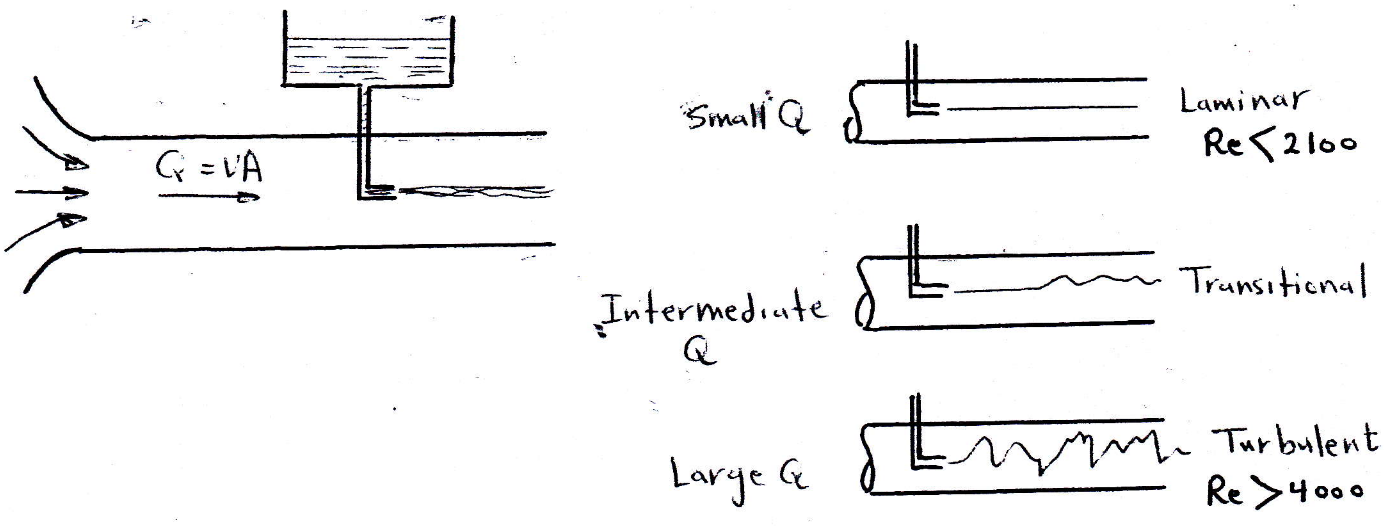
Chapter -5-

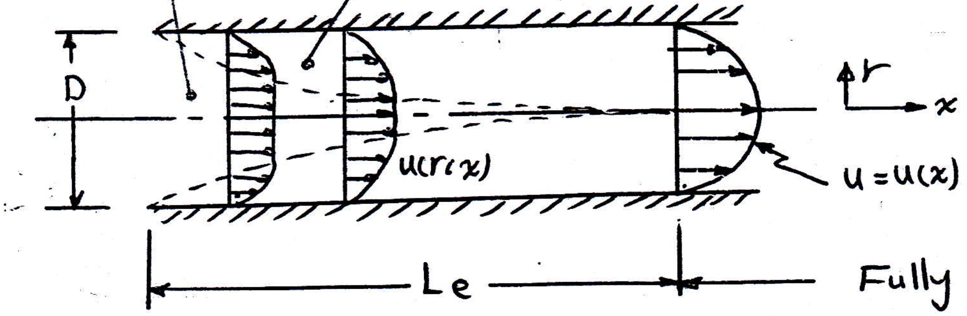
Flow in Pipes

5.1. Laminar and Turbulent Flows



Reynolds Experiment

5.2 Entrance Region and Fully Developed Flow



Fully developed flow $\frac{\partial u}{\partial x} = 0$

$\tau, \frac{\partial p}{\partial x} = \text{constant}$

$$\frac{L_e}{D} = 0.06\, Re_D \quad \text{-----} \quad (5.1) \quad \text{(Laminar)}$$

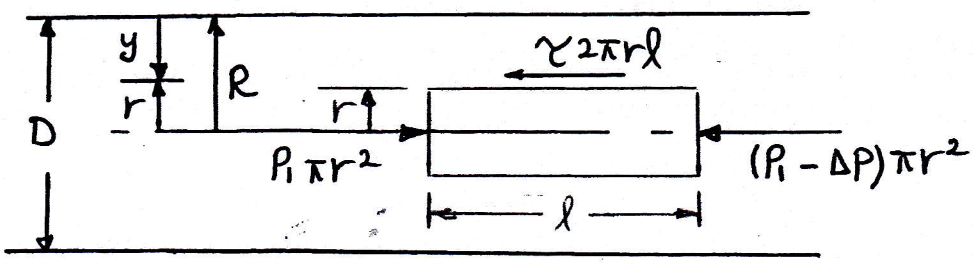
$$\frac{L_e}{D} = 4.4\, Re_D^{1/6} \quad \text{------} \quad (5.2) \quad \text{(Turbulent)}$$

5.3. Fully Developed Laminar Flow

$\Delta P = P_1 - P_2 \qquad \Delta P > 0$

$\sum F_x = \cancel{m a_x}$

Fully developed flow



$$\cancel{P_1}\cancel{\pi} r^2 - (\cancel{P_1} - \Delta P)\cancel{\pi} r^2 - \tau\, 2\cancel{\pi} r l = 0 \longrightarrow$$

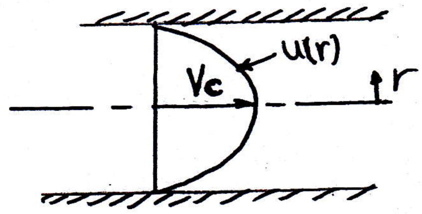
$$\boxed{\frac{\Delta P}{l} = \frac{2\tau}{r}} \quad \text{-------} \quad (5.3)$$

$$\therefore \frac{\Delta P}{l} = \underbrace{-\mu \frac{du}{dr}}_{\tau} \frac{2}{r} \quad \xrightarrow[\text{Integration}]{\text{By}} \quad \int du = -\frac{\Delta P}{2\mu l}\int r\,dr$$

$$u(r) = -\frac{\Delta P}{4\mu l} r^2 + C \qquad \text{at } r = \frac{D}{2} \quad u = 0 \longrightarrow C = \frac{\Delta P D^2}{16\mu l}$$

$$\therefore u(r) = \frac{\Delta P D^2}{16\mu l} - \frac{\Delta P}{4\mu l} r^2 = \frac{\Delta P D^2}{16\mu l}\left[1 - \frac{4r^2}{D^2}\right]$$

$$\therefore \boxed{u(r) = \frac{\Delta P D^2}{16\mu l}\left[1 - \left(\frac{r}{R}\right)^2\right]} \quad \text{------} \quad (5.4)$$



at $r = 0 \qquad u(r=0) = V_c$ Centerline Velocity $\rightarrow \boxed{V_c = \frac{\Delta P D^2}{16\mu l}}$ ---- (5.5)

$$\boxed{\therefore u(r) = V_c\left[1 - \left(\frac{r}{R}\right)^2\right]} \quad \text{------} \quad (5.6)$$

$$Q = \int_0^R u(r)\, dA = \int_0^R u(r)\, 2\pi r\, dr = 2\pi V_c \int_0^R \left(1 - \left(\frac{r}{R}\right)^2\right) r\, dr$$

$$\boxed{Q = \frac{\pi R^2 V_c}{2}} \quad \text{----} \quad (5.7)$$

$$Q = VA \longrightarrow V = \frac{Q}{\pi R^2} = \frac{\cancel{\pi R^2} V_c}{2}\,\frac{1}{\cancel{\pi R^2}}$$

$V = V_{avg}$

$$\therefore \boxed{V = \frac{V_c}{2}} \quad ----(5.8)$$

Sub. of equ.(5.5) into equ.(5.7), gives:-

$$\boxed{Q = \frac{\pi R^2}{2}\,\frac{\Delta P D^2}{16\mu l} = \frac{\pi D^4 \Delta P}{128\mu l}} \quad ----(5.9)$$

Poiseuille's Law

$$\frac{\Delta P}{\frac{1}{2}\rho V^2} = \frac{\frac{Q\,128\mu l}{\pi D^4}}{\frac{1}{2}\rho V^2} = \frac{\frac{V\,\frac{\cancel{\pi}}{4}\cancel{D^2}\,128\mu l}{\cancel{\pi} D^4}}{\frac{1}{2}\rho V^2} = \frac{(32\mu l V/D^2)}{\frac{1}{2}\rho V^2} = 64\left(\frac{\mu}{\rho V D}\right)\left(\frac{l}{D}\right)$$

$$\therefore \frac{\Delta P}{\frac{1}{2}\rho V^2} = \frac{64}{Re}\left(\frac{l}{D}\right)$$

$$\text{or} \quad \boxed{\Delta P = f\,\frac{l}{D}\,\frac{\rho V^2}{2}} \quad ----(5.10)$$

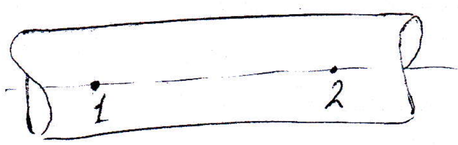
where

$$\boxed{f = \frac{64}{Re}} \quad ------(5.11)$$

Darcy friction factor $(Re < 2100)$

By applying the energy equation between section 1 and 2

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{Z_1} = \frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2} + h_L$$



$$h_L = \frac{P_1 - P_2}{\gamma} = \frac{\Delta P}{\gamma} \quad ----(5.12)$$

By equation (5.10)

$$\boxed{h_L = f\,\frac{l}{D}\,\frac{V^2}{2g}} \quad ------(5.13)$$

5.4. Turbulent Flow in Pipes

Friction factor

$f = f(V, D, \rho, \mu) \xrightarrow{\text{Laminar}} f = f(Re)$

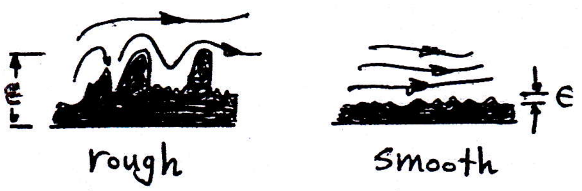
For turbulent flow

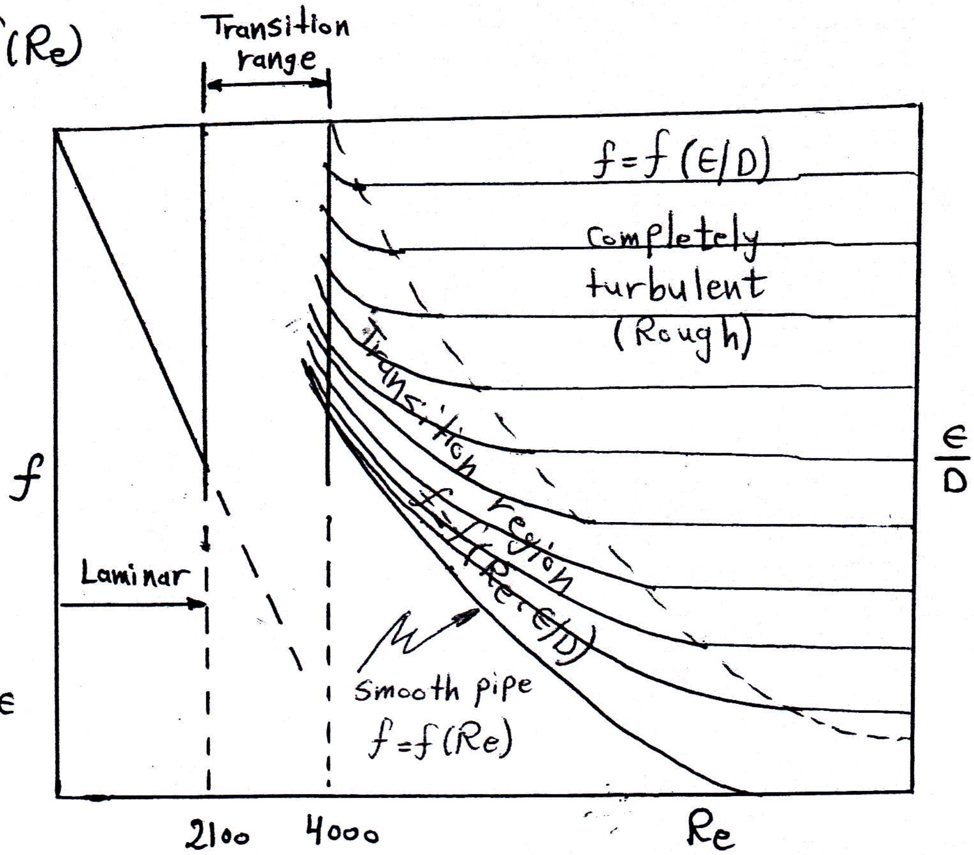
$f = f(V, D, \rho, \mu, \epsilon)$

ϵ: roughness of the wall

By dimensional analysis

$$\boxed{f = f\left(Re, \frac{\epsilon}{D}\right)} \quad ----(5.14)$$





5.5 Pipe Flow Problems

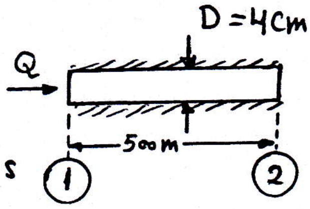
Problem	Known	Unknown
I	Q, D, ϵ	h_L or ΔP
II	D, h_L, ϵ	Q
III	Q, ϵ, h_L	D

<u>Example</u> Water at 20°C is transported for 500m in a 4cm diameter pipe ($\epsilon = 0.046$ mm) with a flowrate of 0.003 m^3/s. Find Pressure drop over the 500m length of pipe.

E.E between 1 and 2

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{Z_1} = \frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2} + h_L$$

$$P_1 - P_2 = \rho f \frac{L}{D} \frac{V^2}{2}$$



at 20°C $\nu = 10^{-6}$ m^2/s

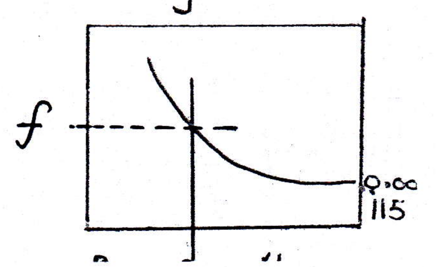
$V = Q/A = 0.003/\pi\, 0.02^2 = 2.39$ m/s

$$Re = \frac{VD}{\nu} = \frac{2.39 \times 0.04}{10^{-6}} = \underline{\underline{9.6 \times 10^4}}$$

$$\frac{\epsilon}{D} = \frac{0.046}{40} = \underline{\underline{0.00115}}$$

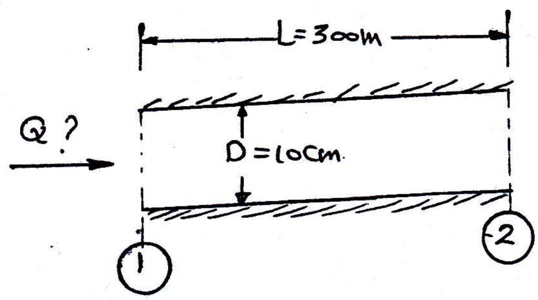
Moody

$f = 0.023$



$\therefore \quad P_1 - P_2 =$

Example. A pressure drop of 700 Kpa is measured over a 300 m length of horizontal 10 cm diamder pipe ($\epsilon = 0.046$ mm) that transport oil ($S = 0.9$, $\nu = 10^{-5}\ m^2/s$). Find the flowrate.



$\epsilon = 0.046$ mm

$\Delta P = 700$ kpa

Energy equation between 1 and 2

$$\frac{P_1 - P_2}{\gamma} = h_L$$

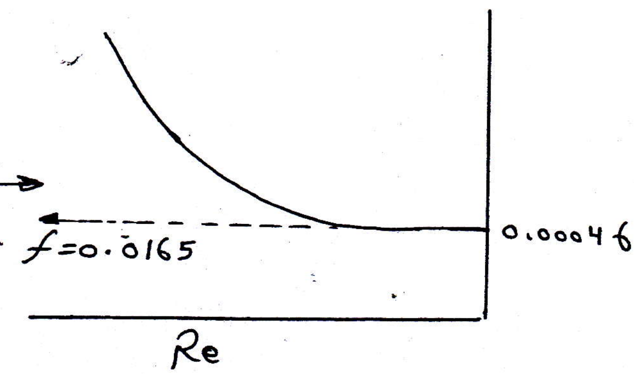
$$P_1 - P_2 = \gamma h_L$$

$$700 * 10^3 = \rho \cancel{g} f \frac{L}{D} \frac{V^2}{2\cancel{g}}$$

$$700 * 10^3 = 10^3 * 0.9 * f * \frac{300}{0.1} \frac{V^2}{2} \quad \text{------ (1)}$$

$$\frac{e}{D} = \frac{0.046}{100} = 0.00046$$

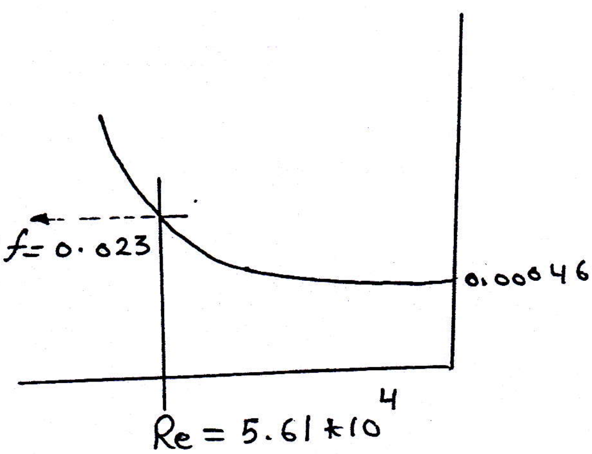
Assume rough pipe



$$\therefore 700 * 10^3 = 10^3 * 0.9 * 0.0165 \frac{300}{0.1} \frac{V^2}{2}$$

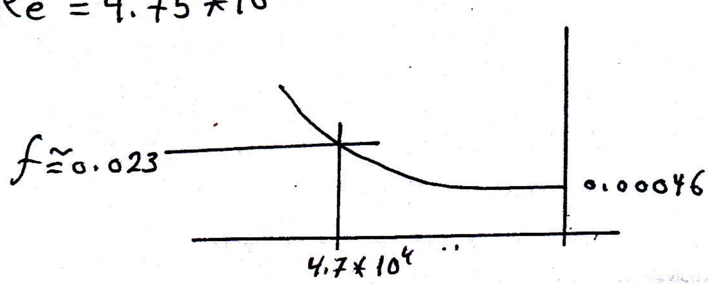
$V = 5.61$ m/s

$$\therefore Re = \frac{VD}{\nu} = \frac{5.61 * 0.1}{10^{-5}} = 5.61 * 10^4 \xrightarrow{\text{Moody}}$$



$\therefore$ From equation (1) $V = 4.75$ m/s

$Re = 4.75 * 10^4$

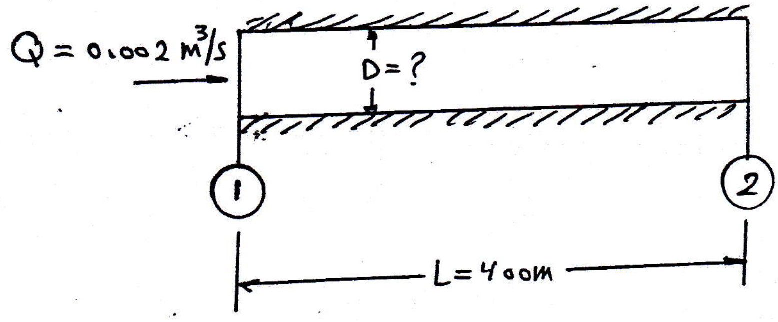


$\therefore V^* = 4.75$ m/s

$$Q = 4.75 * \pi * (0.05)^2 = 0.0373\ \frac{m^3}{s}$$

Example A tube is used to transport water at a rate of $0.002\ m^3/s$ at $20^\circ C$. What is the diameter of the tube so that the head loss not exceed $30\ m$? the length of tube is $L = 400\ m$, $\epsilon = 0.0015\ mm$.

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$



$$30 = f \frac{400}{D} \frac{V^2}{2g}$$

$$V = \frac{Q}{A} = \frac{4Q}{D^2 \pi}$$

$$\therefore 30 = f \frac{400}{D} \frac{1}{2g} \left(\frac{4Q}{D^2 \pi}\right)^2$$

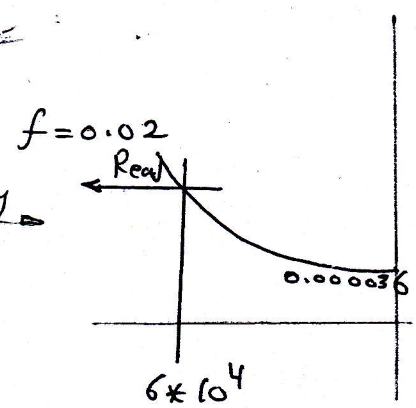
$$D^5 = 4.4 * 10^{-6} f \quad \text{-----} (1)$$

$\epsilon = 0.0015\ mm$

Assume $f = 0.03$

$D = 0.042\ m$

$$\left.\begin{aligned} Re &= \frac{VD}{\nu} = \frac{4Q}{\pi D \nu} = \frac{4 * 0.002}{\pi * 0.042 * 10^{-6}} = \underline{6.06 * 10^4} \\ \frac{\epsilon}{D} &= \frac{0.0015}{42} = \underline{0.000036} \end{aligned}\right\} \xrightarrow{\text{Moody}}$$

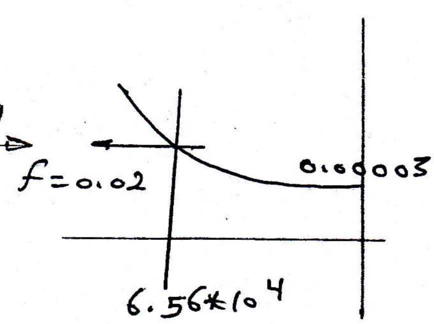


from equation(1)

$D = \underline{0.0388\ m}$

Again

$$\left.\begin{aligned} Re &= \frac{4Q}{\pi D \nu} = \frac{4 * 0.002}{\pi\, 0.0388 * 10^{-6}} = \underline{6.56 * 10^4} \\ \frac{\epsilon}{D} &= \frac{0.0015}{38.8} = \underline{0.000039} \end{aligned}\right\} \xrightarrow{\text{Moody}}$$



$f = 0.02$

$\therefore D^* = 0.0388\ m = 38.8\ mm$

5.6. Minor Losses

Losses due Components of pipe System (Valves, bends, ---) are termed **minor losses**.

$$\Delta P = K_L \frac{1}{2} \rho V^2$$

where K_L is the loss Coefficient.

or

$$\boxed{h_L = K_L \frac{V^2}{2g}} \quad \text{------- (5.15)}$$

$K_L = \phi$ (geometry)

Example Water drain from a pressurized tank through a pipe Syste shown in the figure. The head of the turbine is 116 m. If the entrance effects are negligible. Find the flow rate. $\rho = 999\ kg/m^3$

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + Z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{Z_2} + h_t + \underbrace{f \frac{L}{D} \frac{V^2}{2g}}_{\text{Major}} + \underbrace{\sum K_L \frac{V^2}{2g}}_{\text{minor}}$$

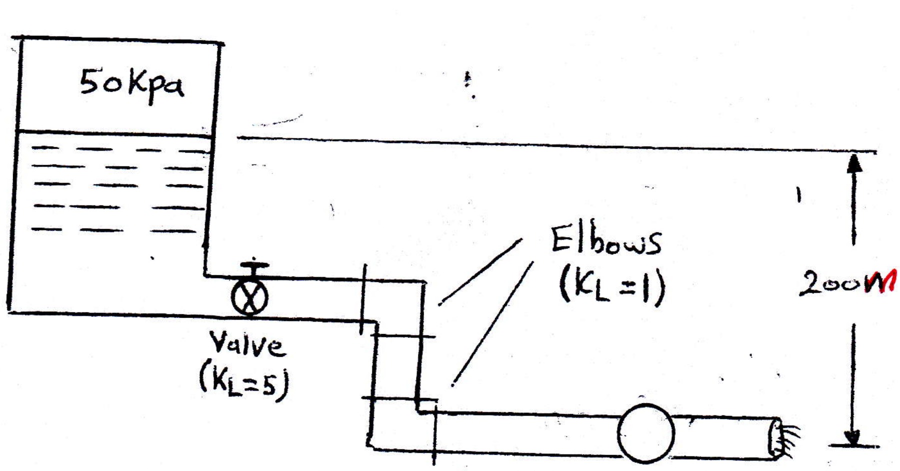
$$\frac{50 \ast 10^3}{9.81 \ast 999} + 200 = 116 + \frac{V^2}{2g}\left[1 + f\frac{L}{D} + \sum K_L\right]$$

$$748 = V^2\left[1 + 0.0356 \frac{200}{0.1} + 5 + 2\right]$$

$V = 4.698\ m/s$

$Q = VA = 4.698 \frac{\pi}{4}(0.1)^2$

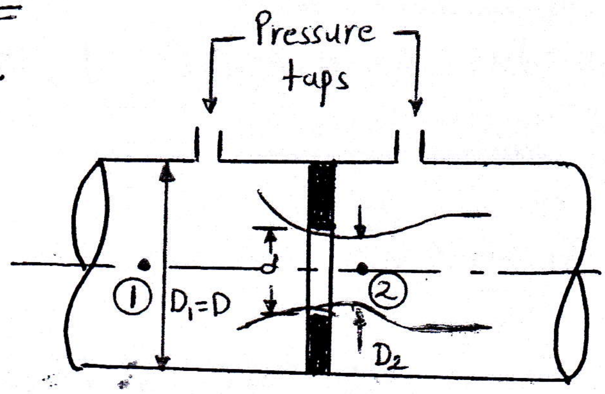
$= \underline{0.037\ m^3/s}$



L = 200 m
D = 0.1 m
$f = 0.0356$

5.7. Pipe Flowrate Measurement

orifice meter losses due to separation



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2} + h_L$$

assume $h_L = 0$

$$\frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_1^2}{2}$$

$$V_1 A_1 = V_2 A_2 \longrightarrow V_1 = \frac{A_2}{A_1} V_2$$

$$\therefore \frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_2^2 \left(\frac{A_2}{A_1}\right)^2}{2} \longrightarrow \frac{2(P_1 - P_2)}{\rho} = V_2^2 \left(1 - \left(\frac{A_2}{A_1}\right)^2\right)$$

$$\therefore V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \quad \text{where } \beta = \frac{D_2}{D_1} \qquad \text{Here } D_2 = d,\ D_1 = D$$

$$\therefore Q_{ideal} = V_2 A_2 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$

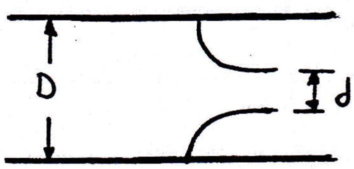
The swirl flow near the orifice plate introduce a head loss that Can not be calculated theoretically. Thus, an orifice <u>discharge Coefficient</u> <u>C_o</u> is used to take these effects into account.

$$\therefore C_o = \frac{Q}{Q_{ideal}} \longrightarrow Q = C_o Q_{ideal} = C_o A_o \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \qquad \text{------(5.16)}$$

where $A_o = \frac{\pi}{4} d^2$ (area of hole in plate), $\beta = d/D$. (C_o from figure 8.41

Nozzle meter

Secondary flow Separation is less than the orifice.



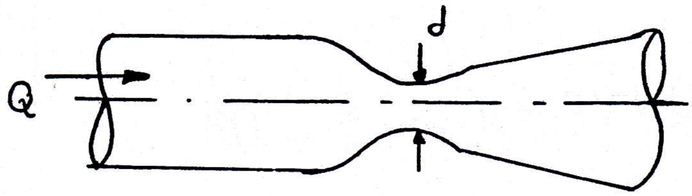
$$\boxed{Q = C_n A_n \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}} \qquad \text{----(5.17)}$$

C_n nozzle discharge coefficient (8.43)

$A_n = \frac{\pi}{4} d^2$, $\beta = d/D$ ($C_n > C_o$) (less

Venturi meter

Losses due to friction along the wall (No separation losses).



$$\boxed{Q = C_v Q_{ideal} = C_v A_T \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}} \qquad \text{---(5.18)}$$

$A_T = \frac{\pi}{4} d^2$, $\beta = d/D$, C_v Venturi discharge Coefficient