Shear Stress

$$\tau = \frac{V}{A}$$

where V is the resultant shearing force which passes through the centroid of the area A being sheared.

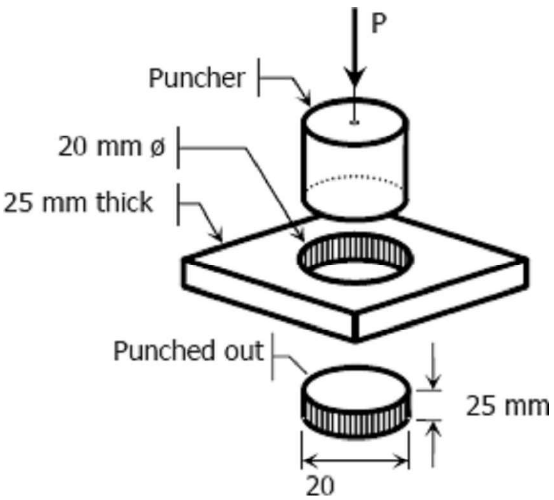
Solution to Problem 115 Shear Stress

Strength of Materials 4th Edition by Pytel and Singer

Given:
Required diameter of hole = 20 mm
Thickness of plate = 25 mm
Shear strength of plate = 350 MN/m^2

Required: Force required to punch a 20-mm-diameter hole

Solution 115



The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P.

$$V = \tau A$$
$$P = 350\,[\,\pi(20)(25)\,]$$
$$P = 549\,778.7\text{ N}$$
$$P = 549.8\text{ kN} \rightarrow \textit{answer}$$

Solution to Problem 116 Shear Stress

Strength of Materials 4th Edition by Pytel and Singer

Given:
Shear strength of plate = 40 ksi
Allowable compressive stress of punch = 50 ksi
The figure below:

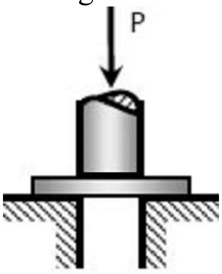


Figure 1-11c

Required:

a. Maximum thickness of plate to punch a 2.5 inches diameter hole
b. Diameter of smallest hole if the plate is 0.25 inch thick

Solution 116

a. **Maximum thickness of plate:**
Based on puncher strength:

$$P = \sigma A$$
$$P = 50\,[\,\tfrac{1}{4}\pi(2.52)\,]$$
$$P = 78.125\pi \text{ kips} \;\rightarrow \textit{Equivalent shear force of the plate}$$

Based on shear strength of plate:

$$V = \tau A \;\;\rightarrow\;\; V = P$$
$$78.125\pi = 40\,[\,\pi(2.5t)\,]$$
$$t = 0.781 \text{ inch} \;\rightarrow \textbf{\textit{answer}}$$

b. **Diameter of smallest hole:**
Based on compression of puncher:

$$P = \sigma A$$
$$P = 50(\tfrac{1}{4}\pi d^2)$$
$$P = 12.5\pi d^2 \;\rightarrow \textit{Equivalent shear force for plate}$$

Based on shearing of plate:

$$V = \tau A \;\;\rightarrow\;\; V = P$$
$$12.5\pi d^2 = 40[\,\pi d(0.25)\,]$$
$$d = 0.8 \text{ in} \;\rightarrow \textbf{\textit{answer}}$$

Solution to Problem 117 Shear Stress

Strength of Materials 4th Edition by Pytel and Singer

Given:
Force P = 400 kN
Shear strength of the bolt = 300 MPa

The figure below:

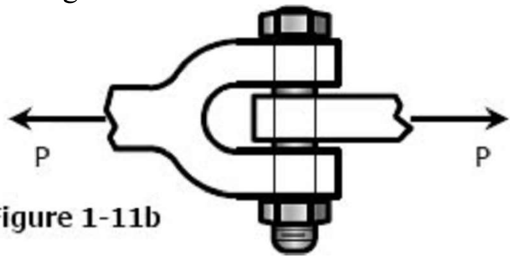


Figure 1-11b

Required: Diameter of the smallest bolt

Solution 117
The bolt is subject to double shear.

$$V = \tau A$$

$$400(1000) = 300[\,2(\tfrac{1}{4}\pi d^2)\,]$$

$$d = 29.13\text{ mm} \ \rightarrow \textit{answer}$$

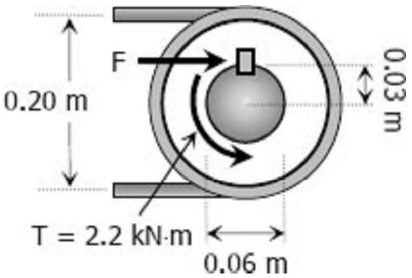
Solution to Problem 118 Shear Stress

Strength of Materials 4th Edition by Pytel and Singer

Given:
Diameter of pulley = 200 mm
Diameter of shaft = 60 mm
Length of key = 70 mm
Applied torque to the shaft = 2.5 kN · m
Allowable shearing stress in the key = 60 MPa

Required: Width b of the key

Solution 118

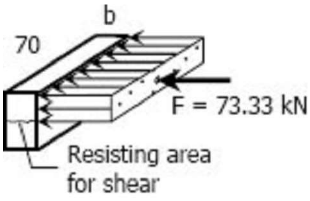


$$T = 0.03F$$

$$2.2 = 0.03F$$

$$F = 73.33\text{ kN}$$

$$V = \tau A$$



FBD of Pin

Where:

$$V = F = 73.33\text{ kN}$$
$$A = 70b$$
$$\tau = 60\text{ MPa}$$

$$73.33(1000) = 60(70b)$$
$$b = 17.46\text{ mm} \;\rightarrow \textbf{\textit{answer}}$$

Solution to Problem 119 Shear Stress

Strength of Materials 4th Edition by Pytel and Singer

Given:
Diameter of pin at B = 20 mm

Required: Shearing stress of the pin at B

Solution 119

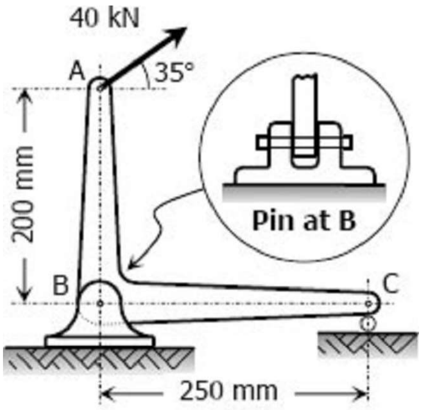


Figure P-119

From the FBD:

$$\Sigma M_C = 0$$
$$0.25R_{BV} = 0.25(40\sin 35°) + 0.2(40\cos 35°)$$
$$R_{BV} = 49.156\text{ kN}$$

$$\Sigma F_H = 0$$
$$R_{BH} = 40\cos 35°$$
$$R_{BH} = 32.766\text{ kN}$$

$$R_B = \sqrt{R_{BH}^2 + R_{BV}^2}$$
$$R_B = \sqrt{32.766^2 + 49.156^2}$$
$$R_B = 59.076\text{ kN} \ \rightarrow \text{shear force of pin at B}$$

$$V_B = \tau_B A \ \rightarrow \text{double shear}$$
$$59.076(1000) = \tau_B \left\{ 2 \left[\tfrac{1}{4}\pi(20^2) \right] \right\}$$
$$\tau_B = 94.02\text{ MPa} \ \rightarrow \textbf{\textit{answer}}$$

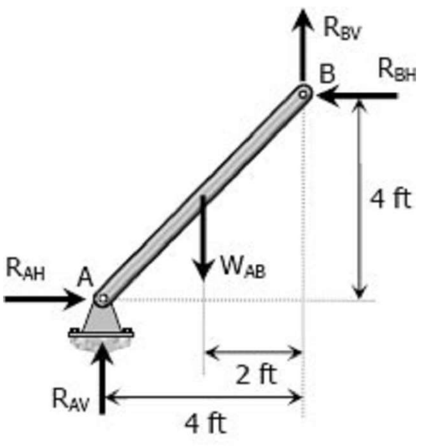
Solution to Problem 120 Shear Stress

Strength of Materials 4th Edition by Pytel and Singer

Given:
Unit weight of each member = 200 lb/ft
Maximum shearing stress for pin at A = 5 000 psi

Required: The smallest diameter pin that can be used at A

Solution 120
For member AB:



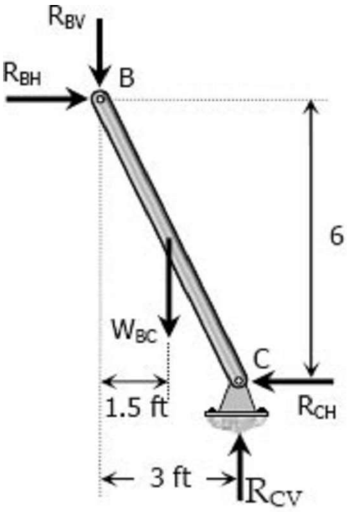
FBD of Member AB

Length, $L_{AB} = \sqrt{4^2 + 4^2} = 5.66\text{ ft}$
Weight, $W_{AB} = 5.66(200) = 1\ 132\text{ lb}$

$$\Sigma M_A = 0$$

$$4R_{BH} + 4R_{BV} = 2W_{AB}$$
$$4R_{BH} + 4R_{BV} = 2(1132)$$
$$R_{BH} + R_{BV} = 566 \rightarrow \text{Equation (1)}$$

For member BC:



FBD of member BC

Length, $L_{BC} = \sqrt{3^2 + 6^2} = 6.71 \text{ ft}$
Weight, $W_{BC} = 6.71(200) = W_{BC} = 1342 \text{ lb}$

$$\Sigma M_C = 0$$
$$6R_{BH} = 1.5W_{BC} + 3R_{BV}$$
$$6R_{BH} - 3R_{BV} = 1.5(1342)$$
$$2R_{BH} - R_{BV} = 671 \rightarrow \text{Equation (2)}$$

Add equations (1) and (2)

$$\begin{array}{ccccccl} R_{BH} & + & R_{BV} & = & 566 & \rightarrow & \text{Equation (1)} \\ R_{BH} & - & R_{BV} & = & 671 & \rightarrow & \text{Equation (2)} \\ 3R_{BH} & + & R_{BV} & = & 1237 & & \end{array}$$
$$R_{BH} = 412.33 \text{ lb}$$

From equation (1):

$$412.33 + R_{BV} = 566$$
$$R_{BV} = 153.67 \text{ lb}$$

From the FBD of member AB
$$\Sigma F_H = 0$$
$$R_{AH} = R_{BH} = 412.33 \text{ lb}$$

$$\Sigma F_V = 0$$
$$R_{AV} + R_{BV} = W_{AB}$$
$$R_{AV} + 153.67 = 1132$$
$$R_{AV} = 978.33 \text{ lb}$$

$$R_A = \sqrt{R_{AH}^2 + R_{AV}^2}$$
$$R_A = \sqrt{412.33^2 + 978.33^2}$$
$$R_A = 1061.67 \text{ lb} \;\rightarrow \text{shear force of pin at A}$$

$$V = \tau A$$
$$1061.67 = 5000(\tfrac{1}{4}\pi d^2)$$
$$d = 0.520 \text{ in} \;\rightarrow \textbf{\textit{answer}}$$

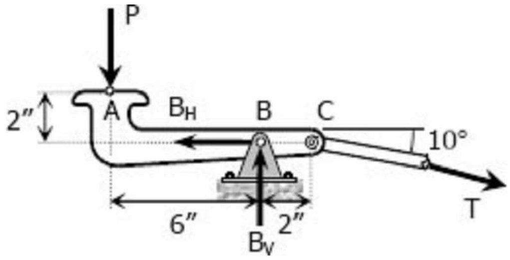
Solution to Problem 121 Shear Stress

Strength of Materials 4th Edition by Pytel and Singer

Given:
Allowable shearing stress in the pin at B = 4000 psi
Allowable axial stress in the control rod at C = 5000 psi
Diameter of the pin = 0.25 inch
Diameter of control rod = 0.5 inch
Pin at B is at single shear

Required: The maximum force P that can be applied by the operator

Solution 121



$$\Sigma M_B = 0$$
$$6P = 2T \sin 10° \quad \rightarrow \text{Equation (1)}$$

$$\Sigma F_H = 0$$
$$B_H = T cos 10°$$

From Equation (1), $T = \dfrac{3P}{\sin 10°}$

$$B_H = \left(\frac{3P}{\sin 10°}\right) \cos 10°$$
$$B_H = 3 \cot 10° \, P$$

$$\Sigma F_V = 0$$
$$B_V = T \sin 10° + P$$

From Equation (1), $T \sin 10° = 3P$

$$B_V = 3P + P$$
$$B_V = 4P$$

$$R_B^2 = B_H^2 + B_V^2$$
$$R_B^2 = (3 \cot 10° \, P)^2 + (4P)^2$$
$$R_B^2 = 305.47P^2$$
$$R_B = 17.48P$$
$$P = \frac{R_B}{17.48} \quad \rightarrow \text{Equation (2)}$$

Based on tension of rod (equation 1):

$$P = \tfrac{1}{3} T \sin 10°$$
$$P = \tfrac{1}{3}[\, 5000 \times \tfrac{1}{4}\pi(0.5)^2 \,] \sin 10°$$
$$P = 56.83 \text{ lb}$$

Based on shear of rivet (equation 2):

$$P = \frac{4000[\, \tfrac{1}{4}\pi(0.25)^2 \,]}{17.48}$$
$$P = 11.23 \text{ lb}$$

Safe load $P = 11.23$ lb $\rightarrow$ ***answer***

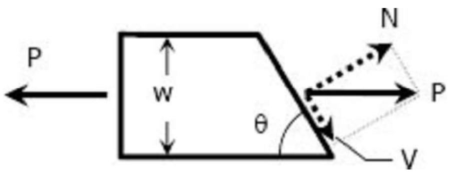
Solution to Problem 122 Shear Stress

Strength of Materials 4th Edition by Pytel and Singer

Given:
Width of wood = w
Thickness of wood = t
Angle of Inclination of glued joint = θ
Cross sectional area = A

Required: Show that shearing stress on glued joint $\tau = P\ \sin 2\theta \ /\ 2A$

Solution 122



Shear area, $A_{\text{shear}} = t(w \csc\theta)$
Shear area, $A_{\text{shear}} = tw \csc\theta$
Shear area, $A_{\text{shear}} = A\ \csc\theta$
Shear force, $V = P\ \cos\theta$

$$V = \tau A_{\text{shear}}$$
$$P\ \cos\theta = \tau(A \csc\theta)$$
$$\tau = \frac{P\ \sin\theta\ \cos\theta}{A}$$
$$\tau = \frac{P(2\sin\theta\ \cos\theta)}{2A}$$
$\tau = P\ \sin 2\theta \ /\ 2A$ (***ok!***)

Solution to Problem 123 Shear Stress

Strength of Materials 4th Edition by Pytel and Singer