Ministry of Higher Education and
Scientific Research
Tikrit University
Engineering Collage –Al shirqat

Fundamentals of Electrical engineering

Lecture 5

Source Transformation &Mesh and Nodal analysis

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General objectives of converting electrical sources (voltage source and current source)

- **Simplifying Circuit Analysis:** Converting sources makes it easier to analyze electrical circuits using methods such as Ohm’s Law and Kirchhoff’s Laws.
- **Enabling the Use of Different Analysis Methods:** This conversion allows for selecting the most suitable method for circuit analysis, such as Mesh Analysis or Nodal Analysis.
- **Flexibility in Design and Practical Applications:** In some cases, using a current source or a voltage source may be more efficient depending on the circuit’s requirements.
- **Applying Various Theorems:** Such as Thevenin’s Theorem and Norton’s Theorem, where converting sources is sometimes necessary to obtain an equivalent circuit model.

General objectives of super mesh (maxwell) theory

- **Reducing the Number of Equations in Circuit Analysis:** The Super mesh method helps minimize the number of equations required when analyzing electrical circuits, making the solution process easier.
- **Handling Interdependent Currents in Circuits:** This theory is used when there is a current source shared between two meshes, allowing them to be merged into a single equation.
- **Enhancing Analysis Efficiency:** By using Supermesh, large and complex circuits can be simplified, saving time and effort.
- **Applicable to Multi-Source Circuits:** This theory helps efficiently analyze circuits containing multiple current or voltage sources.

Specific objectives:

- **Conversion of Electrical Sources (Voltage Source and Current Source)**

- Understand the relationship between voltage sources and current sources and how to convert between them using internal resistance.
- Simplify electrical circuits by converting between the two sources to facilitate circuit analysis.
- Use source conversion in network analysis to easily calculate currents and voltages in the circuit.
- Explain the concept of Thevenin resistance and Norton resistance as part of electrical analysis.

- **Supermesh Theory**

- Apply Supermesh Theory to analyze electrical circuits containing current sources.
- Simplify electrical equations when a shared current source exists between two mesh loops.
- Improve calculation efficiency and reduce the number of equations needed for circuit analysis.
- Understand the relationship between Kirchhoff's Laws (KVL and KCL) and Supermesh Theory to determine currents in complex circuits.

Source Transformation

A **source transformation** is the process of replacing a voltage source $\boldsymbol{v_s}$ in series with a resistor **R** by a current source $\boldsymbol{i_s}$ in parallel with a resistor **R**, or vice versa.

$$\boxed{v_s = i_s R}\quad \text{or} \quad \boxed{i_s = \frac{v_s}{R}}$$

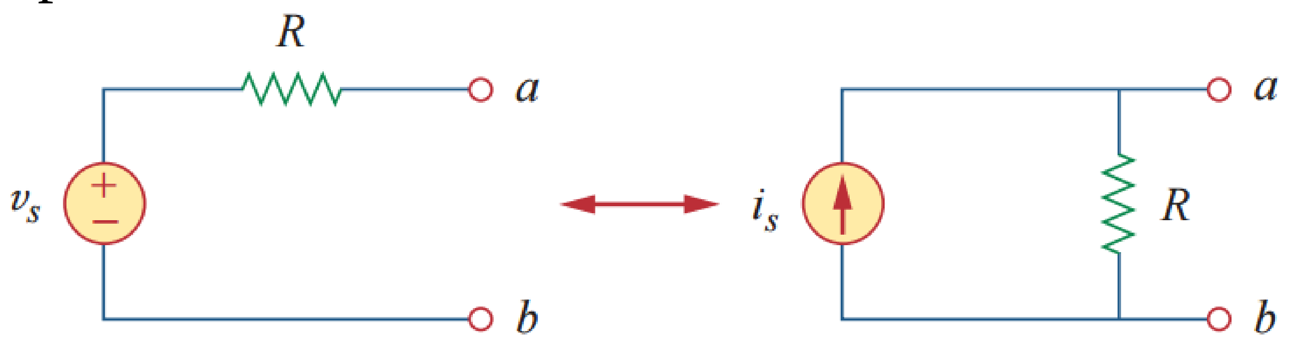


Figure 3-11 Transformation of independent sources.

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable. As shown in Fig. 3-12.

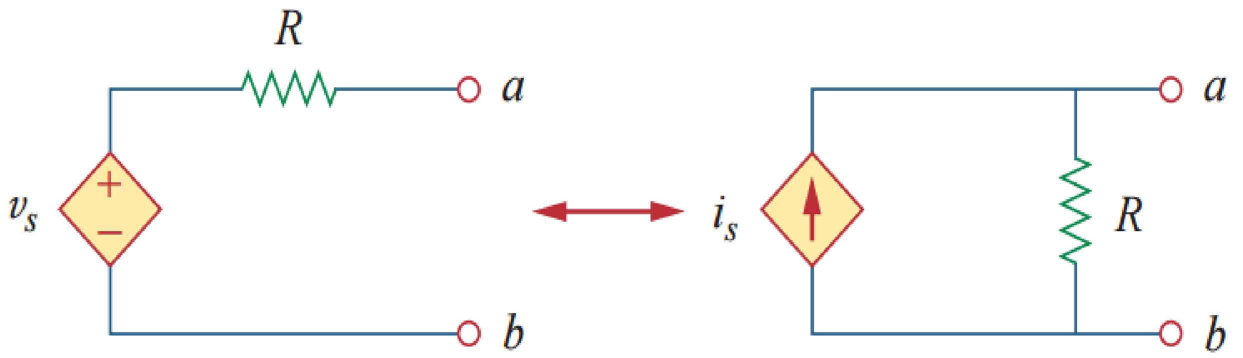


Figure 3-12 Transformation of dependent sources.

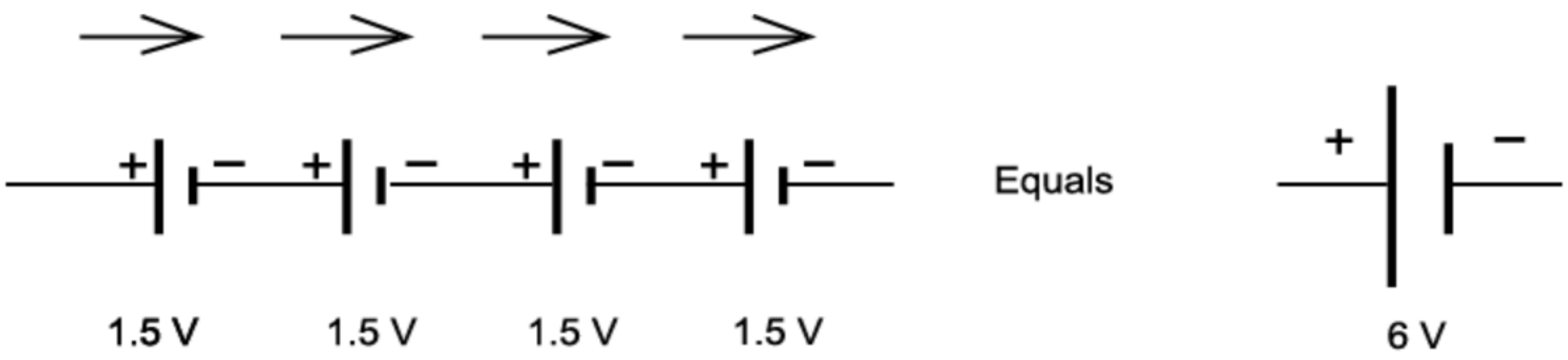
Voltage Source Circuits

Multiple power sources can be connected in series or parallel in order to meet the different voltage or current output requirements for various applications:

- ➢Power sources are connected in series to increase the voltage output.
- ➢Power sources are connected in parallel to increase the current capacity.

❖ Series Sources:

Example:

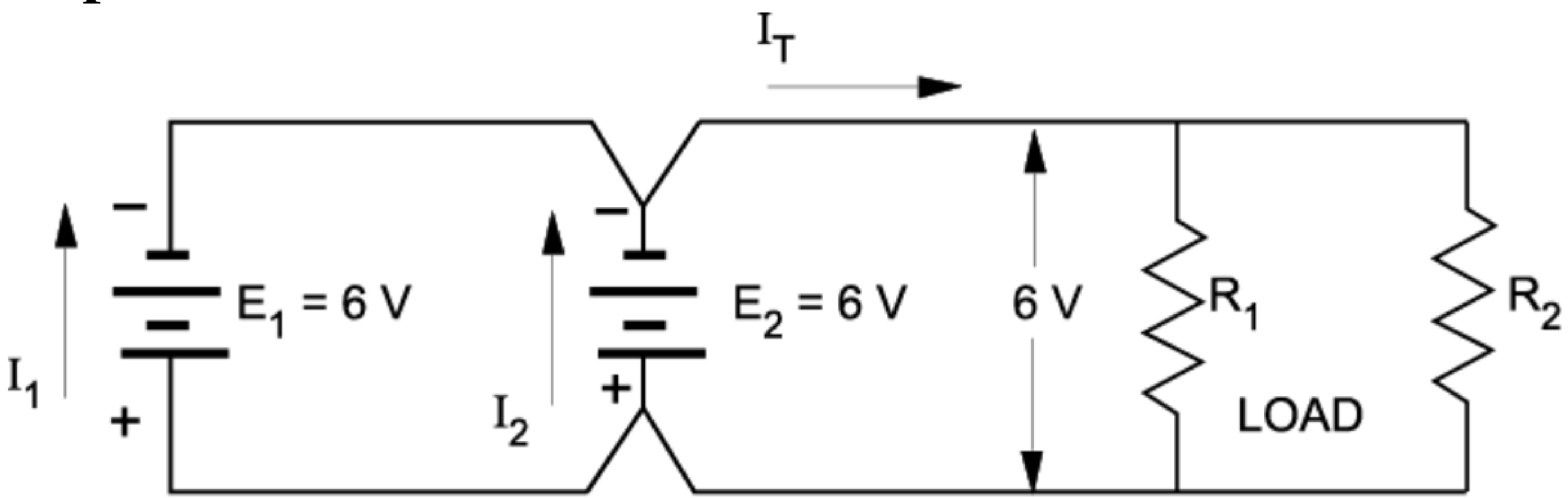


Voltage Source Circuits

❖Parallel Sources:

Voltage sources are connected in parallel whenever it is necessary to deliver a current output greater than the current output that a single source of supply can provide, without increasing voltage across a load.

Example:



Determinates

Consider the two simultaneous equations :

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

Where (x and y) are the unknown variables, and (a_1, a_2, b_1, b_2, c_1, and c_2) are constants. Using the determinants the values of the variables can be obtained as :

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad \text{and} \quad y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Where:

$$\Delta = a_1b_2 - b_1a_2 \ ,$$

$$\Delta_1 = c_1b_2 - b_1c_2 \ ,$$

and $$\Delta_2 = a_1c_2 - c_1a_2$$

Determinates

Consider the following three simultaneous equations :

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

The unknown variables (x, y, and z) are determined as follows :

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

Determinates

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

$$\Delta = a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)$$

$$\Delta_1 = d_1(b_2c_3 - c_2b_3) - b_1(d_2c_3 - c_2d_3) + c_1(d_2b_3 - b_2d_3)$$

$$\Delta_2 = a_1(d_2c_3 - c_2d_3) - d_1(a_2c_3 - c_2a_3) + c_1(a_2d_3 - d_2a_3)$$

$$\Delta_3 = a_1(b_2d_3 - d_2b_3) - b_1(a_2d_3 - d_2a_3) + d_1(a_2b_3 - b_2a_3)$$

Determinates

Ex: Calculated the x and y.

$$-x + 2y = 3$$

$$3x - 2y = -2$$

Solution:

$$\Delta = \begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix} = (-1) * (-2) - 2 * 3 = -4$$

$$\Delta_1 = \begin{vmatrix} 3 & 2 \\ -2 & -2 \end{vmatrix} = 3 * (-2) - 2 * (-2) = -2$$

$$\Delta_2 = \begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix} = (-1) * (-2) - 3 * 3 = -7$$

$$\Rightarrow x = \frac{\Delta_1}{\Delta} = \frac{-2}{-4} = \frac{1}{2} \quad and \quad y = \frac{\Delta_2}{\Delta} = \frac{-7}{-4} = \frac{7}{4}$$

Determinates

Ex: Calculated the x, y, and z.

$$x - 2z = -1$$
$$3y + z = 2$$
$$x + 2y + 3z = 0$$

Solution:

$$x + 0y - 2z = -1$$
$$0x + 3y + z = 2$$
$$x + 2y + 3z = 0$$

$$\Delta = \begin{vmatrix} 1 & 0 & -2 \\ 0 & 3 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 1(3*3-1*2)+(-2)(0*2-3*1) = 13$$

$$\Delta_1 = \begin{vmatrix} -1 & 0 & -2 \\ 2 & 3 & 1 \\ 0 & 2 & 3 \end{vmatrix} = -1(3*3-1*2)+(-2)(2*2-3*0) = -15$$

$$\Delta_2 = \begin{vmatrix} 1 & -1 & -2 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{vmatrix} = 9$$

Determinates

$$\Delta_3 = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 3 & 2 \\ 1 & 2 & 0 \end{vmatrix} = -1$$

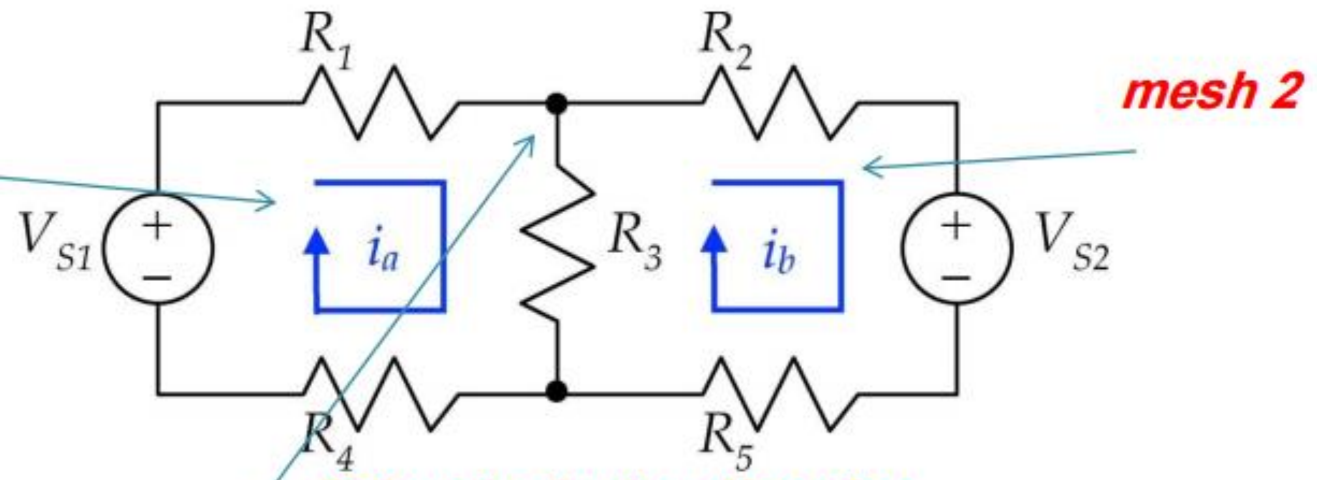
$$\therefore x = \frac{\Delta_1}{\Delta} = \frac{-15}{13} \qquad y = \frac{\Delta_2}{\Delta} = \frac{9}{13} \qquad z = \frac{\Delta_3}{\Delta} = -\frac{1}{13}$$

Determinates

يمكن حساب المحددات من خلال الة الحاسبة

SCAN ME

Mesh-analysis/Loop-analysis/Maxwell's Current Analysis



Take clockwise direction

According to the way that we have defined the currents, we see that i_a flows in V_{S1}, R_1, and R_4. and i_a flow in V_{S2}, R_2 ,and R_5.

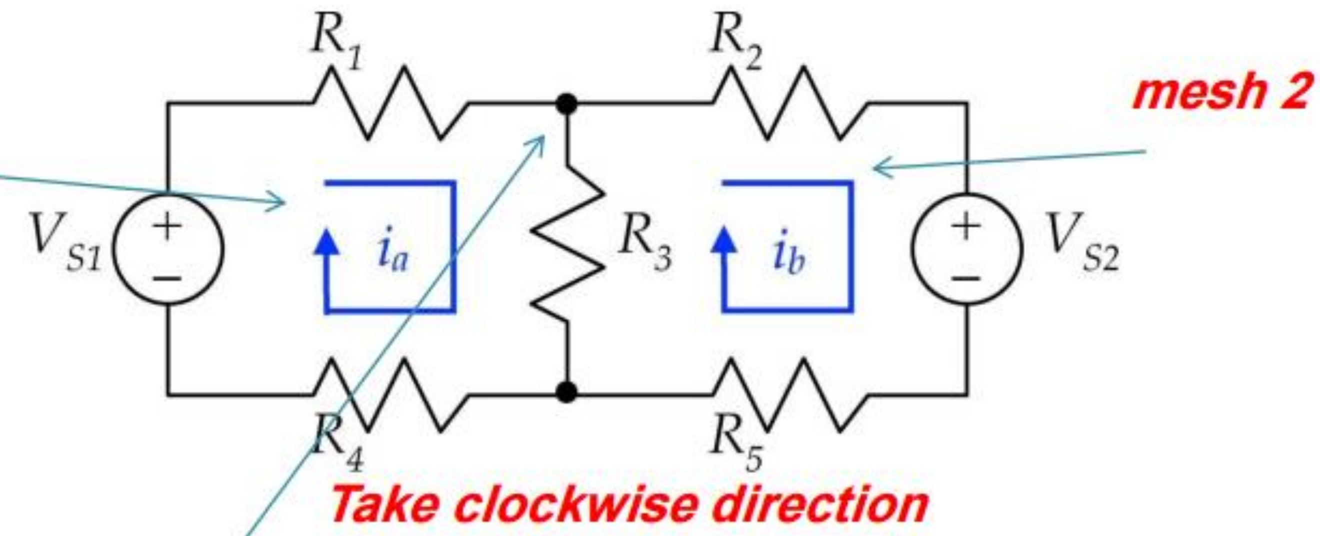
But what about R3 which is common?

In the common branch, the current is the difference of i_a and i_b.

***Note:**

Related to nodes consider only the node with three and more connection.

Mesh-analysis/Loop-analysis/Maxwell's Current Analysis



According to the way that we have defined the currents, we see that i_a flows in V_{S1}, R_1, and R_4. and i_a flow in V_{S2}, R_2 ,and R_5.

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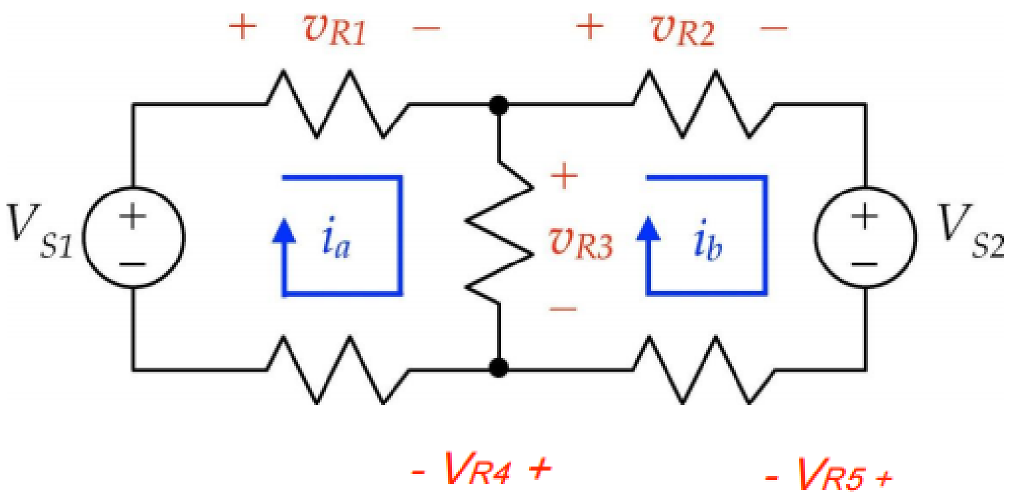
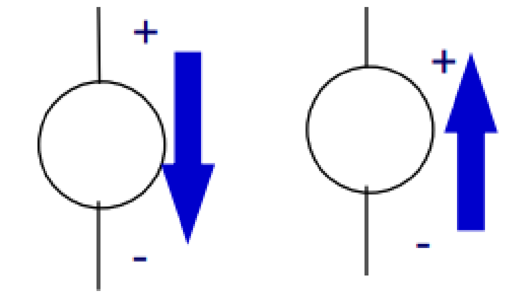
Mesh-analysis/Loop-analysis/Maxwell's Current Analysis





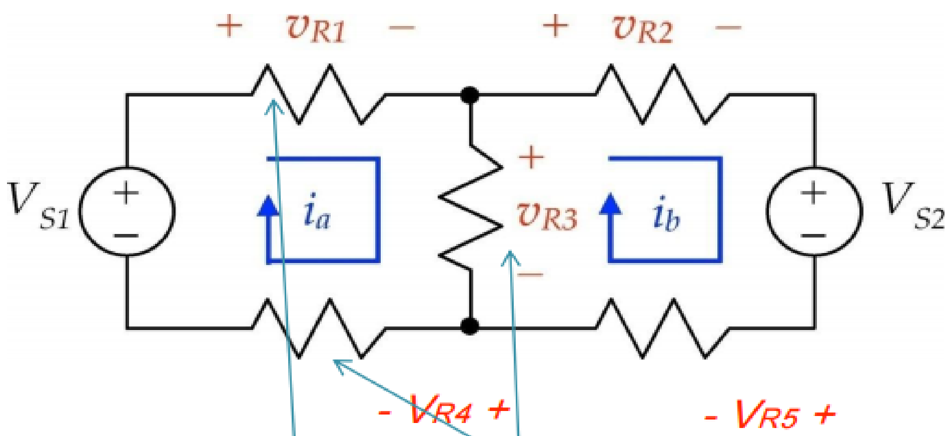
With the mesh currents defined, we can define resistor voltages.

Next, **use <u>KVL</u> around each mesh (loop)**.

From i_a mish ➡ $$V_{S1} - v_{R1} - v_{R3} - v_{R4} = 0$$

From i_b mish ➡ $$v_{R3} - v_{R2} - V_{S2} - v_{R5} = 0$$

Mesh-analysis/Loop-analysis/Maxwell's Current Analysis



$$V_{S1} - v_{R1} - v_{R3} - v_{R4} = 0$$

$$v_{R3} - v_{R2} - V_{S2} - v_{R5} = 0$$

Finally, write resistor voltages in terms of mesh currents(using ohm law)

$$\boxed{V_{S1} - i_a R_1 - (i_a - i_b) R_3 - i_a R_4 = 0}$$ first mesh equation

$$\boxed{(i_a - i_b)\, R_3 - i_b R_2 - V_{S2} - i_b R_5 = 0}$$ Second mesh equation

Rearrange equation for i_a and i_b

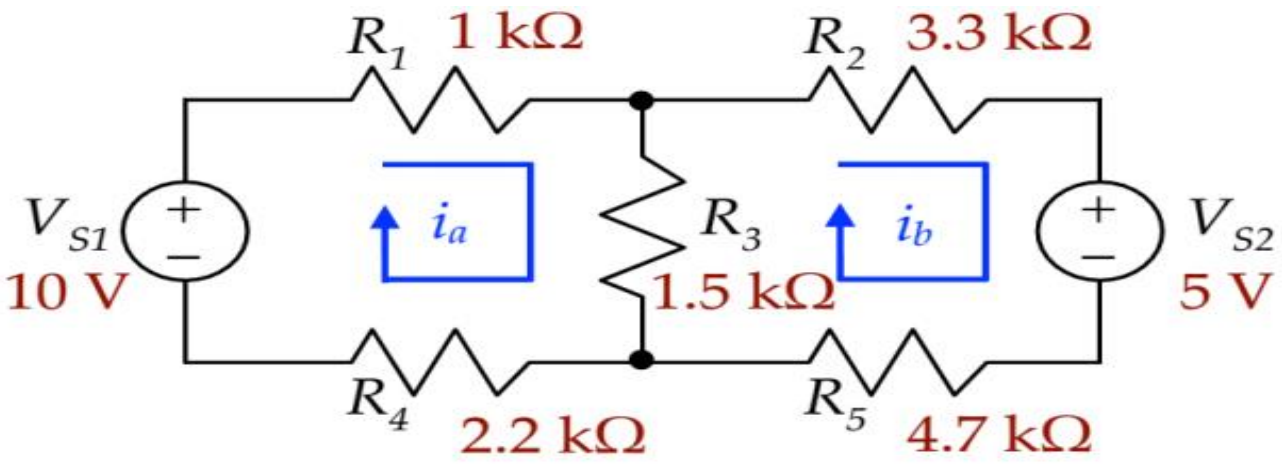
$$i_a\,(R_1 + R_3 + R_4) - i_b R_3 = V_{S1}$$

$$i_a R_3 - i_b\,(R_2 + R_3 + R_5) = V_{S2}$$

2 equations, 2 unknowns!

Solve for i_a and i_b.

Mesh-analysis/Loop-analysis/Maxwell's Current Analysis



$$i_a\left(R_1 + R_3 + R_4\right) - i_b R_3 = V_{S1}$$

$$i_a R_3 - i_b\left(R_2 + R_3 + R_5\right) = V_{S2}$$

$$i_a\,(4.7\text{k}\Omega) - i_b\,(1.5\text{k}\Omega) = 10\text{V}$$

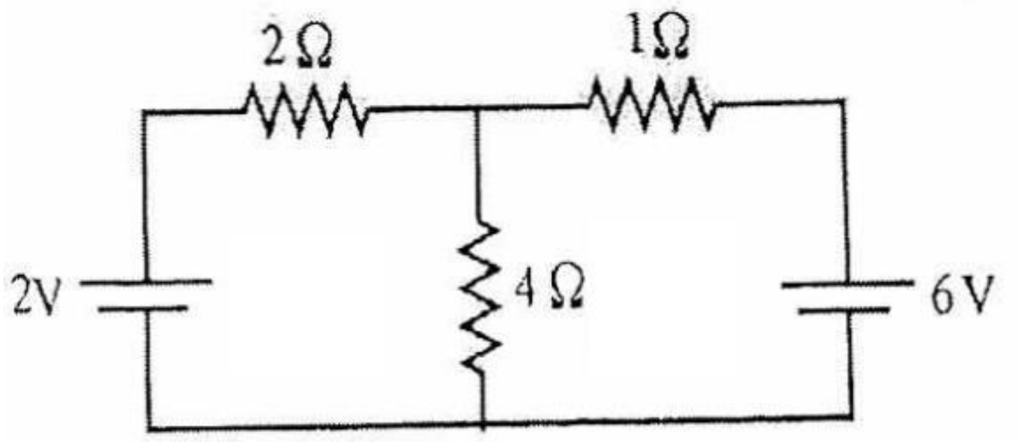
$$i_a\,(1.5\text{k}\Omega) - i_b\,(9.5\text{k}\Omega) = 5\text{V}$$

solving: $i_a = 2.06$ mA and $i_b = -0.200$ mA.

Now you can find all the voltages and powers, if needed

Mesh-analysis/Loop-analysis/Maxwell's Current Analysis

EX: Calculated the current through (4Ω) by using (Mesh-Maxwell) Analysis.



Solution:

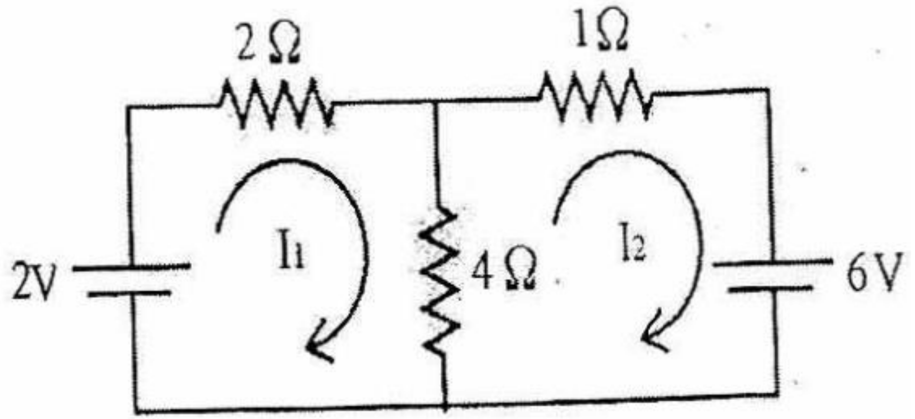
$(2{+}4)\ I_1 - 4I_2 = 2$ .....(1)

$-4I_1 = (4{+}1)\ I_2 = -6$...(2)

$\Delta = \begin{vmatrix} 6 & -4 \\ -4 & 5 \end{vmatrix} = [6 \times 5 - (-4 \times -4)] = 14$

$\Delta_1 = \begin{vmatrix} 2 & -4 \\ -6 & 5 \end{vmatrix} = [2 \times 5 - (-4 \times -6)] = -\ 14$

$\Delta_2 = \begin{vmatrix} 6 & 2 \\ -4 & -6 \end{vmatrix} = [6 \times -6 - (2 \times -4)] = -\ 28$

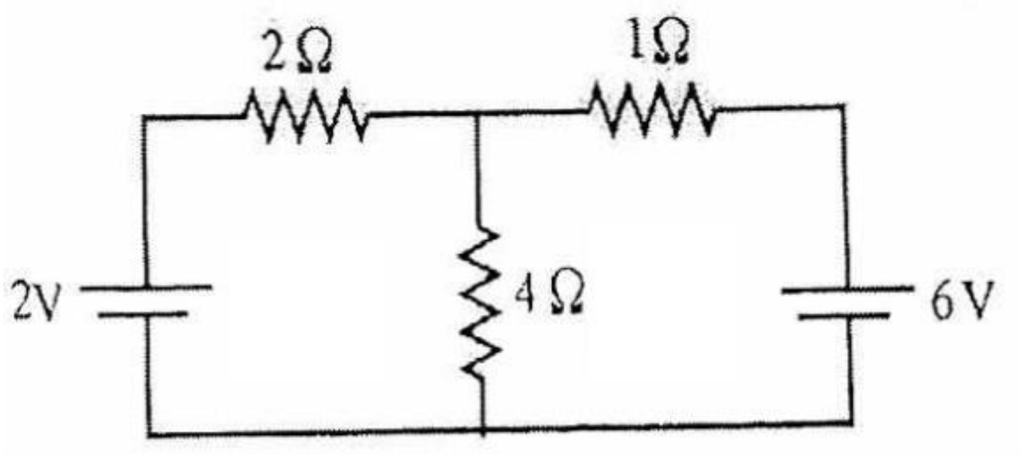


$I_1 = \dfrac{-14}{14} = -1\ A \quad , I_2 = \dfrac{-28}{14} = -2\ A$

$I(4\Omega) = I_1 - I_2 = -1 - (-2) = 1\ A$ in the direction of (I_1)

Mesh-analysis/Loop-analysis/Maxwell's Current Analysis

EX: Calculated the current through (4Ω) by using (Mesh-Maxwell) Analysis.



Solution:

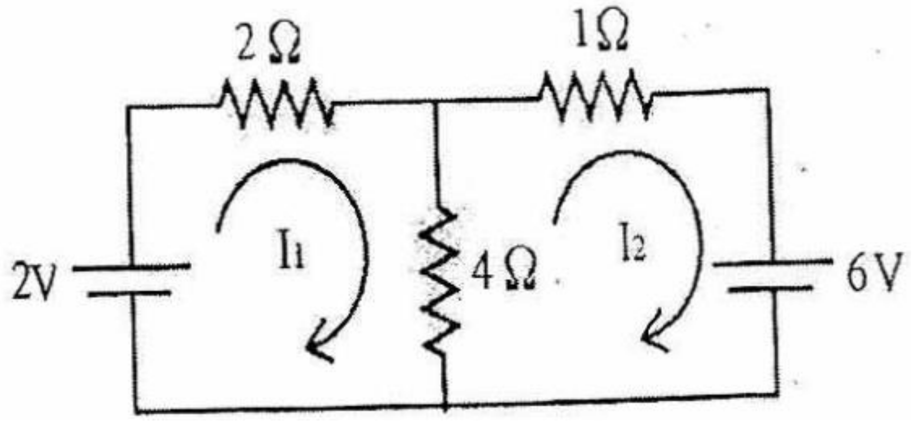
(2+4) $I_1 - 4I_2$= 2 .….(1)

$-4I_1$=(4+1) I_2= −6…(2)

$\Delta=\begin{vmatrix} 6 & -4 \\ -4 & 5 \end{vmatrix}$=[6 × 5 − (−4 × −4)]=14

$\Delta_1=\begin{vmatrix} 2 & -4 \\ -6 & 5 \end{vmatrix}$=[2 × 5 − (−4 × −6)]= − 14

$\Delta_2=\begin{vmatrix} 6 & 2 \\ -4 & -6 \end{vmatrix}$=[6 × −6 − (2 × −4)]= − 28

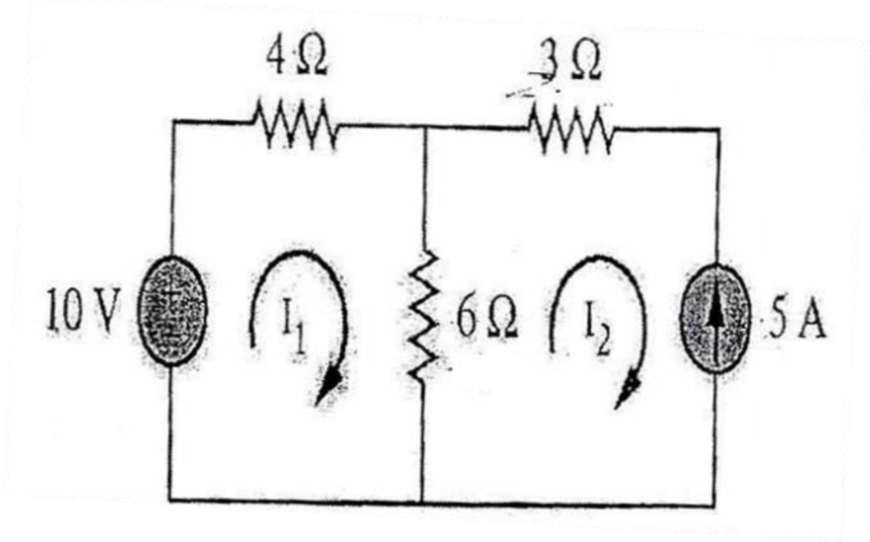


$I_1 = \frac{-14}{14} = -1\ A \quad ,I_2 = \frac{-28}{14} = -2\ A$

I(4Ω)=$I_1 - I_2 = -1 - (-2) = 1\ A$ in the direction of (I_1)

Maxwell's current analysis: special case

EX: Calculated the loop currents and current through (6Ω) by using (Mesh-Maxwell) Analysis.



Solution:

For loop 1

(4+6) $I_1 - 6I_2 = 10$

$10I_1 - 6I_2 = 10$ ......(1)

For loop 2

$I_2 = -5$A

$10I_1 - 6(-5) = 10 \Rightarrow 10I_1 - (-30) = 10$

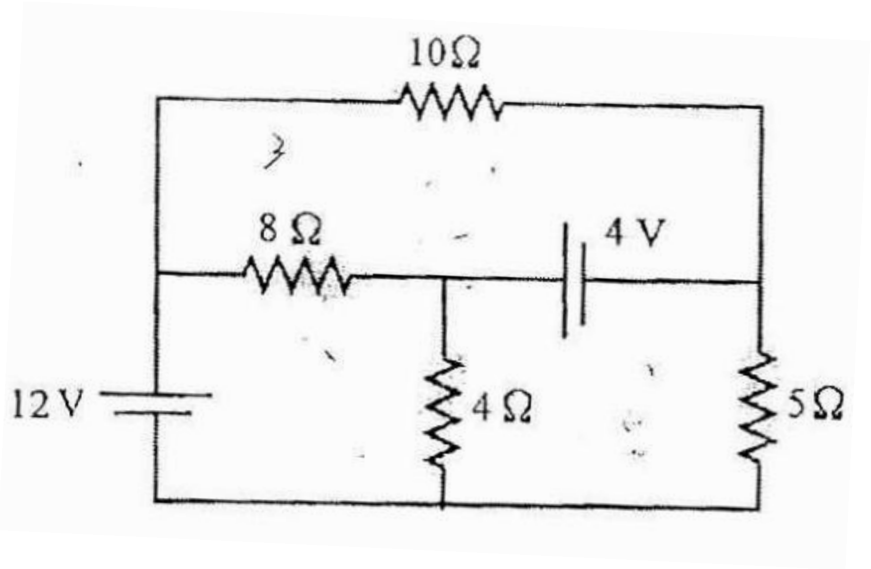
$10I_1 + 30 = 10 \Rightarrow 10I_1 = 10 - 30 \Rightarrow 10I_1 = -20$

$$I_1 = \frac{-20}{10} = -2\ A$$

$I(6\Omega) = I_1 - I_2 = -2 - (-5) = -2 + 5 = 3\ A$ in the direction of (I_1)

Maxwell's current analysis

Homework: Find the loop currents and current through (4Ω) by using (Mesh-Maxwell) Analysis.



Answer: $I_1 = 1.8A,\ I_2 = 0.3555A\ , I_2 = 1.0222A$

Nodal analysis (format approach)

We will now employ Kirchhoff's current law to develop a method referred to as **nodal analysis**.

A node is defined as a junction of two or more branches. If we now define one node of any network as a reference (that is, a point of zero potential or ground), the remaining nodes of the network will all have a fixed potential relative to this reference. For a network of N nodes, therefore, there will exist (N -1) nodes with a fixed potential relative to the assigned reference node. Equations relating these nodal voltages can be written by applying Kirchhoff's current law at each of the (N - 1) nodes. To obtain the complete solution of a network, these nodal voltages are then evaluated in the same manner in which loop currents were found in loop analysis.

Nodal analysis (format approach)

The nodal analysis method is applied as follows:

1. Choose a reference node and assign a subscripted voltage label to the (N 1) remaining nodes of the network.

2. The number of equations required for a complete solution is equal to the number of subscripted voltages (N 1). Column 1 of each equation is formed by summing the conductances tied to the node of interest and multiplying the result by that subscripted nodal voltage.

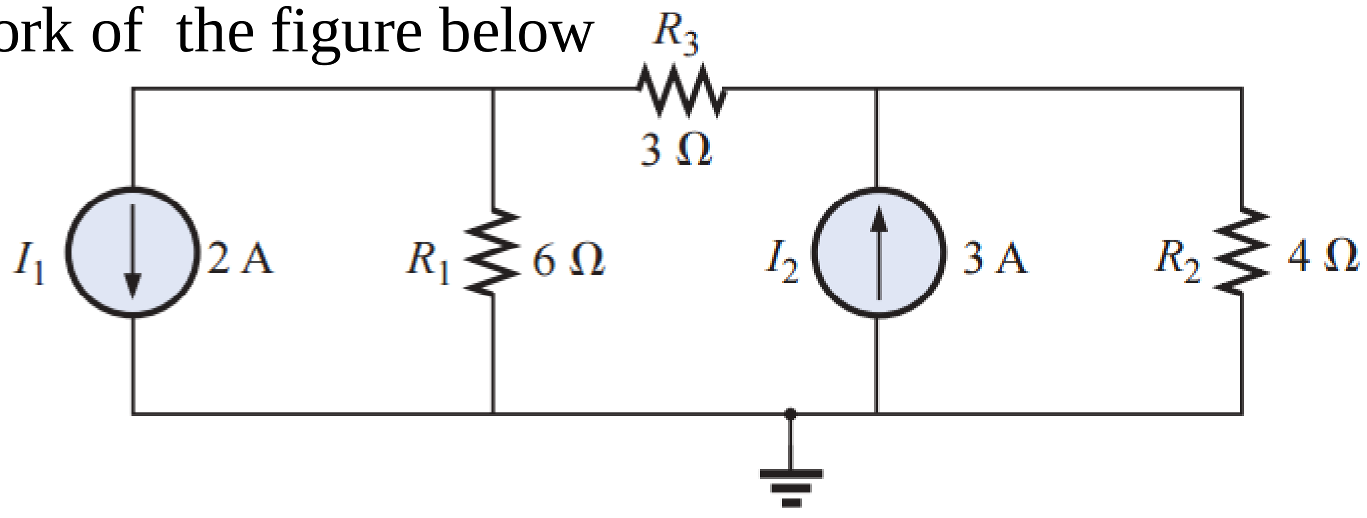
3. We must now consider the mutual terms that, as noted in the preceding example, are always subtracted from the first column. It is possible to have more than one mutual term if the nodal voltage of current interest has an element in common with more than one other nodal voltage. This will be demonstrated in an example to follow. Each mutual term is the product of the mutual conductance and the other nodal voltage tied to that conductance.

4. The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.

5. Solve the resulting simultaneous equations for the desired

voltages.

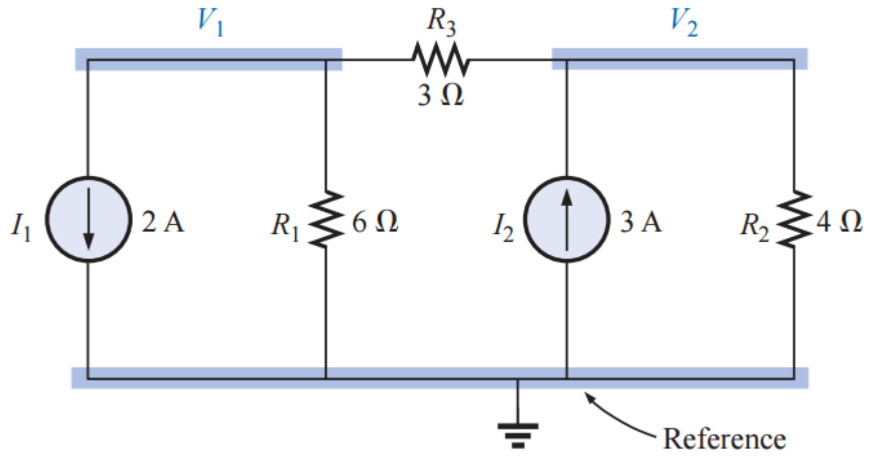
EX: Write the nodal equations for the network of the figure below



Solution:

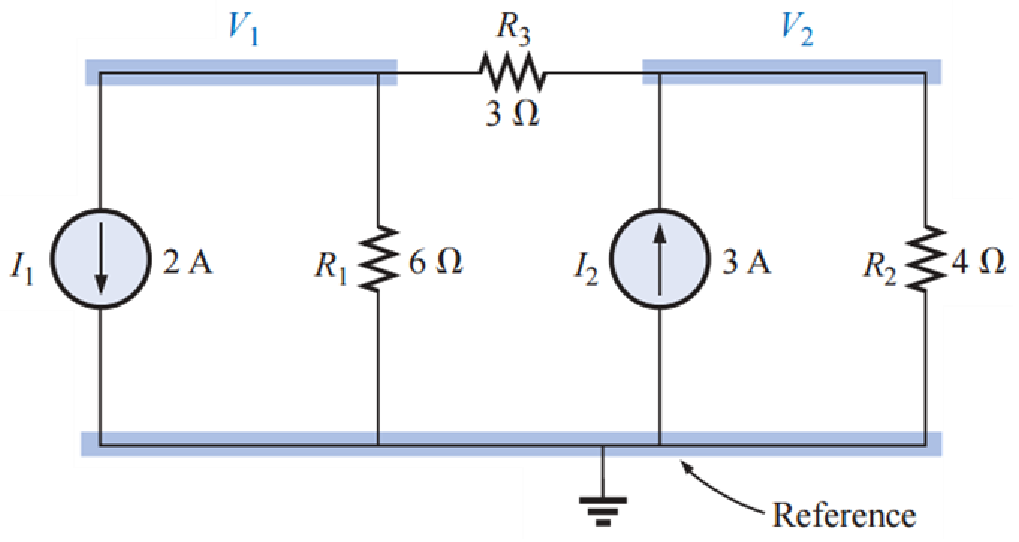
Step 1: The figure is redrawn with assigned subscripted voltages in the figure below.

Steps 2 to 4:



$$V_1:\quad \underbrace{\left(\frac{1}{6\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 1}}} V_1 - \underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_2 = \overset{\substack{\text{Drawing current}\\ \text{from node 1}\\ \downarrow}}{-2\text{ A}}$$

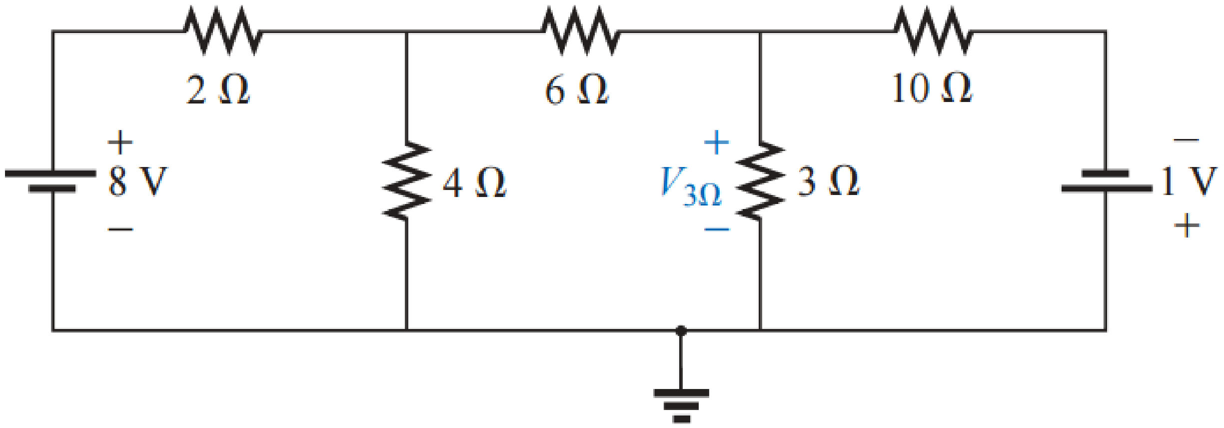
$$V_2:\quad \underbrace{\left(\frac{1}{4\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 2}}} V_2 - \underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_1 = \overset{\substack{\text{Supplying current}\\ \text{to node 2}\\ \downarrow}}{+3\text{ A}}$$



$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

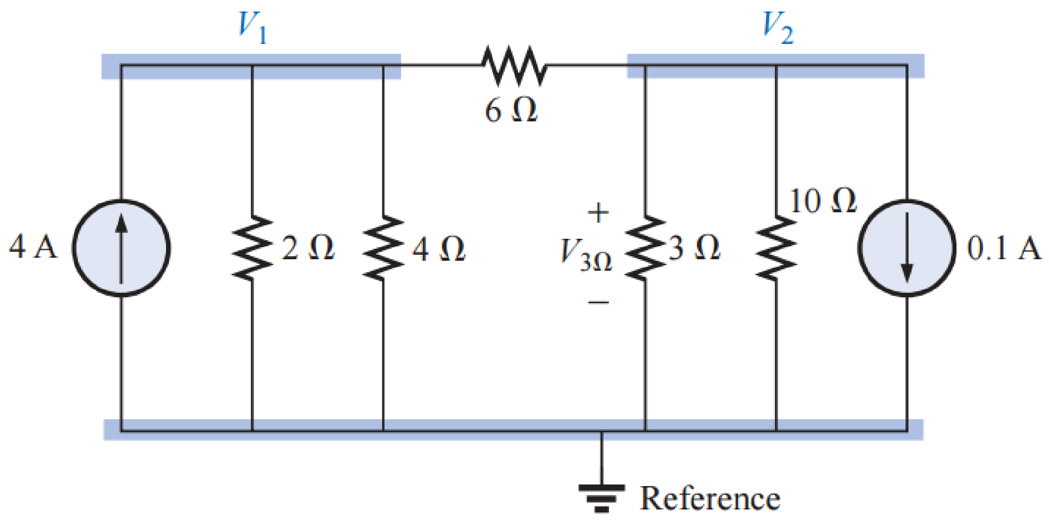
EX: Find the voltage across the 3-Ω resistor of the figure below by nodal analysis.



Solution: Converting sources and choosing nodes

$$\left(\frac{1}{2\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{6\ \Omega}\right)V_1-\left(\frac{1}{6\ \Omega}\right)V_2=+4\ \text{A}$$

$$\left(\frac{1}{10\ \Omega}+\frac{1}{3\ \Omega}+\frac{1}{6\ \Omega}\right)V_2-\left(\frac{1}{6\ \Omega}\right)V_1=-0.1\ \text{A}$$

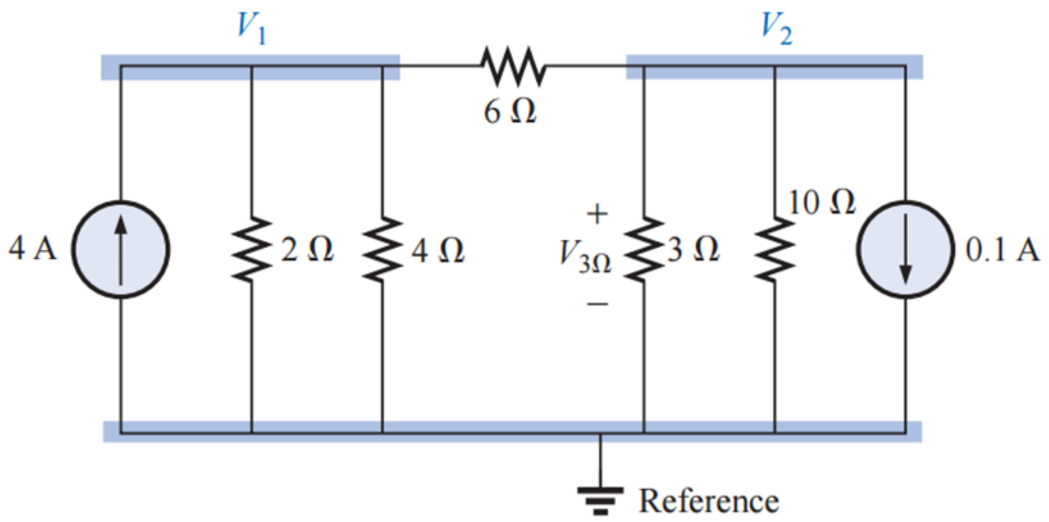


$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$

$$-\frac{1}{6}V_1+\frac{3}{5}V_2=-0.1$$

$$\frac{11}{12}V_1 - \frac{1}{6}V_2 = 4 \qquad \times 12$$

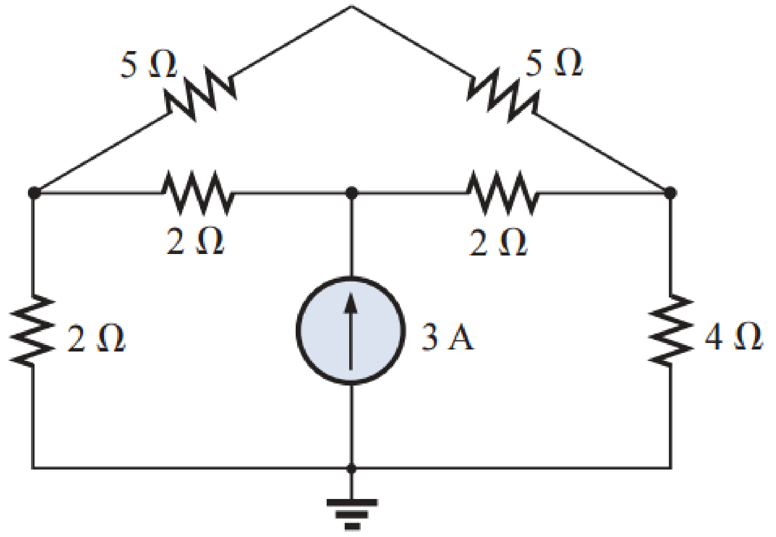
$$-\frac{1}{6}V_1 + \frac{3}{5}V_2 = -0.1 \qquad \times 30$$



$$11V_1 - 2V_2 = +48$$
$$-5V_1 + 18V_2 = -3$$

$$V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33 + 240}{198 - 10} = \frac{207}{188} = \mathbf{1.101\ V}$$

EX: Using nodal analysis, determine the potential across the 4-Ω of the figure below by nodal analysis.

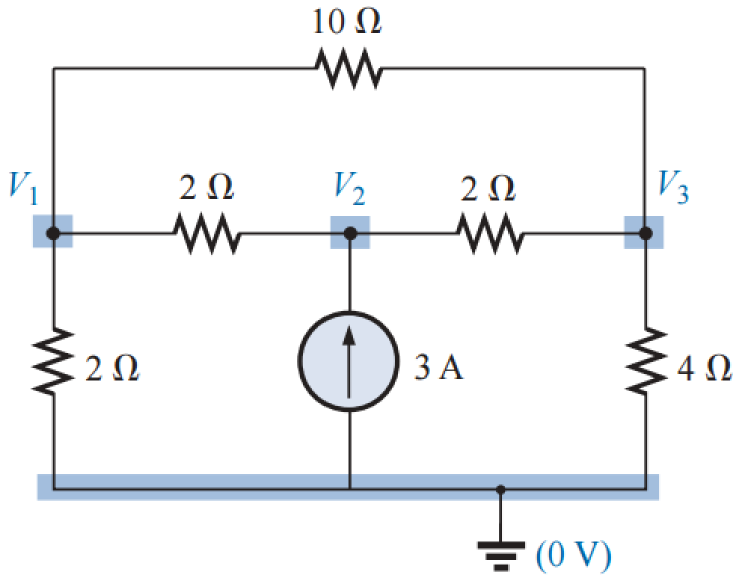


Solution:

$$V_1: \quad \left(\frac{1}{2\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{10\ \Omega}\right)V_1 - \left(\frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{10\ \Omega}\right)V_3 = 0$$

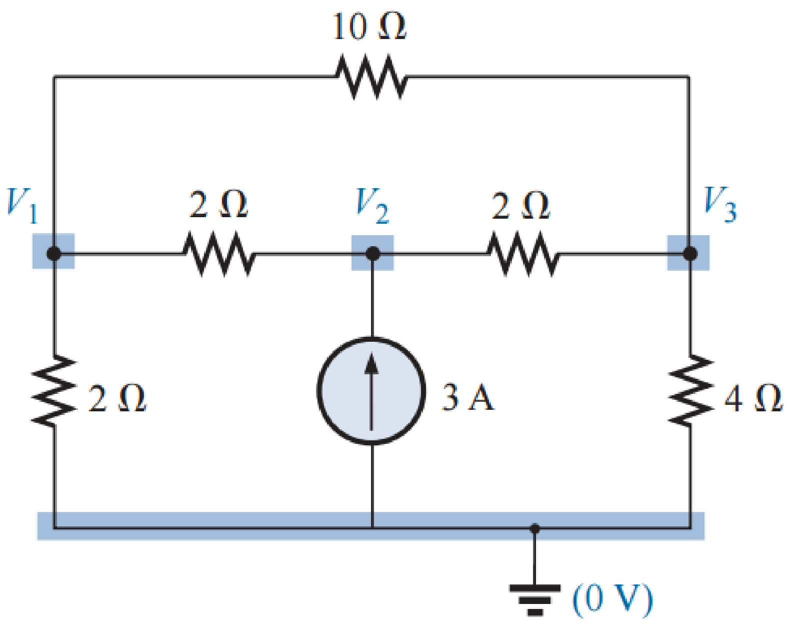
$$V_2: \quad \left(\frac{1}{2\ \Omega} + \frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{2\ \Omega}\right)V_3 = 3\ \text{A}$$

$$V_3: \quad \left(\frac{1}{10\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{4\ \Omega}\right)V_3 - \left(\frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{10\ \Omega}\right)V_1 = 0$$



$$\begin{aligned} 1.1V_1 - 0.5V_2 - 0.1V_3 &= 0 \\ V_2 - 0.5V_1 - 0.5V_3 &= 3 \\ 0.85V_3 - 0.5V_2 - 0.1V_1 &= 0 \end{aligned}$$

For determinants,



$$
\begin{aligned}
1.1V_1 - 0.5V_2 - 0.1V_3 &= 0 \\
-0.5V_1 + 1V_2 - 0.5V_3 &= 3 \\
-0.1V_1 - 0.5V_2 + 0.85V_3 &= 0
\end{aligned}
$$

$$
V_3 = V_{4\Omega} = \frac{\begin{vmatrix} 1.1 & -0.5 & 0 \\ -0.5 & +1 & 3 \\ -0.1 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 1.1 & -0.5 & -0.1 \\ -0.5 & +1 & -0.5 \\ -0.1 & -0.5 & +0.85 \end{vmatrix}} = \mathbf{4.645\ V}
$$

Nodal analysis

SCAN ME

THANK YOU FOR LISTTINIG

شكراً لأصغائكم