Course Outcome 2 (CO2)

Students should be able to understand and evaluate one-dimensional heat flow and in different geometries

Lesson Outcomes from CO2 (Part 3)

- To **derive** the equation for temperature distribution in various geometries
- Thermal Resistance concept – to **derive** expression for various geometries
- To **evaluate** the heat transfer using thermal resistance in various geometries
- To **evaluate** the critical radius of insulation
- To **evaluate** heat transfer from the rectangular fins

CRITICAL RADIUS OF INSULATION

Adding more insulation to a wall or to the attic results:

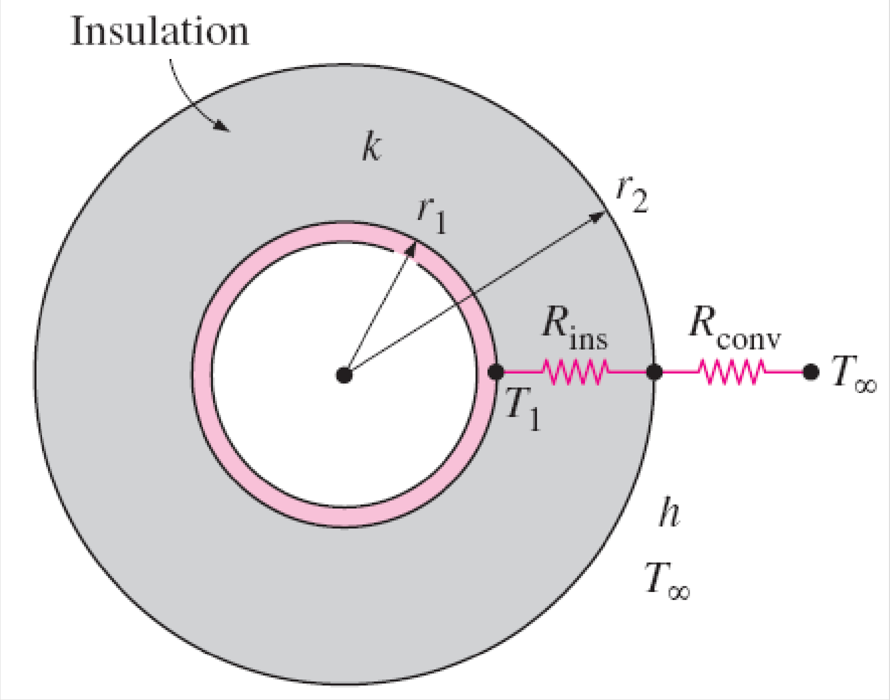
a) decrease heat transfer and,

b) increase the thermal resistance of the wall.

In a cylindrical pipe or a spherical shell, the additional insulation results :

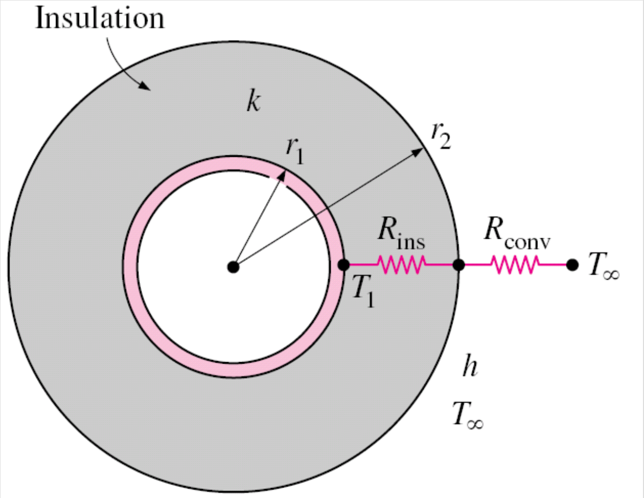
a) increase the conduction resistance but,

b) decreases the convection resistance because of the surface for convection become larger .



An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

- Cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant.
- The pipe is covered with an insulator (k and r_2).
- Convection heat transfer at T_∞ and h.
- The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as



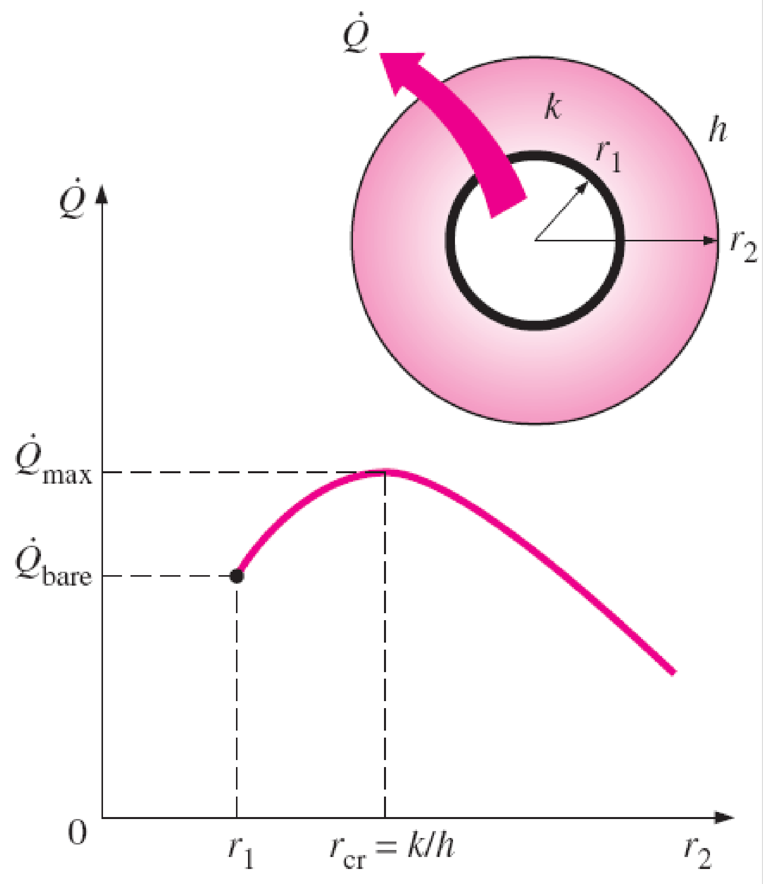
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2 / r_1)}{2\pi L k} + \dfrac{1}{h(2\pi r_2 L)}}$$

The critical radius of insulation for a cylindrical body:

$$r_{cr,\ cylinder} = \frac{k}{h} \qquad \text{(m)}$$

The critical radius of insulation for a spherical shell:

$$r_{cr,\ sphere} = \frac{2k}{h}$$



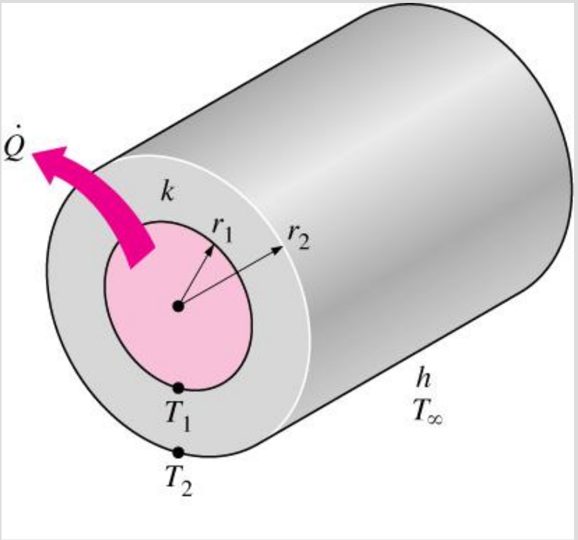
The variation of heat transfer rate with the outer radius of the insulation r_2 when $r_1 < r_{cr}$.

- The rate of heat transfer from the cylinder increases with the addition of insulation for $r_2 < r_{cr}$
- Reaches maximum, when $r_2 = r_{cr}$
- Decreases when $r_2 > r_{cr}$
- Thus, insulating the pipe may actually increase the rate of heat transfer instead of decreasing it when $r_2 < r_{cr}$.

Problem

A 3 mm diameter and 5 m long electrical wire is tightly wrapped with a 2 mm thick plastic cover whose thermal conductivity is *k = 0.15 W/m°C. Electrical measurements indicate that a current of 10A passes* through the wire and there is a voltage drop of 8V along the wire.

If the insulating wire is exposed to a medium at *T_∞ = 30°C with a heat transfer coefficient of h = 24 W/m^2 °C, find the temperature at the* interface of the wire and the plastic cover in steady state operation. Also, determine the effect of doubling the thickness of the plastic cover on the interface temperature.



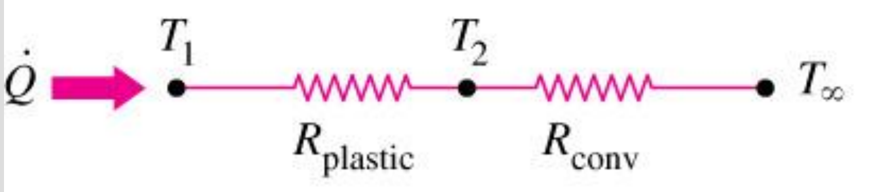
Solution

Assumptions:

1. Heat transfer is steady state.
2. Heat transfer is 1-D,
3. Thermal properties are constant.
4. The thermal contact resistance at the interface is negligible.
5. Heat transfer coefficient accounts for the radiation effects, if any.
6. Heat generation *is uniform*

Properties:

k= 0.15 W/m^{0}C, *h*= 12 W/m^2 ^{0}C



Analysis:

Rate of heat transfer from the wire is equal to the heat generated within the wire,

$$\dot{Q} = \dot{W}_e = VI = 8 \times 10 = 80 \text{ W}$$

- Outer surface Area

$$A = (2\pi r_2)L = (2\pi \times 0.0035) \times 5 = 0.11 \ \mathrm{m}^2$$

- Thermal resistances

$$R_{\mathrm{conv}} = \frac{1}{hA_2} = \frac{1}{12 \times 0.11} = 0.76 \ {}^0\mathrm{C/W}$$

$$R_{\mathrm{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi \times 0.15 \times 5} = 0.18 \ {}^0\mathrm{C/W}$$

$$\mathrm{R}_{\mathrm{total}} = R_{\mathrm{conv}} + R_{\mathrm{plastic}} = 0.76 + 018 = 0.94 \ {}^0\mathrm{C/W}$$

- The interface temperature can be determined

$$\dot{Q} = \frac{\mathrm{T}_1 - \mathrm{T}_\infty}{R_{\mathrm{total}}} \Rightarrow 80 = \frac{\mathrm{T}_1 - 30}{0.94} \Rightarrow \mathrm{T}_1 = 103 \ {}^0\mathrm{C}$$

- Critical radius

$$\mathrm{r}_{\mathrm{cr}} = \frac{k}{h} = \frac{0.15}{12} = 00125 \,\mathrm{m} = 12.5 \,\mathrm{mm}$$

- Doubling thickness?

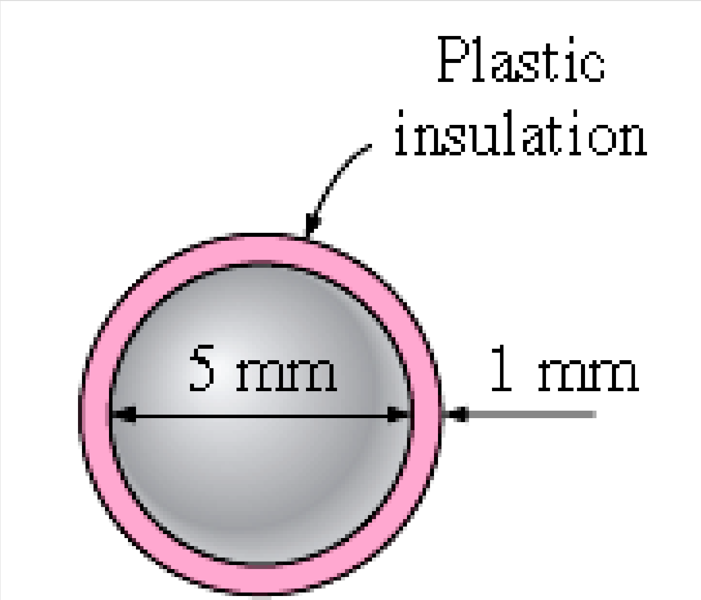
$\mathrm{r}_{2,\mathrm{new}} = 6$ mm, which is less than r_{cr}

Thus increasing the heat transfer until the outer radius reaches r_{cr}

Problem

A 5-mm-diameter spherical ball at 50 ^{0}C is covered by a 1-mm-thick plastic insulation (*k=0.13 W/m ·^{0}C). The ball is* exposed to a medium at 15 ^{0}C, with a combined convection and radiation heat transfer coefficient of 20 W/m^2·^{0}C.

Determine if the plastic insulation on the ball will help or hurt heat transfer from the ball.



Solution

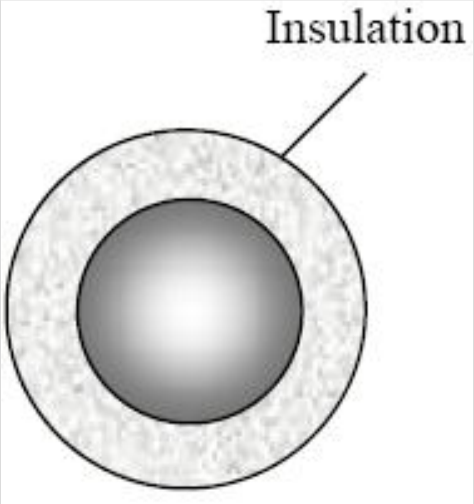
Assumptions : 1. steady state, 2. one-dimensional , 3. Thermal properties are constant. 4 The thermal contact resistance at the interface is negligible.

Properties: k = 0.13 W/m·°C.

Analysis: The critical radius of plastic insulation is

$$r_{cr} = \frac{2k}{h} = \frac{2 \times 0.13}{20} = 0.013\,\mathrm{m} = 13\,\mathrm{mm}$$

which is greater than r_2



Since the outer radius of the ball with insulation is smaller than critical radius of insulation, plastic insulation will increase heat transfer from the wire.

HEAT TRANSFER FROM FINNED SURFACES

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

Newton's law of cooling: The rate of heat transfer from a surface to the surrounding medium

When T_s and T_∞ are fixed, there are *two ways* to increase the rate of heat transfer:

- Increase the *convection heat transfer coefficient h.* This may require the installation of a pump or fan or add larger size of fan. Not adequate.
- Increase the *surface area A_s* by attaching to the surface *extended surfaces* called ***fins*** made of highly conductive materials such as aluminum.

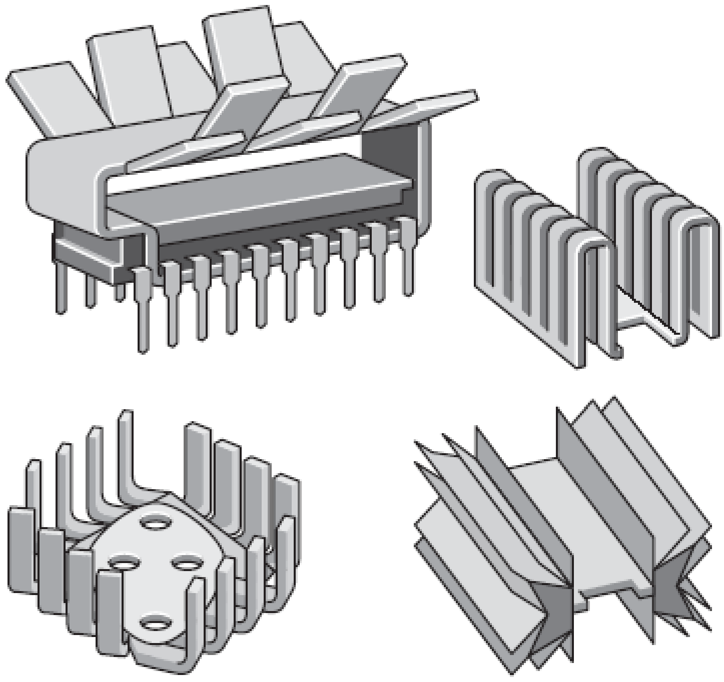
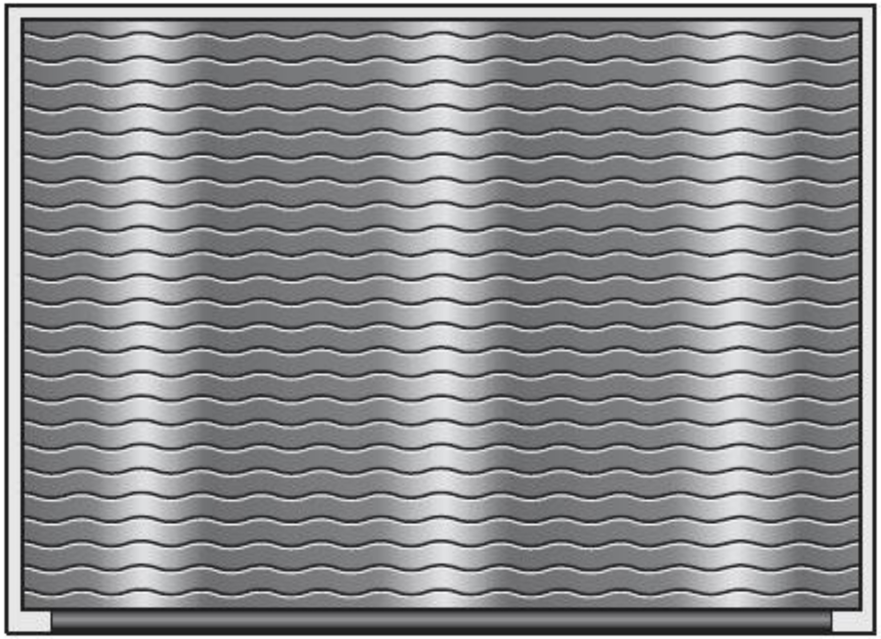


FIGURE 3–35
Some innovative fin designs.

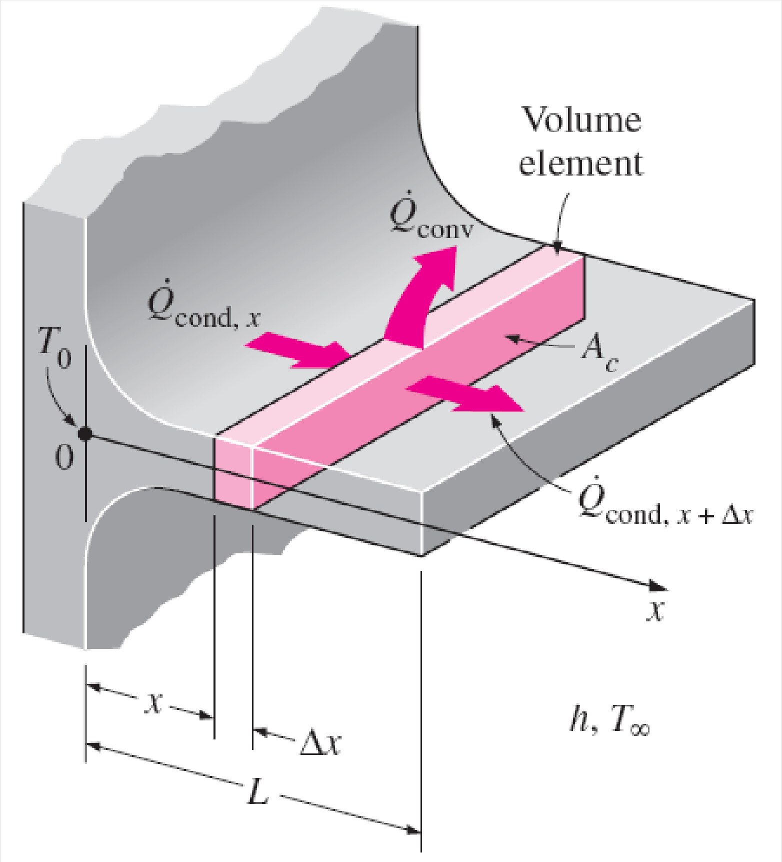
FIGURE 3–33
Presumed cooling fins on dinosaur stegosaurus. (© Alamy RF.)

The thin plate fins of a car radiator greatly increase the rate of heat transfer to the air.



Fin Equation

$$\begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x\end{pmatrix} = \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element}\end{pmatrix}$$



Volume element of a fin at location *x* having a length of Δ*x*, cross-sectional area of A_c, and perimeter of *p.*

$$\dot{Q}_{\text{cond, }x} = \dot{Q}_{\text{cond, }x+\Delta x} + \dot{Q}_{\text{conv}}$$

$$\dot{Q}_{\text{conv}} = h(p\,\Delta x)(T - T_\infty)$$

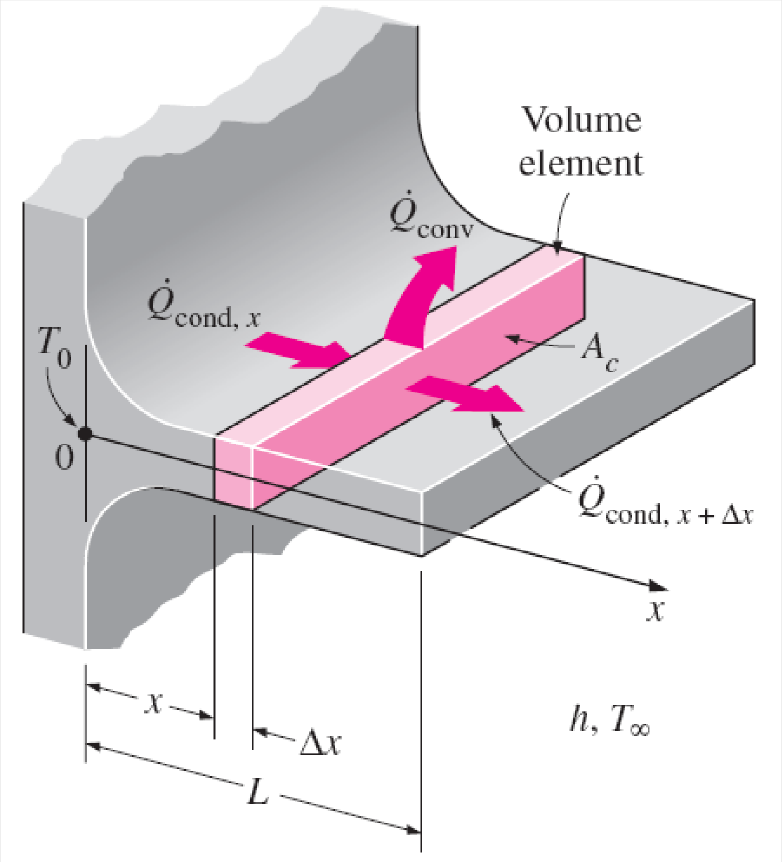
$$\frac{\dot{Q}_{\text{cond, }x+\Delta x} - \dot{Q}_{\text{cond, }x}}{\Delta x} + hp(T - T_\infty) = 0$$

$$\Delta x \to 0$$

$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0$$

Fin Equation

$$\begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x\end{pmatrix} = \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element}\end{pmatrix}$$



Volume element of a fin at location *x* having a length of Δx, cross-sectional area of A_c, and perimeter of *p*.

$$\dot{Q}_{cond} = -kA_c\frac{dT}{dx}$$

$$\frac{d}{dx}\left(kA_c\frac{dT}{dx}\right) - hp(T - T_\infty) = 0$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

where $m^2 = \dfrac{hp}{kA_c}$ and

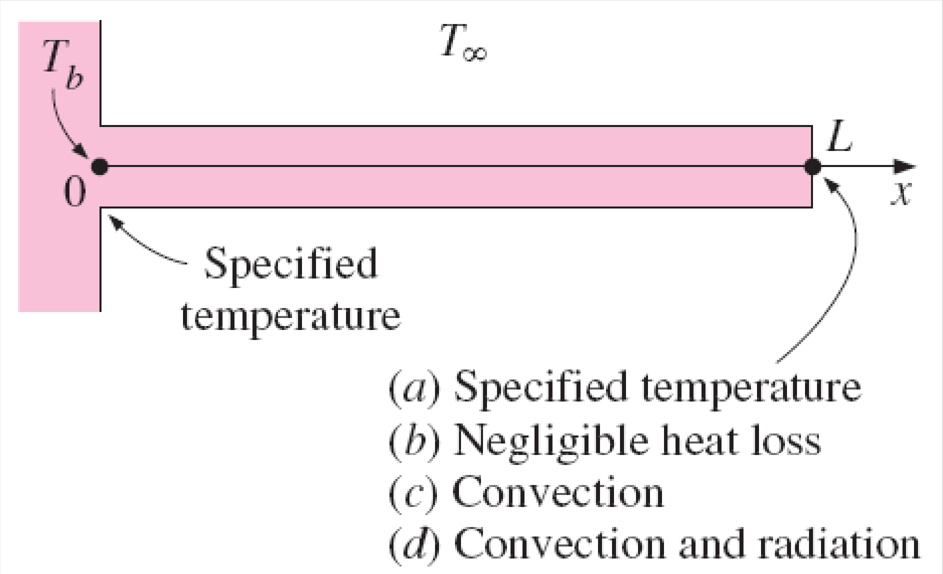
$\theta = T - T_\infty$ Temperature excess

The general solution of the differential equation

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary condition at fin base

$$\theta(0) = \theta_b = T_b - T_\infty$$

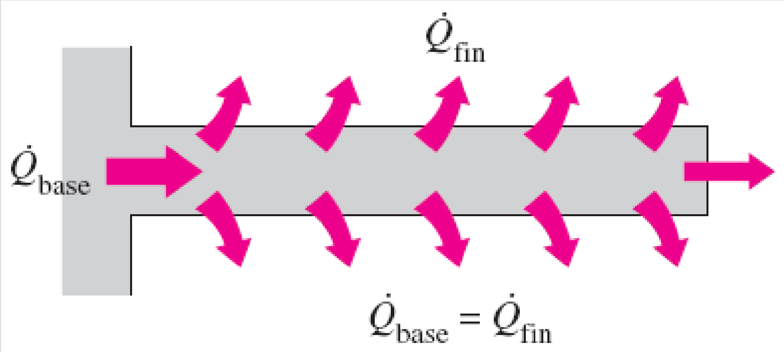


Boundary conditions at the fin base and the fin tip.

1 Infinitely Long Fin
($T_{\text{fin tip}} = T_\infty$)

Boundary condition at fin tip

$$\theta(L) = T(L) - T_\infty = 0 \qquad L \rightarrow \infty$$



Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

The variation of temperature along the fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}} \qquad m = \sqrt{hp/kA_c}$$

The steady rate of *heat transfer* from the entire fin

$$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$