br} + 0.8$$
$$\frac{R_1\,(500)}{900(70\,000)} = \frac{(150\,000 - R_1)(250)}{2000(200\,000)} + \frac{(240\,000 - R_1)(350)}{1200(83\,000)} + 0.8$$
$$\frac{R_1}{126\,000} = \frac{150\,000 - R_1}{1\,600\,000} + \frac{7(240\,000 - R_1)}{1\,992\,000} + 0.8$$
$$\tfrac{1}{63}R_1 = \tfrac{1}{800}(150\,000 - R_1) + \tfrac{7}{996}(240\,000 - R_1) + 1600$$
$$(\tfrac{1}{63} + \tfrac{1}{800} + \tfrac{7}{996})R_1 = \tfrac{1}{800}(150\,000) + \tfrac{7}{996}(240\,000) + 1600$$
$$R_1 = 143\,854 \text{ N} = 143.854 \text{ kN}$$

$P_{al} = R_1 = 143.854\text{ kN}$
$P_{st} = 150 - R_1 = 150 - 143.854 = 6.146\text{ kN}$
$P_{br} = R_2 = 240 - R_1 = 240 - 143.854 = 96.146\text{ kN}$

$\sigma = P/A$

$\sigma_{al} = 143.854(1000)/900$
$\sigma_{al} = 159.84\text{ MPa} \rightarrow$ ***answer***

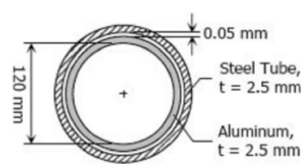
$\sigma_{st} = 6.146(1000)/2000$
$\sigma_{st} = 3.073\text{ MPa} \rightarrow$ ***answer***

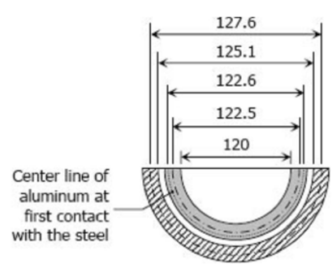
$\sigma_{br} = 96.146(1000)/1200$
$\sigma_{br} = 80.122\text{ MPa} \rightarrow$ ***answer***

Solution to Problem 249 Statically Indeterminate

There is a radial clearance of 0.05 mm when a steel tube is placed over an aluminum tube. The inside diameter of the aluminum tube is 120 mm, and the wall thickness of each tube is 2.5 mm. Compute the contact pressure and tangential stress in each tube when the aluminum tube is subjected to an internal pressure of 5.0 MPa.

Solution 249





Internal pressure of aluminum tube to cause contact with the steel:

$$\delta_{al} = \left(\frac{\sigma L}{E}\right)_{al}$$

$$\pi(122.6 – 122.5) = \frac{\sigma_1 (122.5\pi)}{70\,000}$$

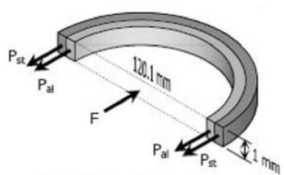
$$\sigma_1 = 57.143\text{ MPa}$$

$$\frac{p_1 D}{2t} = 57.143$$

$$\frac{p_1(120)}{2(2.5)} = 57.143$$

$p_1 = 2.381\ \text{ MPa} \;\rightarrow$ pressure that causes aluminum to contact with the steel, further increase of pressure will expand both aluminum and steel tubes.

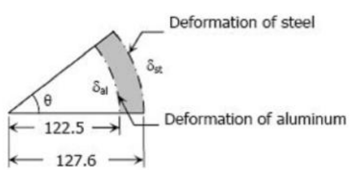
Let p_c = contact pressure between steel and aluminum tubes



FBD for p ≥ 2.381 MPa

$$2P_{st} + 2P_{al} = F$$
$$2P_{st} + 2P_{al} = 5.0(120.1)(1)$$
$$P_{st} + P_{al} = 300.25 \;\rightarrow\; \text{Equation (1)}$$

The relationship of deformations is (from the figure):



Geometric relation of deformations

$$\delta_{st} = 127.6\theta$$
$$\theta = \delta_{st}/127.6$$

$$\delta_{al} = 122.5\theta$$
$$\delta_{al} = 122.5(\delta_{st}/127.6)$$
$$\delta_{al} = 0.96\,\delta_{st}$$
$$\left(\frac{PL}{AE}\right)_{al} = 0.96\left(\frac{PL}{AE}\right)_{st}$$
$$\frac{P_{al}(122.5\pi)}{2.5(70\,000)} = 0.96\left[\frac{P_{st}(127.6)}{2.5(200\,000)}\right]$$

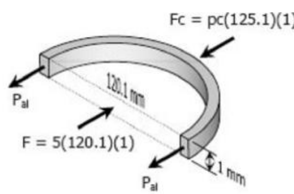
$$P_{al} = 0.35P_{st} \rightarrow \text{Equation (2)}$$

From Equation (1)

$$P_{st} + 0.35P_{st} = 300.25$$
$$P_{st} = 222.41 \text{ N}$$

$$P_{al} = 0.35(222.41)$$
$$P_{al} = 77.84 \text{ N}$$

Contact Force



$$F_c + 2P_{st} = F$$
$$p_c(125.1)(1) + 2(77.84) = 5(120.1)(1)$$
$$p_c = 3.56 \text{ MPa} \rightarrow \textit{answer}$$

Solution to Problem 250 Statically Indeterminate

In the assembly of the bronze tube and steel bolt shown in Fig. P-250, the pitch of the bolt thread is p = 1/32 in.; the cross-sectional area of the bronze tube is 1.5 in.^{2} and of steel bolt is 3/4 in.^{2} The nut is turned until there is a compressive stress of 4000 psi in the bronze tube. Find the stresses if the nut is given one additional turn. How many turns of the nut will reduce these stresses to zero? Use $E_{br} = 12 \times 10^6$ psi and $E_{st} = 29 \times 10^6$ psi.

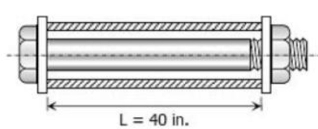
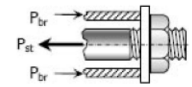


Figure P-250

Solution 250



$$P_{st} = P_{br}$$
$$A_{st}\,\sigma_{st} = P_{br}\,\sigma_{br}$$
$$\tfrac{3}{4}\sigma_{st} = 1.5\,\sigma_{br}$$
$$\sigma_{st} = 2\sigma_{br}$$

For one turn of the nut:

$$\delta_{st} + \delta_{br} = \tfrac{1}{32}$$
$$\left(\frac{\sigma L}{E}\right)_{st} + \left(\frac{\sigma L}{E}\right)_{br} = \frac{1}{32}$$
$$\frac{\sigma_{st}(40)}{29 \times 10^6} + \frac{\sigma_{br}(40)}{12 \times 10^6} = \frac{1}{32}$$
$$\sigma_{st} + \tfrac{29}{12}\sigma_{br} = 22\,656.25$$
$$2\sigma_{br} + \tfrac{29}{12}\sigma_{br} = 22\,656.25$$
$$\sigma_{br} = 5129.72 \text{ psi}$$
$$\sigma_{st} = 2(5129.72) = 10259.43 \text{ psi}$$

Initial stresses:

$$\sigma_{br} = 4000 \text{ psi}$$
$$\sigma_{st} = 2(4000) = 8000 \text{ psi}$$

Final stresses:

$$\sigma_{br} = 4000 + 5129.72 = 9\,129.72 \text{ psi} \quad \rightarrow \textit{answer}$$
$$\sigma_{st} = 2(9129.72) = 18\,259.4 \text{ psi} \quad \rightarrow \textit{answer}$$

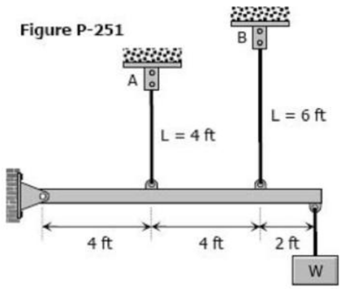
Required number of turns to reduce σ_{br} to zero:

$$n = \frac{9129.72}{5129.72} = 1.78 \text{ turns}$$

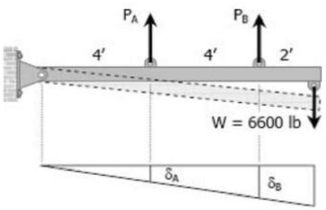
The nut must be turned back by **1.78 turns**

Solution to Problem 251 Statically Indeterminate

The two vertical rods attached to the light rigid bar in Fig. P-251 are identical except for length. Before the load W was attached, the bar was horizontal and the rods were stress-free. Determine the load in each rod if W = 6600 lb.



Solution 251



$$\Sigma M_{pin\ support} = 0$$
$$4P_A + 8P_B = 10(6600)$$
$$P_A + 2P_B = 16500 \ \rightarrow \text{equation (1)}$$

By ratio and proportion

$$\frac{\delta_A}{4} = \frac{\delta_B}{8}$$
$$\delta_A = 0.5\delta_B$$
$$\left(\frac{PL}{AE}\right)_A = 0.5\left(\frac{PL}{AE}\right)_B$$
$$\frac{P_A(4)}{AE} = \frac{0.5P_B(4)}{AE}$$
$$P_A = 0.75P_B$$

From equation (1)

$$0.75P_B + 2P_B = 16500$$
$$P_B = 6000 \text{ lb} \ \rightarrow \textit{answer}$$

$$P_A = 0.75(6000)$$
$$P_A = 4500 \text{ lb} \ \rightarrow \textit{answer}$$

Solution to Problem 252 Statically Indeterminate

Problem 252

The light rigid bar ABCD shown in Fig. P-252 is pinned at B and connected to two vertical rods. Assuming that the bar was initially horizontal and the rods stress-free, determine the stress in each rod after the load after the load P = 20 kips is applied.

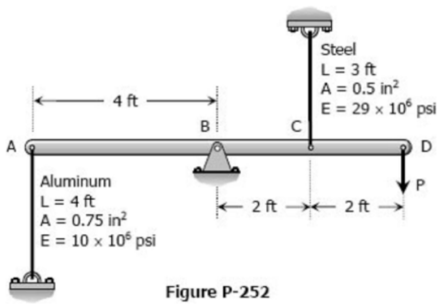
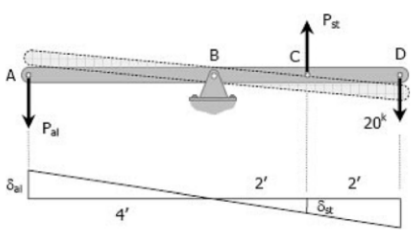


Figure P-252

Solution 252

$$\Sigma M_B = 0$$
$$4P_{al} + 2P_{st} = 4(20\,000)$$
$$4(\sigma_{al} A_{al}) + 2\sigma_{st} A_{st} = 80\,000$$
$$4\,[\,\sigma_{al}(0.75)\,] + 2\,[\,\sigma_{st}(0.5)\,] = 80\,000$$
$$3\sigma_{al} + \sigma_{st} = 80\,000 \;\rightarrow\; \text{equation (1)}$$



$$\frac{\delta_{st}}{2} = \frac{\delta_{al}}{4}$$
$$\delta_{st} = 0.5\delta_{al}$$
$$\left(\frac{\sigma L}{E}\right)_{st} = 0.5\left(\frac{\sigma L}{E}\right)_{al}$$

$$\frac{\sigma_{st}(3)}{29 \times 10^6} = 0.5 \left[\frac{\sigma_{al}(4)}{10 \times 10^6} \right]$$
$$\sigma_{st} = \frac{29}{15}\sigma_{al}$$

From equation (1)

$$3\sigma_{al} + \frac{29}{15}\sigma_{al} = 80\,000$$
$$\sigma_{al} = 16\,216.22 \text{ psi}$$
$$\sigma_{al} = 16.22 \text{ ksi} \quad \rightarrow \textbf{\textit{answer}}$$

$$\sigma_{st} = \frac{29}{15}(16.22)$$
$$\sigma_{st} = 31.35 \text{ ksi} \quad \rightarrow \textbf{\textit{answer}}$$

Solution to Problem 253 Statically Indeterminate

As shown in Fig. P-253, a rigid beam with negligible weight is pinned at one end and attached to two vertical rods. The beam was initially horizontal before the load W = 50 kips was applied. Find the vertical movement of W.

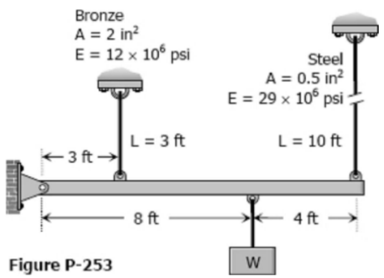
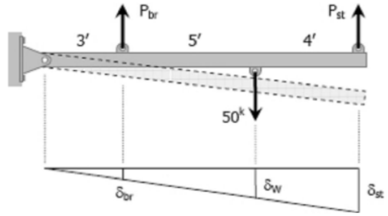


Figure P-253

Solution 253

$$\Sigma M_{pin\ support} = 0$$
$$3P_{br} + 12P_{st} = 8(50\,000)$$
$$3P_{br} + 12P_{st} = 400\,000 \quad \rightarrow \text{Equation (1)}$$



$$\frac{\delta_{st}}{12} = \frac{\delta_{br}}{3}$$
$$\delta_{st} = 4\delta_{br}$$
$$\left(\frac{PL}{AE}\right)_{st} = 4\left(\frac{PL}{AE}\right)_{br}$$
$$\frac{P_{st}(10)}{0.5(29\times10^6)} = 4\left[\frac{P_{br}(3)}{2(12\times10^6)}\right]$$
$$P_{st} = 0.725P_{br}$$

From equation (1)

$$3P_{br} + 12(0.725P_{br}) = 400\,000$$
$$P_{br} = 34\,188.03\text{ lb}$$

$$\delta_{br} = \left(\frac{PL}{AE}\right)_{br} = \frac{34\,188.03(3\times12)}{2(12\times10^6)}$$
$$\delta_{br} = 0.0513\text{ in}$$

$$\frac{\delta_W}{8} = \frac{\delta_{br}}{3}$$
$$\delta_W = \tfrac{8}{3}\delta_{br}$$
$$\delta_W = \tfrac{8}{3}(0.0513)$$
$$\delta_W = 0.1368\text{ in} \quad \rightarrow \textit{answer}$$

Check by δ_{st}:

$$P_{st} = 0.725P_{br} = 0.725(34\,188.03)$$
$$P_{st} = 24\,786.32\text{ lb}$$

$$\delta_{st} = \left(\frac{PL}{AE}\right)_{st} = \frac{24\,786.32(10 \times 12)}{0.5(29 \times 10^6)}$$
$$\delta_{br} = 0.2051 \text{ in}$$

$$\frac{\delta_W}{8} = \frac{\delta_{st}}{12}$$
$$\delta_W = \tfrac{2}{3}\delta_{st}$$
$$\delta_W = \tfrac{2}{3}(0.2051)$$
$$\delta_W = 0.1368 \text{ in} \quad \rightarrow \textit{ok!}$$

Solution to Problem 254 Statically Indeterminate

As shown in Fig. P-254, a rigid bar with negligible mass is pinned at O and attached to two vertical rods. Assuming that the rods were initially stress-free, what maximum load P can be applied without exceeding stresses of 150 MPa in the steel rod and 70 MPa in the bronze rod.

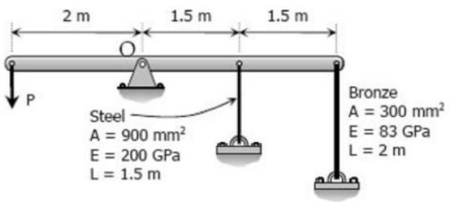
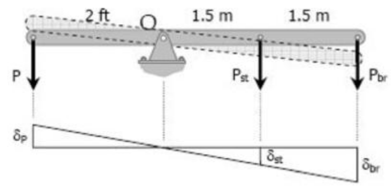


Figure P-254

Solution 254

$$\Sigma M_O = 0$$
$$2P = 1.5P_{st} + 3P_{br}$$
$$2P = 1.5(\sigma_{st} A_{st}) + 3(\sigma_{br} A_{br})$$
$$2P = 1.5\,[\,\sigma_{st}\,(900)\,] + 3\,[\,\sigma_{br}\,(300)\,]$$
$$2P = 1350\sigma_{st} + 900\sigma_{br}$$
$$P = 675\sigma_{st} + 450\sigma_{br}$$



$$\frac{\delta_{br}}{3} = \frac{\delta_{st}}{1.5}$$
$$\delta_{br} = 2\delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{br} = 2\left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{br}(2)}{83} = 2\left[\frac{\sigma_{st}(1.5)}{200}\right]$$
$$\sigma_{br} = 0.6225\sigma_{st}$$

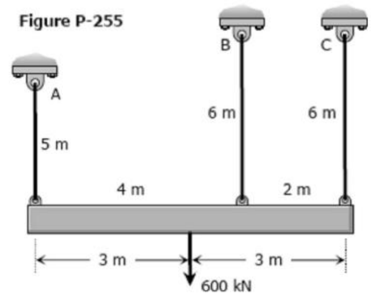
When $\sigma_{st} = 150\,\text{MPa}$
$$\sigma_{br} = 0.6225(150)$$
$$\sigma_{br} = 93.375\,\text{MPa} > 70\,\text{MPa} \;\textit{(not ok!)}$$

When $\sigma_{br} = 70\,\text{MPa}$
$$70 = 0.6225\sigma_{st}$$
$$\sigma_{st} = 112.45\,\text{MPa} < 150\,\text{MPa} \;\textit{(ok!)}$$

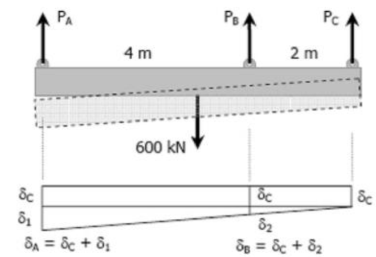
Use $\sigma_{st} = 112.45\,\text{MPa}$ and $\sigma_{br} = 70\,\text{MPa}$
$$P = 675\sigma_{st} + 450\sigma_{br}$$
$$P = 675(112.45) + 450(70)$$
$$P = 107\,403.75\,\text{N}$$
$$P = 107.4\,\text{kN} \rightarrow \textbf{\textit{answer}}$$

Solution to Problem 255 Statically Indeterminate

Shown in Fig. P-255 is a section through a balcony. The total uniform load of 600 kN is supported by three rods of the same area and material. Compute the load in each rod. Assume the floor to be rigid, but note that it does not necessarily remain horizontal.



Solution 255



$\delta_B = \delta_C + \delta_2$
$\delta_2 = \delta_B - \delta_C$
$\dfrac{\delta_1}{6} = \dfrac{\delta_2}{2}$
$\delta_1 = 3\delta_2$

$\delta_A = \delta_C + \delta_1$
$\delta_A = \delta_C + 3\delta_2$
$\delta_A = \delta_C + 3(\delta_B - \delta_C)$
$\delta_A = 3\delta_B - 2\delta_C$
$\left(\dfrac{PL}{AE}\right)_A = 3\left(\dfrac{PL}{AE}\right)_B - 2\left(\dfrac{PL}{AE}\right)_C$
$\dfrac{P_A(5)}{AE} = \dfrac{3P_B(6)}{AE} - \dfrac{2P_C(6)}{AE}$
$P_A = 3.6P_B - 2.4P_C \rightarrow$ Equation (1)

$\Sigma F_V = 0$
$P_A + P_B + P_C = 600$
$(3.6P_B - 2.4P_C) + P_B + P_C = 600$
$4.6P_B - 1.4P_C = 600 \rightarrow$ Equation (2)

$\Sigma M_A = 0$
$4P_B + 6P_C = 3(600)$

$P_B = 450 - 1.5P_C \rightarrow$ Equation (3)

Substitute $P_B = 450 - 1.5\,P_C$ to Equation (2)

$$4.6(450 - 1.5P_C) - 1.4P_C = 600$$
$$8.3P_C = 1470$$
$$P_C = 177.11\ \text{kN} \rightarrow \textit{answer}$$

From Equation (3)

$$P_B = 450 - 1.5(177.11)$$
$$P_B = 184.34\ \text{kN} \rightarrow \textit{answer}$$

From Equation (1)

$$P_A = 3.6(184.34) - 2.4(177.11)$$
$$P_A = 238.56\ \text{kN} \rightarrow \textit{answer}$$

Solution to Problem 256 Statically Indeterminate

Three rods, each of area 250 mm^2, jointly support a 7.5 kN load, as shown in Fig. P-256. Assuming that there was no slack or stress in the rods before the load was applied, find the stress in each rod. Use E_{st} = 200 GPa and E_{br} = 83 GPa.

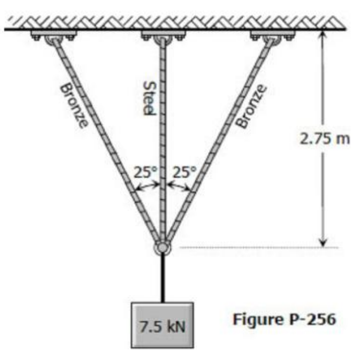
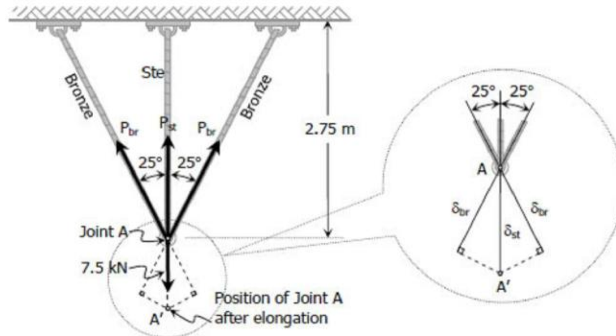


Figure P-256

Solution 256

$$\cos 25^\circ = \frac{2.75}{L_{br}}$$
$$L_{br} = 3.03 \text{ m}$$



$$\Sigma F_V = 0$$
$$2P_{br}\cos 25^\circ + P_{st} = 7.5(1000)$$
$$P_{st} = 7500 - 1.8126 P_{br}$$
$$\sigma_{st} A_{st} = 7500 - 1.8126 \sigma_{br} A_{br}$$
$$\sigma_{st}(250) = 7500 - 1.8126\,[\,\sigma_{br}(250)\,]$$
$$\sigma_{st} = 30 - 1.8126\sigma_{br} \;\rightarrow\; \textbf{Equation (1)}$$

$$\cos 25^\circ = \frac{\delta_{br}}{\delta_{st}}$$
$$\delta_{br} = 0.9063\delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{br} = 0.9063\left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{br}(3.03)}{83} = 0.0963\left[\frac{\sigma_{st}(2.75)}{200}\right]$$
$$\sigma_{br} = 0.3414\sigma_{st} \;\rightarrow\; \textbf{Equation (2)}$$

From Equation (1)
$$\sigma_{st} = 30 - 1.8126(0.3414\sigma_{st})$$
$$\sigma_{st} = 18.53 \text{ MPa} \;\rightarrow\; \textit{answer}$$

From Equation (2)
$$\sigma_{br} = 0.3414(18.53)$$
$$\sigma_{br} = 6.33 \text{ MPa} \;\rightarrow\; \textit{answer}$$

Solution to Problem 257 Statically Indeterminate

Three bars AB, AC, and AD are pinned together as shown in Fig. P-257. Initially, the assembly is stress free. Horizontal movement of the joint at A is prevented by a short horizontal strut AE. Calculate the stress in each bar and the force in the strut AE when the assembly is used to support the load W = 10 kips. For each steel bar, A = 0.3 in.^{2} and E = 29 × 10^6 psi. For the aluminum bar, A = 0.6 in.^{2} and E = 10 × 10^6 psi.

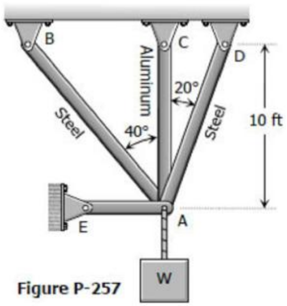


Figure P-257

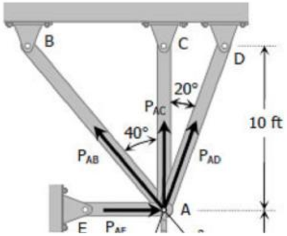
Solution 257

$$\cos 40^\circ = \frac{10}{L_{AB}};\ \ L_{AB} = 13.05\ \text{ft}$$

$$\cos 20^\circ = \frac{10}{L_{AD}};\ \ L_{AD} = 10.64\ \text{ft}$$

$$\Sigma F_V = 0$$
$$P_{AB}\cos 40^\circ + P_{AC} + P_{AD}\cos 20^\circ = 10(1000)$$
$$0.7660P_{AB} + P_{AC} + 0.9397P_{AD} = 10000 \rightarrow \textbf{Equation (1)}$$

$$\delta_{AB} = \cos 40^\circ \delta_{AC} = 0.7660\delta_{AC}$$
$$\left(\frac{PL}{AE}\right)_{AB} = 0.7660\left(\frac{PL}{AE}\right)_{AC}$$
$$\frac{P_{AB}(13.05)}{0.3(29\times 10^6)} = 0.7660\left[\frac{P_{AC}(10)}{0.6(10\times 10^6)}\right]$$
$$P_{AB} = 0.8511P_{AC} \rightarrow \textbf{Equation (2)}$$



This formula neglects the curvature of the spring. This is used for light spring where the ratio d/4R is small.

For heavy springs and considering the curvature of the spring, a more precise formula is given by: (A.M. Wahl Formula)

$$\tau = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

where m is called the **spring index** and (4m – 1)/(4m – 4) is the **Wahl Factor**.

The elongation of the bar is

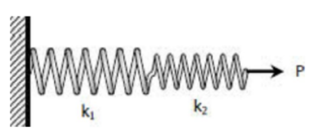
$$\delta = \frac{64PR^3 n}{Gd^4}$$

Notice that the deformation δ is directly proportional to the applied load P. The ratio of P to δ is called the **spring constant** k and is equal to

$$k = \frac{P}{\delta} = \frac{Gd^4}{64R^3 n} \text{ N/mm}$$

Springs in Series

For two or more springs with spring laid in series, the resulting spring constant k is given by

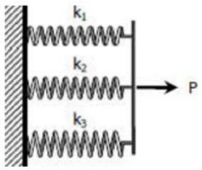


$$1/k = 1/k_1 + 1/k_2 + \ldots$$

where k_1, k_2,... are the spring constants for different springs.

Springs in Parallel

For two or more springs in parallel, the resulting spring constant is



$$k = k_1 + k_2 + \ldots$$

Solution to Problem 343 | Helical Springs

Determine the maximum shearing stress and elongation in a helical steel spring composed of 20 turns of 20-mm-diameter wire on a mean radius of 90 mm when the spring is supporting a load of 1.5 kN. Use Eq. (3-10) and G = 83 GPa.

Problem 343

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right) \rightarrow \text{Equation (3-10)}$$

Where:
P = 1.5 kN = 1500 N; R = 90 mm
d = 20 mm; n = 20 turns
m = 2R/d = 2(90)/20 = 9

$$\tau_{max} = \frac{16(1500)(90)}{\pi(20^3)}\left(\frac{4(9)-1}{4(9)-4} + \frac{0.615}{9}\right)$$
$\tau_{max} = 99.87\text{ MPa}$ ***answer***

$$\delta = \frac{64PR^3n}{Gd^4} = \frac{64(1500)(90^3)(20)}{83\,000(20^4)}$$
$\delta = 105.4\text{ mm}$ ***answer***

Solution to Problem 344 | Helical Springs

Determine the maximum shearing stress and elongation in a bronze helical spring composed of 20 turns of 1.0-in.-diameter wire on a mean radius of 4 in. when the spring is supporting a load of 500 lb. Use Eq. (3-10) and G = 6 × 10^6 psi.

Solution 344

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right) \rightarrow \text{Equation (3-10)}$$

Where
P = 500 lb; R = 4 in
d = 1 in; n = 20 turns
m = 2R/d = 2(4)/1 = 8

$$\tau_{max} = \frac{16(500)(4)}{\pi(1^3)}\left[\frac{4(8)-1}{4(9)-4} + \frac{0.615}{8}\right]$$
$\tau_{max} = 12\,060.3\text{ psi} = 12.1\text{ ksi}$ ***answer***

$$\delta = \frac{64PR^3n}{Gd^4} = \frac{64(500)(4^3)(20)}{(6\times10^6)(1^4)}$$
$\delta = 6.83\text{ in}$ ***answer***

Solution to Problem 345 | Helical Springs

A helical spring is fabricated by wrapping wire 3/4 in. in diameter around a forming cylinder 8 in. in diameter. Compute the number of turns required to permit an elongation of 4 in. without exceeding a shearing stress of 18 ksi. Use Eq. (3-9) and G = 12 × 10^6 psi.

Solution 345

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right) \rightarrow \text{Equation (3-9)}$$

$$18\,000 = \frac{16P(4)}{\pi(3/4^3)}\left[1 + \frac{3/4}{4(4)}\right]$$

$$P = 356.07 \text{ lb}$$

$$\delta = \frac{64PR^3n}{Gd^4}$$

$$4 = \frac{64(356.07)(4^3)n}{(12 \times 10^6)(3/4)^3}$$

$$n = 13.88 \text{ say 14 turns}$$ ***answer***

Solution to Problem 346 | Helical Springs

Compute the maximum shearing stress developed in a phosphor bronze spring having mean diameter of 200 mm and consisting of 24 turns of 200-mm diameter wire when the spring is stretched 100 mm. Use Eq. (3-10) and G = 42 GPa.

Solution 346

$$\delta = \frac{64PR^3n}{Gd^4}$$

Where
δ = 100 mm; R = 100 mm
d = 20 mm; n = 24 turns
G = 42 000 MPa

$$100 = \frac{64P(100^3)24}{42\,000(20^4)}$$
$$P = 437.5\text{ N}$$

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right) \rightarrow \text{Equation (3-10)}$$

Where
m = 2R/d = 2(100)/20 = 10

$$\tau_{max} = \frac{16(437.5)(100)}{\pi(20^3)}\left[\frac{4(10)-1}{4(10)-4} + \frac{0.615}{10}\right]$$
$$\tau_{max} = 31.89\text{ MPa} \quad \textbf{\textit{answer}}$$

Solution to Problem 347 | Helical Springs

Two steel springs arranged in series as shown in Fig. P-347 supports a load P. The upper spring has 12 turns of 25-mm-diameter wire on a mean radius of 100 mm. The lower spring consists of 10 turns of 20-mm diameter wire on a mean radius of 75 mm. If the maximum shearing stress in either spring must not exceed 200 MPa, compute the maximum value of P and the total elongation of the assembly. Use Eq. (3-10) and G = 83 GPa. Compute the equivalent spring constant by dividing the load by the total elongation.

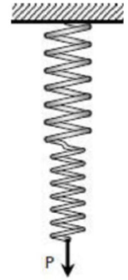
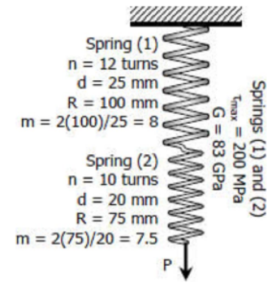


Figure P-347

Solution 347

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right) \rightarrow \text{Equation (3-10)}$$



For Spring (1)

$$200 = \frac{16P(100)}{\pi(25^3)}\left[\frac{4(8)-1}{4(8)-4} + \frac{0.615}{8}\right]$$

$$P = 5182.29\text{ N}$$

For Spring (2)

$$200 = \frac{16P(75)}{\pi(20^3)}\left[\frac{4(7.5)-1}{4(7.5)-4} + \frac{0.615}{7.5}\right]$$

$$P = 3498.28\text{ N}$$

Use $P = 3498.28\text{ N}$ ***answer***

Total elongation:

$$\delta = \delta_1 + \delta_2$$

$$\delta = \left(\frac{64PR^3n}{Gd^4}\right)_1 + \left(\frac{64PR^3n}{Gd^4}\right)_2$$

$$\delta = \frac{64(3498.28)(100^3)12}{83\,000(25^4)} + \frac{64(3498.28)(75^3)10}{83\,000(20^4)}$$

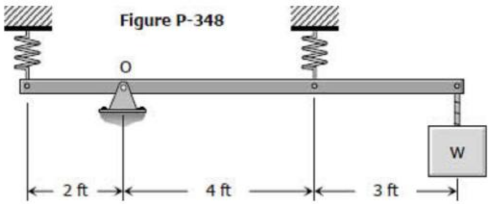
$\delta = 153.99\text{ mm}$ ***answer***

Equivalent spring constant, $k_{equivalent}$:

$$k_{equivalent} = \frac{P}{\delta} = \frac{3498.28}{153.99}$$
$$k_{equivalent} = 22.72 \text{ N/mm} \quad \textbf{\textit{answer}}$$

Solution to Problem 348 | Helical Springs

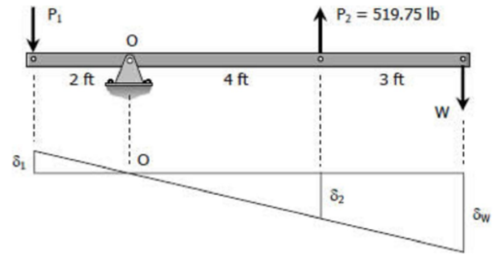
A rigid bar, pinned at O, is supported by two identical springs as shown in Fig. P-348. Each spring consists of 20 turns of 3/4-in-diameter wire having a mean diameter of 6 in. Determine the maximum load W that may be supported if the shearing stress in the springs is limited to 20 ksi. Use Eq. (3-9).



Solution 348

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right) \rightarrow \text{Equation (3-9)}$$
$$20\,000 = \frac{16P(3)}{\pi(3/4)^3}\left[1 + \frac{3/4}{4(3)}\right]$$
$$P = 519.75 \text{ lb}$$

For this problem, the critical spring is the one subjected to tension. Use $P_2 = 519.75$ lb.



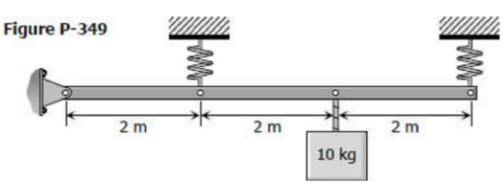
$$\frac{\delta_1}{2} = \frac{\delta_2}{4}$$
$$\delta_1 = \tfrac{1}{2}\delta_2$$
$$\frac{64P_1R^3n}{Gd^4} = \frac{1}{2}\left(\frac{64P_2R^3n}{Gd^4}\right)$$
$$P_1 = \tfrac{1}{2}P_2 = \tfrac{1}{2}(519.75)$$
$$P_1 = 259.875 \text{ lb}$$

$$\Sigma M_O = 0$$
$$7W = 2P_1 + 4P_2$$
$$7W = 2(259.875) + 4(519.75)$$
$$W = 371.25 \text{ lb} \quad \textit{answer}$$

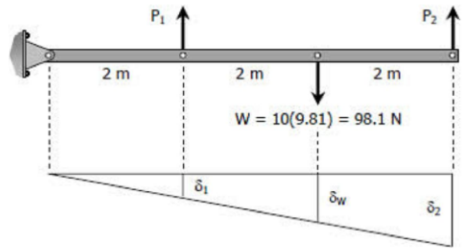
Solution to Problem 349 | Helical Springs

A rigid bar, hinged at one end, is supported by two identical springs as shown in Fig. P-349. Each spring consists of 20 turns of 10-mm wire having a mean diameter of 150 mm. Compute the maximum shearing stress in the springs, using Eq. (3-9). Neglect the mass of the rigid bar.

Figure P-349



Solution 349



$$\frac{\delta_1}{2} = \frac{\delta_2}{6}$$
$$\delta_1 = \tfrac{1}{3}\delta_2$$
$$\frac{64P_1R^3n}{Gd^4} = \frac{1}{3}\left(\frac{64P_2R^3n}{Gd^4}\right)$$
$$P_1 = \tfrac{1}{3}P_2$$

$$\Sigma M_{at\ hinged\ support} = 0$$
$$2P_1 + 6P_2 = 4(98.1)$$
$$2(\tfrac{1}{3}P_2) + 6P_2 = 4(98.1)$$
$$P_2 = 58.86\ \text{N}$$
$$P_1 = \tfrac{1}{3}(58.86) = 19.62\ \text{N}$$

$$\tau = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right) \rightarrow \text{Equation (3-9)}$$

For spring at left:

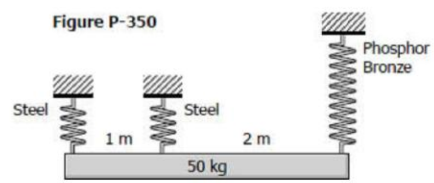
$$\tau_{max1} = \frac{16(19.62)(75)}{\pi(10^3)}\left[1 + \frac{10}{4(75)}\right]$$
$$\tau_{max1} = 7.744\ \text{MPa}\ \textit{answer}$$

For spring at right:

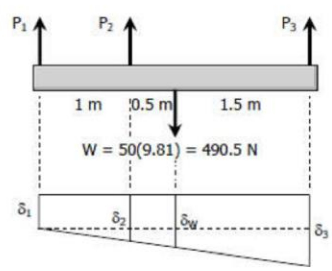
$$\tau_{max2} = \frac{16(58.86)(75)}{\pi(10^3)}\left[1 + \frac{10}{4(75)}\right]$$
$$\tau_{max2} = 23.232 \text{ MPa} \quad \textbf{\textit{answer}}$$

Solution to Problem 350 | Helical Springs

As shown in Fig. P-350, a homogeneous 50-kg rigid block is suspended by the three springs whose lower ends were originally at the same level. Each steel spring has 24 turns of 10-mm-diameter on a mean diameter of 100 mm, and G = 83 GPa. The bronze spring has 48 turns of 20-mm-diameter wire on a mean diameter of 150 mm, and G = 42 GPa. Compute the maximum shearing stress in each spring using Eq. (3-9).



Solution 350



$$\Sigma F_V = 0$$
$$P_1 + P_2 + P_3 = 490.5 \rightarrow \textbf{Equation (1)}$$

$$\Sigma M_1 = 0$$