Ministry of Higher Education and Scientific Research
Tikrit University
Engineering Collage –Al shirqat

FUNDAMENTALS OF ELECTRICAL ENGINEERING

LECTURE 9

MAXIMUM POWER TRANSFER THEOREM

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PREPARED BY
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General Objectives:

➢ **Understanding the Scientific Basis of the Theory:**
- Defining the theory and its basic concept
- Clarifying its practical applications in electrical and electronic systems

➢ **Achieving High Efficiency in Energy Use:**
- Ensuring the maximum transfer of energy from the source to the load
- Improving the performance of electrical and electronic systems based on the principles of the theory

➢ **Enhancing Analytical Thinking and Engineering Skills:**
- Using mathematical models to explain the operation of electrical systems
- Analyzing electrical circuits to achieve the optimal state for energy transfer.

Specific objectives:

- **Mastering the Theoretical Conditions:**
  - Understanding that maximum power transfer occurs when the load resistance equals the internal resistance of the source.
  - Recognizing the cases where the theory applies and where it does not.
- **Practical Application of the Theory:**
  - Calculating the maximum power that can be transferred from the source to the load in various electrical circuits.
- **Training in Circuit Design:**
  - Designing electrical circuits that meet the conditions of the theory.
  - Selecting load and source components to achieve optimal efficiency.
- **Linking Theory to Practical Applications:**
  - Understanding the theory's applications in electrical devices (such as communication and power systems).
  - Analyzing and improving system efficiency in renewable energy applications.

Introduction

- **Maximum Power Transfer Theorem:** The Maximum Power Transfer Theorem is one of the most important theories in electrical circuit analysis. It addresses how to transfer the maximum possible amount of electrical power from a power source to a specific load.
- **Theory Concept:**

The theorem states that the maximum power transfer from a power source to a load occurs when the load resistance R_L is exactly equal to the internal resistance of the source R_S, or the equivalent resistance of the source if it consists of multiple resistances.

- **Practical Applications:**

- Designing communication systems to effectively transmit signals.
- Improving the efficiency of power transfer in electrical power systems.
- Designing electronic circuits to ensure devices operate at optimal efficiency.

Thevenin's Theorem

Solution method:

1. **Calculating the Source's Equivalent Resistance:**

- If you have a complex circuit containing a voltage source and other components (resistors, current sources), start by calculating the source's equivalent resistance. Follow these steps:
- **Eliminate the effect of voltage sources:** Replace internal voltage sources with a **short circuit**.
- **Eliminate the effect of current sources:** Replace internal current sources with an **open circuit**.
- **Calculate the equivalent resistance** between the points where the load will be connected.

Thevenin's Theorem

Solution method:

2. Matching the Resistance Between Source and Load:

Make the load resistance (R_L) equal to the source's equivalent resistance (R_{Th}). This adjustment ensures maximum power is transferred from the source to the load.

3. Calculating the Transferred Power:

The power transferred to the load can be calculated using the following equation:

$$P_{max} = \frac{{v_{Th}}^2}{4R_{Th}}$$

Where:

v_{Th}: Thevenin equivalent voltage.

$4R_{Th}$:Thevenin equivalent resistance

Maximum Power Transfer Theorem

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load (R_L=R_{Th}).

- For maximum power transfer ⇒ $\boldsymbol{R_L = R_{Th}}$

$$\boldsymbol{i} = \frac{v_{Th}}{R_L + R_{Th}}$$

$$P = i^2 R_L = \left(\frac{v_{Th}}{R_L + R_{Th}}\right)^2 . R_L$$

At maximum power transfer ⇒ $\boldsymbol{R_L = R_{Th}}$

$$P_{max} = \left(\frac{v_{Th}}{R_{Th} + R_{Th}}\right)^2 . R_{Th} = \left(\frac{v_{Th}}{2R_{Th}}\right)^2 . R_{Th}$$

$$P_{max} = \frac{{v_{Th}}^2}{4R_{Th}}$$

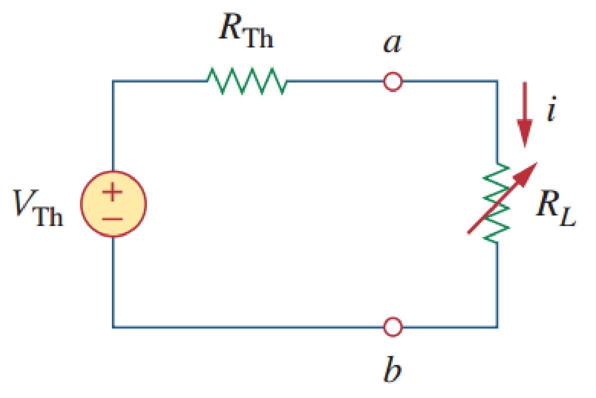


Figure 9-1 The circuit used for maximum power transfer.

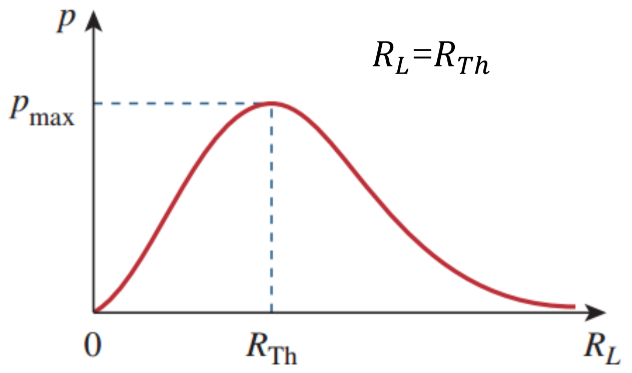


Figure 9-2 Power delivered to the load as a function of R_L

Maximum Power Transfer Theorem

$$P_{max} = \left(\frac{v_{Th}}{R_L + R_{Th}}\right)^2 \times R_L = \frac{(v_{Th})^2}{(R_L + R_{Th})^2} \times R_L$$

$$\frac{dP_L}{dR_L} = 0$$

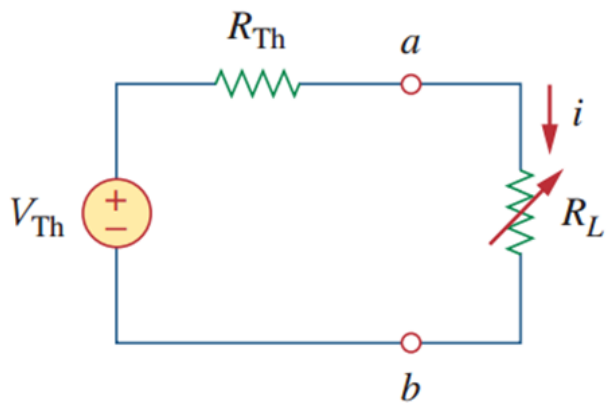
$$\frac{dP_L}{dR_L} = V_{th}^2 \frac{(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L)}{(R_{th} + R_L)^4}$$

$$(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L) = 0$$

$$R_{th}^2 + 2R_{th}R_L + R_L^2 - 2R_L R_{th} - 2R_L^2 = 0$$

$$R_{th}^2 - R_L^2 = 0$$

$$R_{th}^2 = R_L^2 \Rightarrow \boxed{R_{th} = R_L}$$



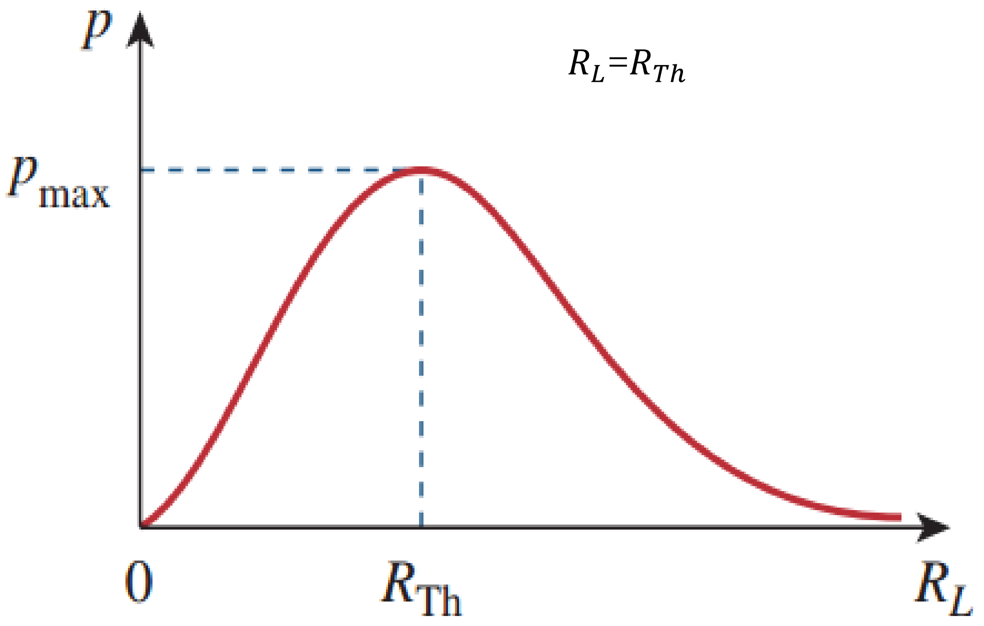
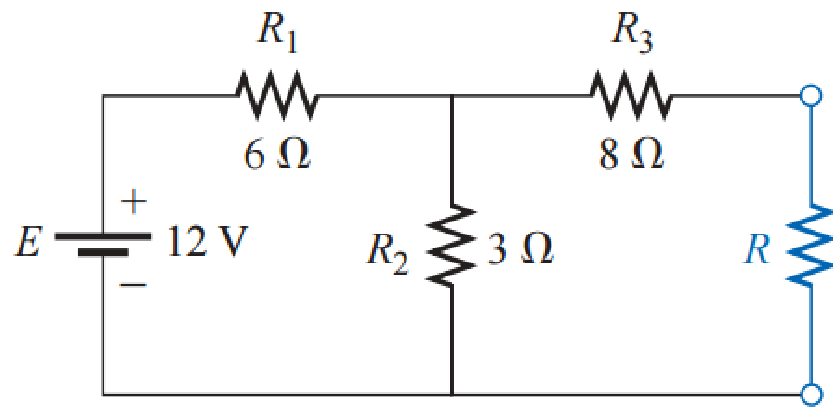


Figure 9-2 Power delivered to the load as a function of R_L

Maximum PowerTransfer Theorem

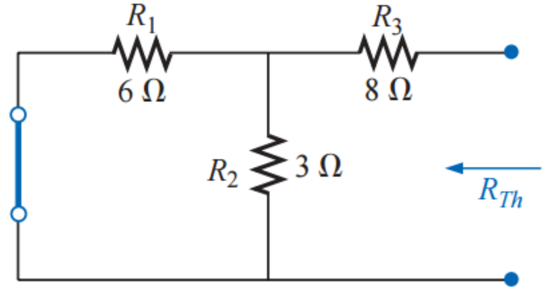
Example 1: For the circuit below, determine the value of R for maximum power to R, and calculate the power delivered under these conditions.



Solution:

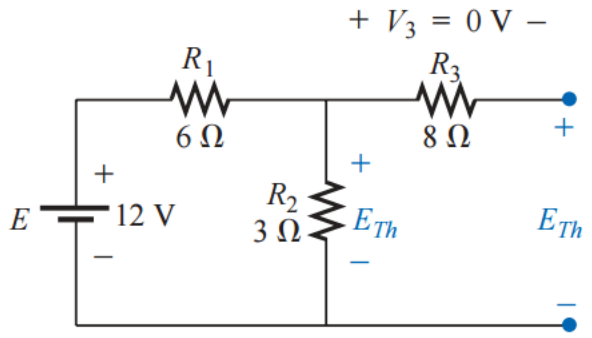
$$R_{Th} = R_3 + R_1 \parallel R_2 = 8\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 8\ \Omega + 2\ \Omega$$

$$R = R_{Th} = \mathbf{10\ \Omega}$$



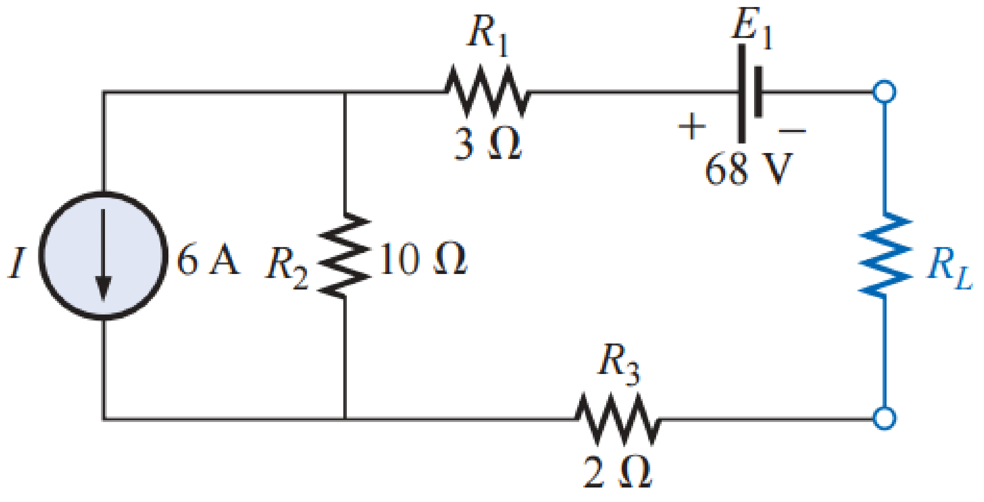
$$E_{Th} = \frac{R_2 E}{R_2 + R_1} = \frac{(3\ \Omega)(12\ \text{V})}{3\ \Omega + 6\ \Omega} = \frac{36\ \text{V}}{9} = \mathbf{4\ V}$$

$$P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(4\ \text{V})^2}{4(10\ \Omega)} = \mathbf{0.4\ W}$$



Maximum PowerTransfer Theorem

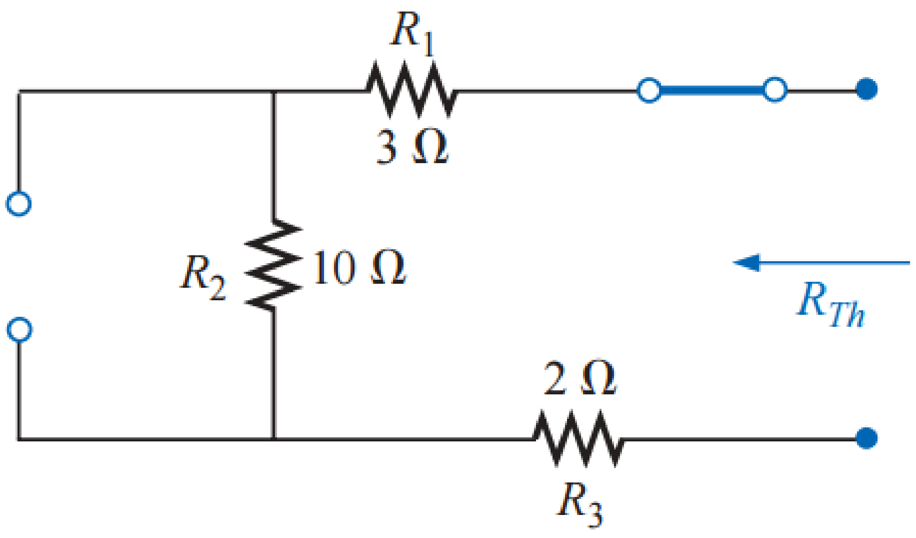
Example 2: Find the value of R_L in the figure below for maximum power to R_L , and determine the maximum power



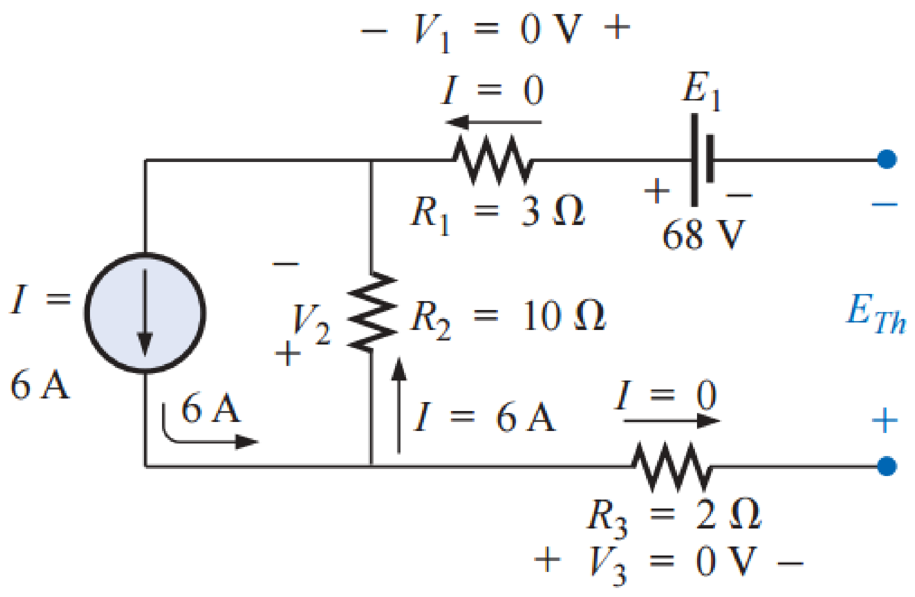
Solution:

$$R_{Th} = R_1 + R_2 + R_3 = 3\ \Omega + 10\ \Omega + 2\ \Omega = 15\ \Omega$$

$$R_L = R_{Th} = \mathbf{15\ \Omega}$$



Maximum PowerTransfer Theorem



$$V_1 = V_3 = 0\text{ V}$$

$$V_2 = I_2R_2 = IR_2 = (6\text{ A})(10\ \Omega) = 60\text{ V}$$

Applying Kirchhoff's voltage law,

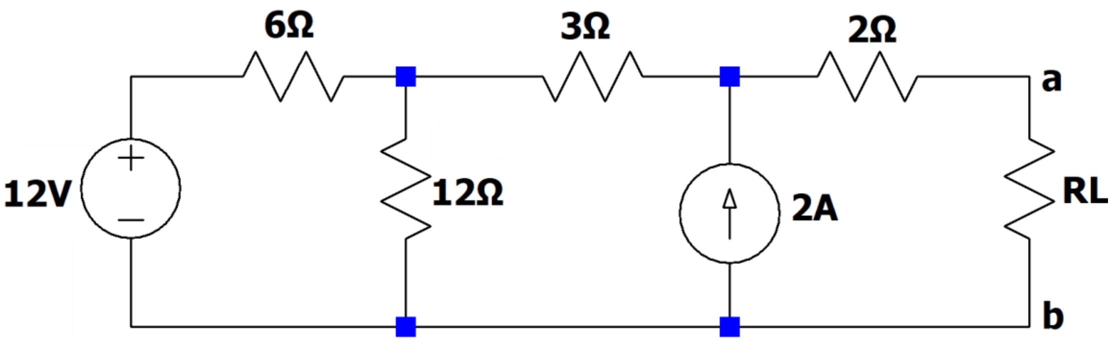
$$\Sigma_{\circlearrowright} V = -V_2 - E_1 + E_{Th} = 0$$

and $$E_{Th} = V_2 + E_1 = 60\text{ V} + 68\text{ V} = 128\text{ V}$$

Thus, $$P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\text{ V})^2}{4(15\ \Omega)} = \mathbf{273.07\ W}$$

Maximum PowerTransfer Theorem

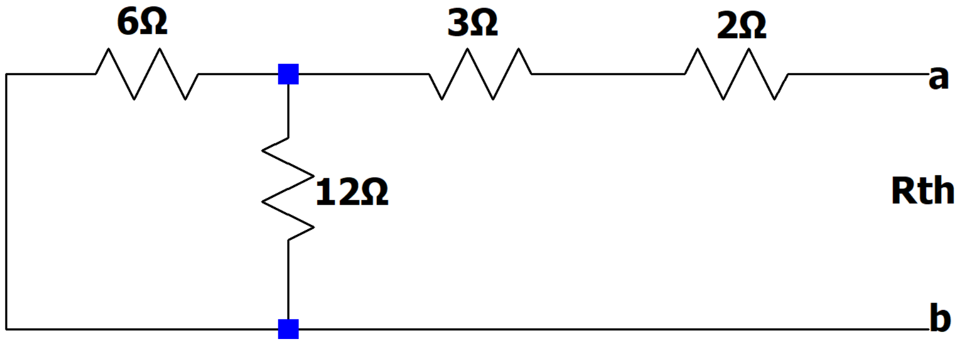
Example 3: Find the value of R_L for maximum power in the figure below, then find the maximum power



Solution:

Calculating the (R_{Th}):

$$R_{Th} = 2 + 3 + (6\|12) = 5 + (\frac{6\times12}{6+12}) = 9\Omega$$



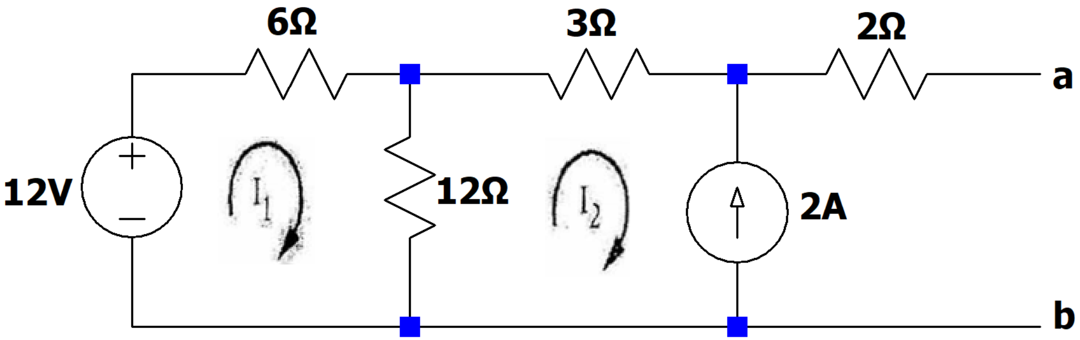
Calculating the (E_{Th}): Using Mesh

For loop 2:
$I_2 = -2$ A

For loop 1
$(6 + 12)I_1 - 12I_2 = 12$

$18I_1 - 12(-2) = 12 \Rightarrow 18I_1 + 24 = 12$

$$I_4 = \frac{-12}{18} = \frac{-2}{3}\text{ A}$$



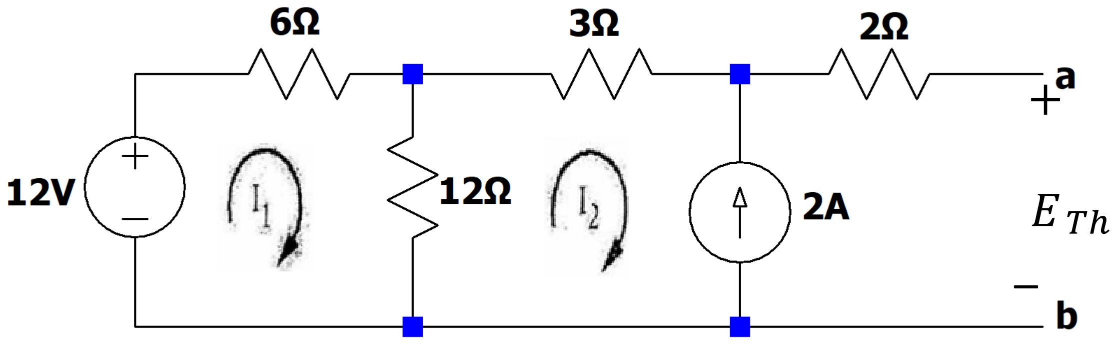
Maximum PowerTransfer Theorem

Applying KVL around the other loop:

$$12 - 6I_1 - 3I_2 - 2(0) - E_{Th} = 0$$

$$12 - 6\left(\frac{-2}{3}\right) - 3(-2) = E_{Th}$$

$$Eth = 22 \text{ V}$$



For maximum power transfer:

$$R_L = R_{Th} = 9\Omega$$

And maximum power is :

$$P_{max} = \frac{{v_{Th}}^2}{4R_{Th}}$$

$$P_{max} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$