Thermal Stress

Temperature changes cause the body to expand or contract. The amount δ_T, is given by

$$\delta_T = \alpha L\,(T_f - T_i) = \alpha L\,\Delta T$$

where α is the coefficient of thermal expansion in m/m°C, L is the length in meter, T_i and T_f are the initial and final temperatures, respectively in °C. For steel, $\alpha = 11.25 \times 10^{-6}$ m/m°C.

If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure. In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as **thermal stress**.

For a homogeneous rod mounted between unyielding supports as shown, the thermal stress is computed as:

deformation due to temperature changes;

$$\delta_T = \alpha L\,\Delta T$$

deformation due to equivalent axial stress;

$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$\delta_T = \delta_P$$

$$\alpha L\,\Delta T = \frac{\sigma L}{E}$$

$$\sigma = E\,\alpha\,\Delta T$$

where σ is the thermal stress in MPa, E is the modulus of elasticity of the rod in MPa.

If the wall yields a distance of x as shown, the following calculations will be made:

$\delta_T = x + \delta_P$

$$\alpha L \Delta T = x + \frac{\sigma L}{E}$$

where σ represents the thermal stress.

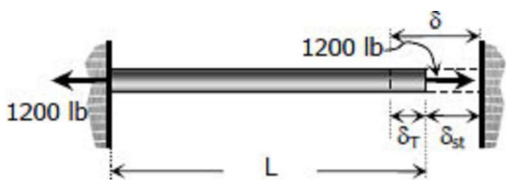
Take note that as the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension.

Solution to Problem 261 Thermal Stress

A steel rod with a cross-sectional area of 0.25 in^2 is stretched between two fixed points. The tensile load at 70°F is 1200 lb. What will be the stress at 0°F? At what temperature will the stress be zero? Assume α = 6.5 × 10^{-6} in/(in·°F) and E = 29 × 10^6 psi.

Solution 261

For the stress at 0°C:



$$\delta = \delta_T + \delta_{st}$$

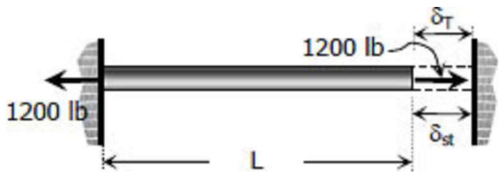
$$\frac{\sigma L}{E} = \alpha L (\Delta T) + \frac{PL}{AE}$$

$$\sigma = \alpha E (\Delta T) + \frac{P}{E}$$

$$\sigma = (6.5 \times 10^{-6})(29 \times 10^{6})(70) + \frac{1200}{0.25}$$

$$\sigma = 17\,995 \text{ psi} = 18 \text{ ksi} \quad \rightarrow \textbf{\textit{answer}}$$

For the temperature that causes zero stress:



$$\delta_T = \delta_{st}$$

$$\alpha L\,(\Delta T) = \frac{PL}{AE}$$

$$\alpha\,(\Delta T) = \frac{P}{AE}$$

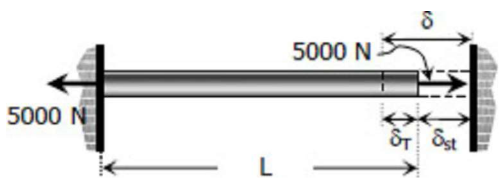
$$(6.5 \times 10^{-6})(T - 70) = \frac{1200}{0.25(29 \times 10^{6})}$$

$$T = 95.46\,°C \;\; \rightarrow \textit{\textbf{answer}}$$

Solution to Problem 262 Thermal Stress

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 µm/(m·°C) and E = 200 GPa.

Solution 262



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma L}{E} = \alpha L\,(\Delta T) + \frac{PL}{AE}$$

$$\sigma = \alpha E\,(\Delta T) + \frac{P}{E}$$

$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

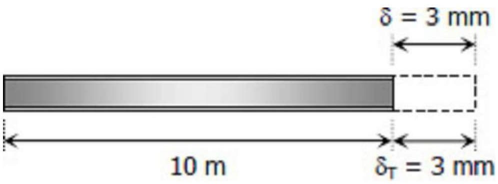
$$A = \frac{5000}{36.4}\; 137.36 \;\; \text{mm^2}$$

$\frac{1}{4}\pi d^2 = 137.36$
d$= 13.22\ \text{mm}$ $\rightarrow$ ***answer***

Solution to Problem 263 Thermal Stress

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume α = 11.7 µm/(m·°C) and E = 200 GPa.

Solution 263



Temperature at which $\delta_T = 3$ mm**:**

$$\delta_T = \alpha L\,(\Delta T)$$

$$\delta_T = \alpha L\,(T_f - T_i)$$

$$3 = (11.7 \times 10^{-6})(10\,000)(T_f - 15)$$

$T_f = 40.64°\text{C}$ $\rightarrow$ ***answer***

Required stress:

$$\delta = \delta_T$$

$$\frac{\sigma L}{E} = \alpha L\,(\Delta T)$$

$$\sigma = \alpha E\,(T_f - T_i)$$

$$\sigma = (11.7 \times 10^{-6})(200\,000)(40.64 - 15)$$

$\sigma = 60\ \text{MPa}$ $\rightarrow$ ***answer***

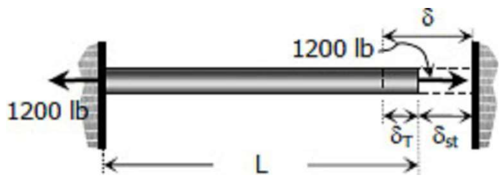
Solution to Problem 264 Thermal Stress

Problem 264

A steel rod 3 feet long with a cross-sectional area of 0.25 in.^{2} is stretched between two fixed points. The tensile force is 1200 lb at 40°F. Using E = 29 × 10^6 psi and α = 6.5 × 10^{-6} in./(in.·°F), calculate (a) the temperature at which the stress in the bar will be 10 ksi; and (b) the temperature at which the stress will be zero.

Solution 264

(a) Without temperature change:



$$\sigma = \frac{P}{A} = \frac{1200}{0.25} = 4800 \text{ psi}$$
$$\sigma = 4.8\,\text{ksi} < 10\,\text{ksi}$$

A drop of temperature is needed to increase the stress to 10 ksi. See figure above.

$$\delta = \delta_T + \delta_{st}$$
$$\frac{\sigma L}{E} = \alpha L(\Delta T) + \frac{PL}{AE}$$
$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$
$$10\,000 = (6.5 \times 10^{-6})(29 \times 10^6)(\Delta T) + \frac{1200}{0.25}$$
$$\Delta T = 27.59\text{°F}$$

Required temperature: *(temperature must drop from 40°F)*

$T = 40 - 27.59 = 12.41\text{°F}$ → ***answer***

(b) From the figure below:

$$\delta = \delta_T$$
$$\frac{PL}{AE} = \alpha L (\Delta T)$$
$$P = \alpha AE(T_f - T_i)$$
$$1200 = (6.5 \times 10^{-6})(0.25)(29 \times 10^6)(T_f - 40)$$
$$T_f = 65.46°\text{F} \rightarrow \textbf{\textit{answer}}$$

Solution to Problem 265 Thermal Stress

A bronze bar 3 m long with a cross sectional area of 320 mm^2 is placed between two rigid walls as shown in Fig. P-265. At a temperature of −20°C, the gap Δ = 25 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use α = 18.0 × 10^{-6} m/(m·°C) and E = 80 GPa.

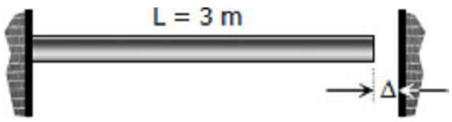
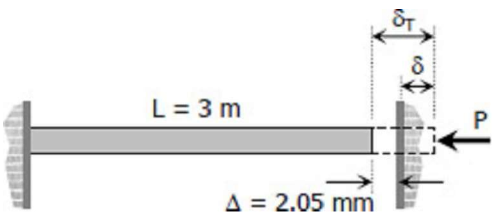


Figure P-265

Problem 265

$$\delta_T = \delta + \Delta$$



$$\alpha L (\Delta T) = \frac{\sigma L}{E} + 2.5$$
$$(18 \times 10^{-6})(3000)(\Delta T) = \frac{35(3000)}{80\,000} + 2.5$$
$$\Delta T = 70.6°\text{C}$$
$$T = 70.6 - 20$$
$$T = 50.6°\text{C} \rightarrow \textbf{\textit{answer}}$$

Solution to Problem 266 Thermal Stress

Calculate the increase in stress for each segment of the compound bar shown in Fig. P-266 if the temperature increases by 100°F. Assume that the supports are unyielding and that the bar is suitably braced against buckling.

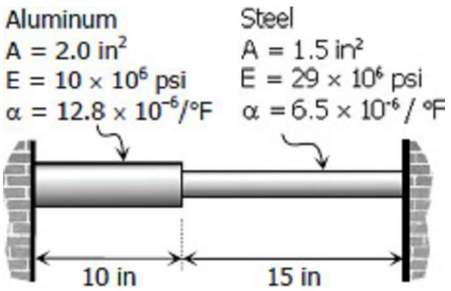
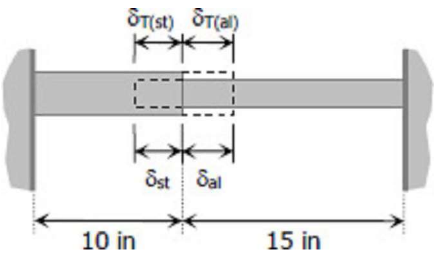


Figure P-266

Problem 266

$$\delta_T = \alpha L \, \Delta T$$



$$\delta_{T(st)} = (6.5 \times 10^{-6})(15)(100)$$
$$\delta_{T(st)} = 0.00975$$

$$\delta_{T(al)} = (12.8 \times 10^{-6})(10)(100)$$
$$\delta_{T(al)} = 0.0128 \text{ in}$$

$$\delta_{st} + \delta_{al} = \delta_{T(st)} + \delta_{T(al)}$$

$$\left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{al} = 0.00975 + 0.0128$$

where $P = P_{st} = P_{al}$

$$\frac{P(15)}{1.5(29 \times 10^6)} + \frac{P(10)}{2(10 \times 10^6)} = 0.02255$$

$$P = 26691.84 \text{ psi}$$

$$\sigma = \frac{P}{A}$$

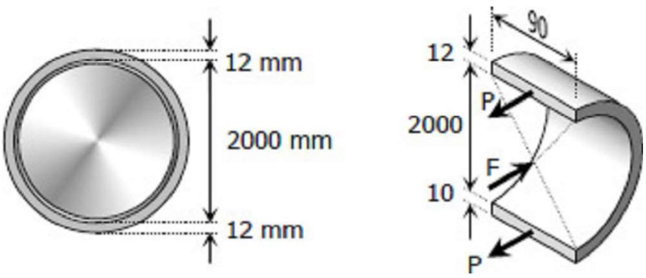
$$\sigma_{st} = \frac{26\,691.84}{1.5} = 17\,794.56 \text{ psi} \ \rightarrow \textbf{\textit{answer}}$$

$$\sigma_{st} = \frac{26\,691.84}{2.0} = 13\,345.92 \text{ psi} \ \rightarrow \textbf{\textit{answer}}$$

Solution to Problem 267 Thermal Stress

At a temperature of 80°C, a steel tire 12 mm thick and 90 mm wide that is to be shrunk onto a locomotive driving wheel 2 m in diameter just fits over the wheel, which is at a temperature of 25°C. Determine the contact pressure between the tire and wheel after the assembly cools to 25°C. Neglect the deformation of the wheel caused by the pressure of the tire. Assume α = 11.7 µm/(m·°C) and E = 200 GPa.

Solution 267



$$\delta = \delta_T$$

$$\frac{PL}{AE} = \alpha L \Delta T$$

$$P = \alpha \Delta T AE$$

$$P = (11.7 \times 10^{-6})(80–25)(90 \times 12)(200\,000)$$

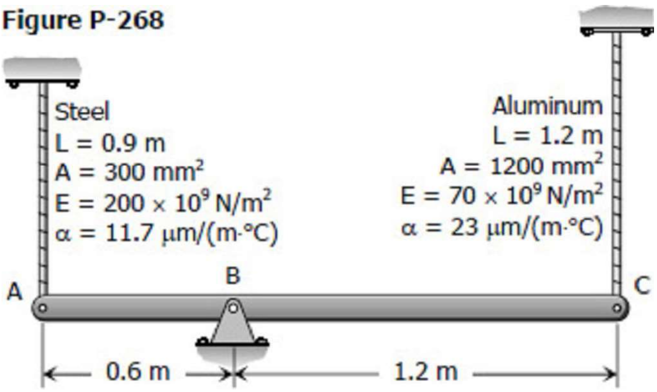
$P = 138\,996\,\text{N}$

$$F = 2P$$
$$pDL = 2P$$
$$p(2000)(90) = 2(138\,996)$$
$$p = 1.5444\,\text{MPa} \rightarrow \textbf{\textit{answer}}$$

Solution to Problem 268 Thermal Stress

The rigid bar ABC in Fig. P-268 is pinned at B and attached to the two vertical rods. Initially, the bar is horizontal and the vertical rods are stress-free. Determine the stress in the aluminum rod if the temperature of the steel rod is decreased by 40°C. Neglect the weight of bar ABC.

Figure P-268



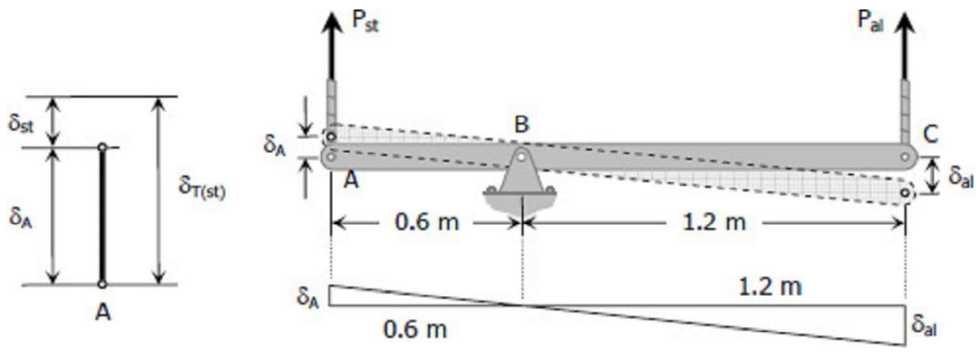
Solution 268

Contraction of steel rod, assuming complete freedom:

$$\delta_{T(st)} = \alpha L \Delta T$$
$$\delta_{T(st)} = (11.7 \times 10^{-6})(900)(40)$$
$$\delta_{T(st)} = 0.4212\,\text{mm}$$

The steel rod cannot freely contract because of the resistance of aluminum rod. The movement of A (referred to as δ_A), therefore, is less than 0.4212 mm. In terms of aluminum, this movement is (by ratio and proportion):

$$\frac{\delta_A}{0.6} = \delta_{al}1.2$$
$$\delta_A = 0.5\delta_{al}$$



$$\delta_{T(st)} – \delta_{st} = 0.5\delta_{al}$$
$$0.4212 – \left(\frac{PL}{AE}\right)_{st} = 0.5\left(\frac{PL}{AE}\right)_{al}$$
$$0.4212 – \frac{P_{st}(900)}{300(200\,000)} = 0.5\left[\frac{P_{al}(1200)}{1\,200(70\,000)}\right]$$
$28080 – P_{st} = 0.4762P_{al}$ $\rightarrow$ **Equation (1)**

$$\Sigma M_B = 0$$
$$0.6P_{st} = 1.2P_{al}$$
$P_{st} = 2P_{al}$ $\rightarrow$ **Equation (2)**

Equations (1) and (2)
$$28\,080 – 2P_{al} = 0.4762P_{al}$$
$$P_{al} = 11\,340\text{ N}$$

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{11\,340}{1200}$$
$\sigma_{al} = 9.45\ \text{MPa}$ $\rightarrow$ ***answer***

Solution to Problem 269 Thermal Stress

Problem 269

As shown in Fig. P-269, there is a gap between the aluminum bar and the rigid slab that is supported by two copper bars. At 10°C, $\Delta = 0.18$ mm. Neglecting the mass of the slab, calculate the stress in each rod when the temperature in the assembly is increased to 95°C. For each copper bar, $A = 500\ mm^2$, $E = 120$ GPa, and $\alpha = 16.8\ \mu m/(m\cdot°C)$. For the aluminum bar, $A = 400\ mm^2$, $E = 70$ GPa, and $\alpha = 23.1\ \mu m/(m\cdot°C)$.

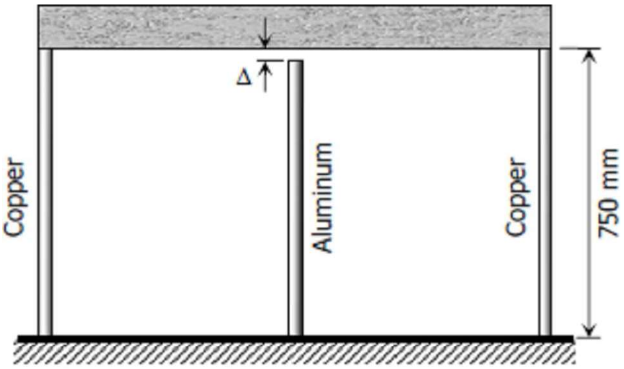


Figure P-269

Solution 269

Assuming complete freedom:

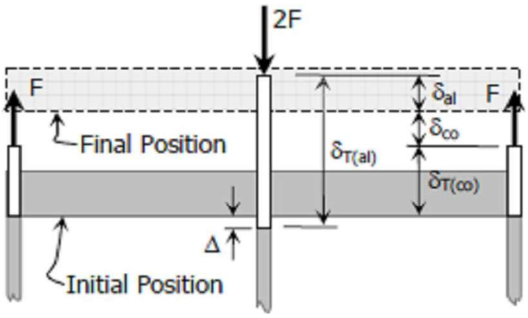
$$\delta_T = \alpha L \Delta T$$

$$\delta_{T(co)} = (16.8 \times 10^{-6})(750)(95–10)$$

$$\delta_{T(co)} = 1.071 \text{ mm}$$

$$\delta_{T(al)} = (23.1 \times 10^{-6})(750–0.18)(95–10)$$

$$\delta_{T(al)} = 1.472 \text{ mm}$$



From the figure:

$$\delta_{T(al)} – \delta_{al} = \delta_{T(co)} + \delta_{co}$$

$$1.472 – \left(\frac{PL}{AE}\right)_{al} = 1.071 + \left(\frac{PL}{AE}\right)_{co}$$

$$1.472 - \frac{2F(750 - 0.18)}{400(70\,000)} = 1.071 + \frac{F(750)}{500(120\,000)}$$

$$0.401 = (6.606 \times 10^{-5})\,F$$

$$F = 6070.37\ \text{N}$$

$$P_{co} = F = 6070.37\ \text{N}$$

$$P_{al} = 2F = 12\,140.74\ \text{N}$$

$$\sigma = P/A$$

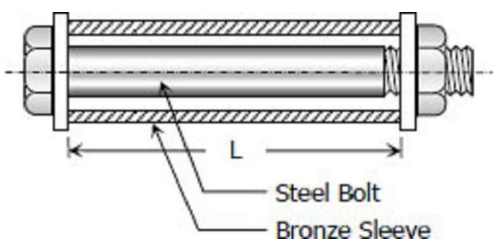
$$\sigma_{co} = \frac{6070.37}{500} = 12.14\ \text{MPa} \ \rightarrow \ \textit{\textbf{answer}}$$

$$\sigma_{al} = \frac{12\,140.74}{400} = 30.35\ \text{MPa} \ \rightarrow \ \textit{\textbf{answer}}$$

Solution to Problem 270 Thermal Stress

A bronze sleeve is slipped over a steel bolt and held in place by a nut that is turned to produce an initial stress of 2000 psi in the bronze. For the steel bolt, A = 0.75 in^2, E = 29 × 10^6 psi, and α = 6.5 × 10^{-6} in/(in·°F). For the bronze sleeve, A = 1.5 in^2, E = 12 × 10^6 psi and α = 10.5 × 10^{-6} in/(in·°F). After a temperature rise of 100°F, find the final stress in each material.

Solution 270

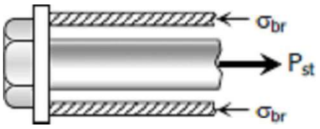


Before temperature change:

$$P_{br} = \sigma_{br}\, A_{br}$$

$$P_{br} = 2000(1.5)$$

$$P_{br} = 3000\ \text{lb} \text{ compression}$$



$\Sigma F_H = 0$
$P_{st} = P_{br} = 3000\text{ lb}$ tension
$\sigma_{st} = P_{st}/A_{st} = 3000/0.75$
$\sigma_{st} = 4000\text{ psi}$ tensile stress

$$\delta = \frac{\sigma L}{E}$$
$$a = \delta_{br} = \frac{2000L}{12 \times 10^6} = 1.67 \times 10^{-4}L \text{ shortening}$$
$$b = \delta_{st} = \frac{4000L}{29 \times 10^6} = 1.38 \times 10^{-4}L \text{ lengthening}$$

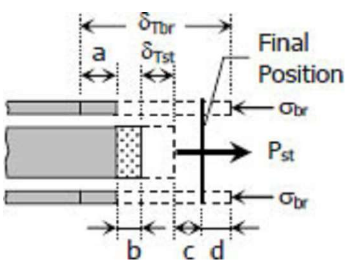
With temperature rise of 100°F: (*Assuming complete freedom*)
$\delta_T = \alpha L \Delta T$
$\delta_{Tbr} = (10.5 \times 10^{-6})L\,(100)$
$\delta_{Tbr} = 1.05 \times 10^{-3}L > a$

$\delta_{Tst} = (6.5 \times 10^{-6})L\,(100)$
$\delta_{Tst} = 6.5 \times 10^{-4}L$

$\delta_{Tbr} - a = 1.05 \times 10^{-3}L - 1.67 \times 10^{-4}L$
$\delta_{Tbr} - a = 8.83 \times 10^{-4}L$

$\delta_{Tst} + b = 6.5 \times 10^{-4}L + 1.38 \times 10^{-4}L$
$\delta_{Tst} + b = 7.88 \times 10^{-4}L$

$\delta_{Tbr} - a > \delta_{Tst} + b$ (see figure below)



$$\delta_{Tbr} – a – d = b + \delta_{Tst} + c$$

$$(1.05 \times 10^{-3}L) – (1.67 \times 10^{-4}L) – \left(\frac{\sigma L}{E}\right)_{br} = (1.38 \times 10^{-4}L) + (6.5 \times 10^{-4}L) + \left(\frac{P}{A}\right.$$

$$(8.83 \times 10^{-4}L) – \frac{\sigma_{br}L}{12 \times 10^6} = (7.88 \times 10^{-4}L) + \frac{P_{st}L}{0.75(29 \times 10^6)}$$

$$9.5 \times 10^{-4} – \frac{P_{br}}{1.5(12 \times 10^6)} = \frac{P_{st}}{0.75(29 \times 10^6)}$$

$$P_{st} = 20\,662.5 – 1.2083P_{br} \rightarrow \textbf{Equation (1)}$$

$$\Sigma F_H = 0$$

$$P_{br} = P_{st} \rightarrow \textbf{Equation (2)}$$

Equations (1) and (2)

$$P_{st} = 20\,662.5 – 1.2083P_{st}$$

$$P_{st} = 9356.74 \text{ lb}$$

$$P_{br} = 9356.74 \text{ lb}$$

$$\sigma = P/A$$

$$\sigma_{br} = \frac{9356.74}{1.5} = 6237.83 \text{ psi} \text{ compressive stress } \textbf{\textit{answer}}$$

$$\sigma_{st} = \frac{9356.74}{0.75} = 12475.66 \text{ psi} \text{ tensile stress } \textbf{\textit{answer}}$$

Solution to Problem 271 Thermal Stress

A rigid bar of negligible weight is supported as shown in Fig. P-271. If W = 80 kN, compute the temperature change that will cause the stress in the steel rod to be 55 MPa. Assume the coefficients of linear expansion are 11.7 µm/(m·°C) for steel and 18.9 µm/(m·°C) for bronze.