Ministry of Higher Education and Scientific Research
Tikrit University
Engineering Collage –Al shirqat

FUNDAMENTALS OF ELECTRICAL ENGINEERING
LECTURE 2
SERIES AND PARALLEL RESISTORS

PREPARED BY
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General objectives

- **Series Connection**

- **Increases total resistance:** When resistors are connected in series, the total resistance increases, reducing the current flowing through the circuit.
- **Divides voltage:** The total voltage is divided among the components connected in series according to their resistance values.
- **Maintains constant current:** The same current flows through all components in a series connection.
- **Used in applications such as:** Voltage dividers, filter circuits, and circuit protection.

- **Parallel Connection**

- **Decreases total resistance:** When resistors are connected in parallel, the total resistance decreases, increasing the total current in the circuit.
- **Divides current:** The total current is divided among the parallel branches according to their resistance values.
- **Maintains constant voltage:** The voltage across all components in parallel remains the same.
- **Used in applications such as:** Increasing power capacity, load distribution, and minimizing power loss.

General objectives

➢ **Voltage Divider**

- **Reduces voltage to a specific level:** Used to generate a voltage lower than the total voltage as needed.
- **Provides reference signals:** Used in electronic circuits as a reference voltage source.
- **Used in applications such as:** Analog circuits, sensors, and measuring devices.

➢ **Current Divider**

- **Distributes current among branches:** It divides the current among multiple resistors connected in parallel.
- **Controls the current flowing through each branch:** Based on the resistance value of each path.
- **Used in applications such as:** DC circuits, filtering techniques, and measuring instruments.

Specific objectives:

- **Series Connection**

- Achieve a higher voltage at the same current.
- Distribute the total voltage across multiple electrical components.
- Maintain the same current in all elements connected in series.
- Improve the performance of systems requiring a higher voltage than a single source can provide.
- Increase the equivalent resistance of the circuit.

- **Parallel Connection**

- Achieve a higher current at the same voltage.
- Distribute the total current across multiple branches to prevent overload.
- Ensure circuit operation continues even if one component fails.
- Reduce the equivalent resistance of the circuit, increasing power transmission efficiency.

Specific objectives:

- **Voltage Divider**

- Reduce the voltage to a suitable level for different loads.
- Provide a reference voltage for control circuits and sensors.
- Protect sensitive components from high voltage.
- Simplify electronic circuits requiring different voltage levels.

- **Current Divider**

- Distribute current among multiple branches in a circuit.
- Reduce the current in each branch to protect components from overloading.
- Improve energy consumption efficiency in electrical circuits.
- Used in measurement and control circuits to determine the appropriate current for each element.

Series Resistors and Voltage Division

The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then

Resistance : $$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

$$R_{eq} = R_1 + R_2$$

Voltage : $$v = v_t = v_1 + v_2$$

Current : $$i = i_t = i_1 = i_2$$

$$i = i_t = \frac{v_t}{R_{eq}}$$

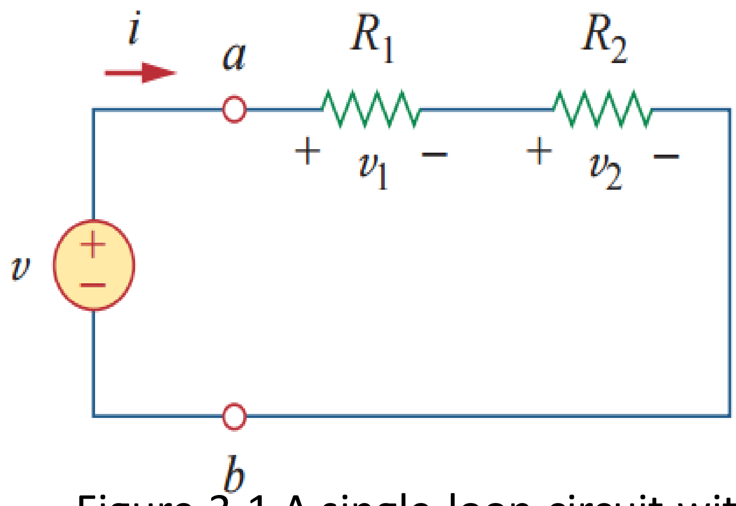


Figure 3-1 A single-loop circuit with two resistors in series.

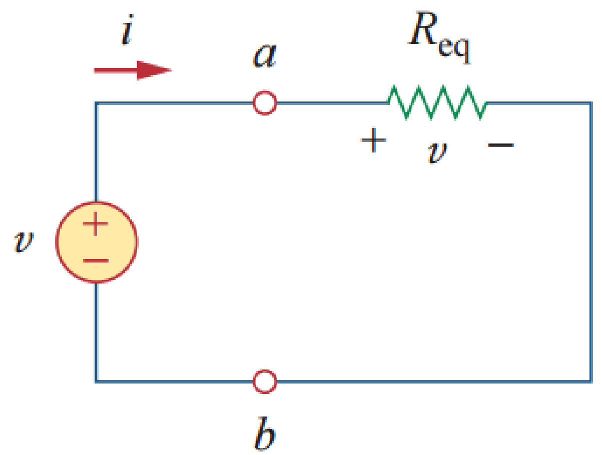
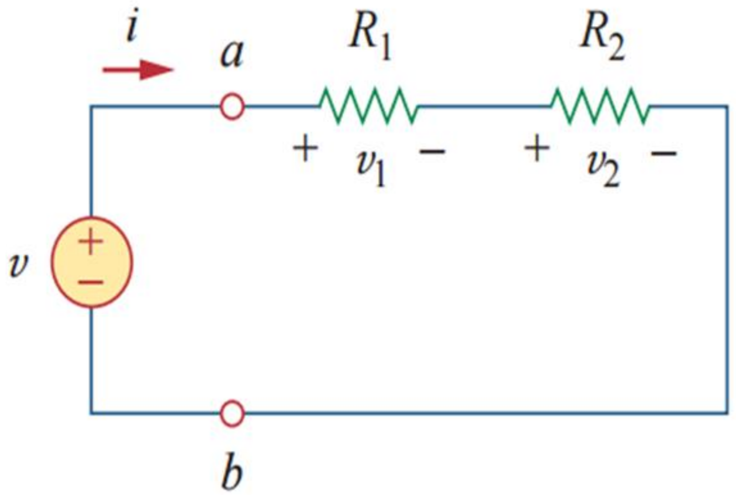


Figure 3-2 Equivalent circuit of the Fig. 3.1 circuit

Series Resistors and Voltage Division

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

$$v_x = v \times \left(\frac{R_x}{R_1 + R_2} \right)$$



Parallel Resistors and Current Division

The **equivalent resistance** of two parallel resistors is equal to the product of their resistances divided by their sum.

the general case of a circuit with N resistors in parallel.

The equivalent resistance is

Resistance :

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

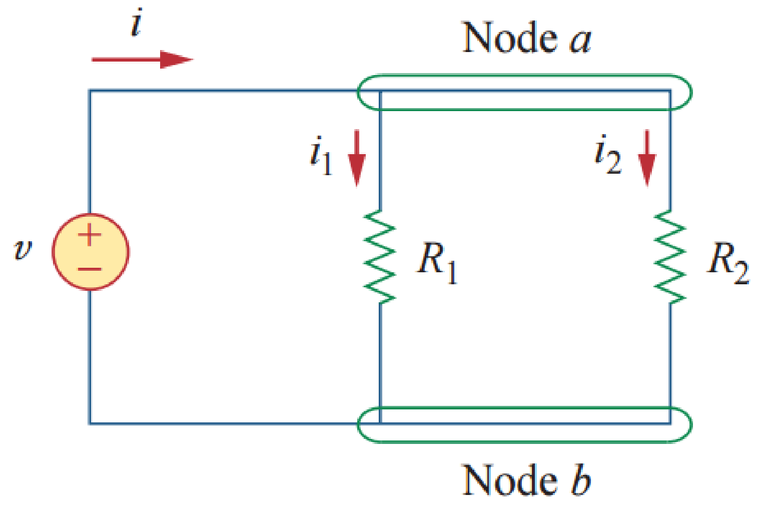


Figure 3-3 A single-loop circuit with two resistors in parallel.

where R_{eq} is the equivalent resistance of the resistors in parallel:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

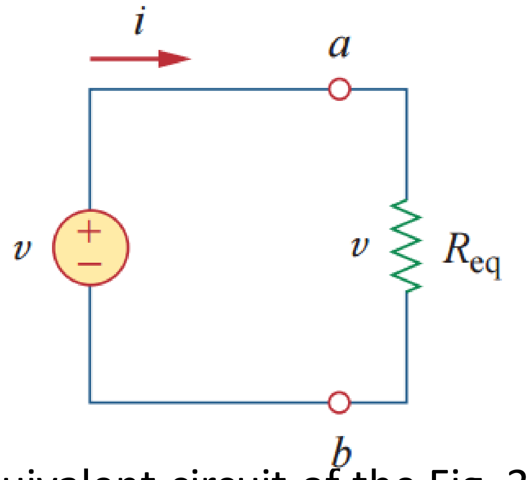


Figure 3-4 Equivalent circuit of the Fig. 3.3 circuit

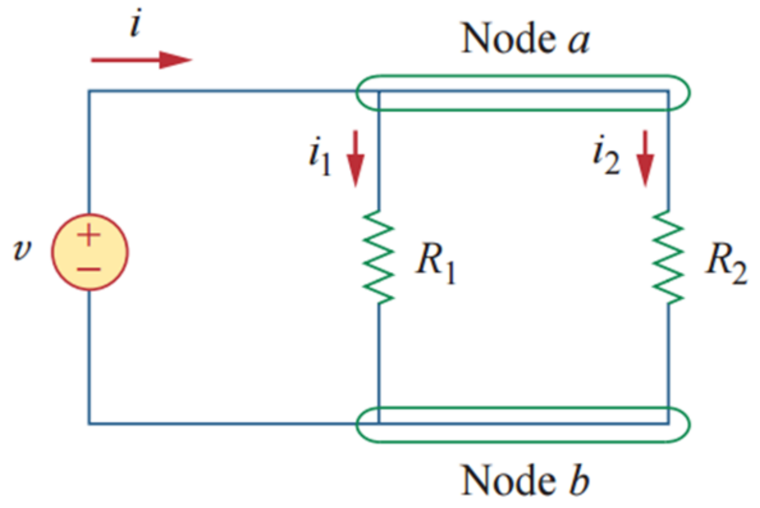
Parallel Resistors and Current Division

Voltage :

$$v = v_t = v_1 = v_2 = i_t \times R_{eq} = i_1 R_1 = i_2 R_2$$

Current :

$$i = i_t = i_1 + i_2$$



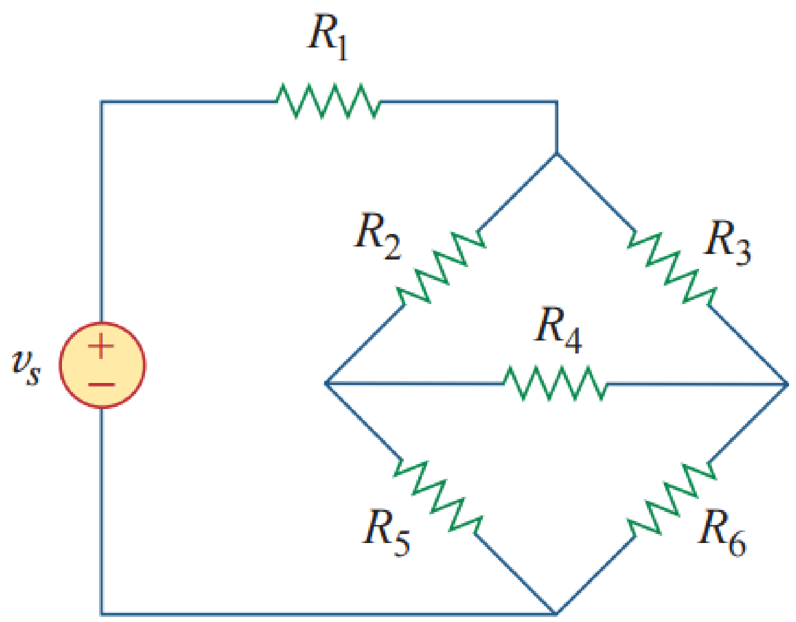
$$i_1 = i_t \times \left(\frac{R_2}{R_1 + R_2}\right)$$

$$i_2 = i_t \times \left(\frac{R_1}{R_1 + R_2}\right)$$

$$i = i_t = \frac{v_t}{R_{eq}}$$

Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit as shown in figure bellow:



In this circuit R_1, R_2, R_3, R_4, R_5 and R_6 are neither in parallel nor in series.

Question :When to convert **delta → star** or vice versa?

- ✅ **Delta → Star**: Used when reducing starting current or lowering the voltage on windings is needed. This is common in motor starting applications to reduce the inrush current.
- ✅ **Star → Delta**: Used when operating equipment at full power after startup or when a higher voltage is required. This is typically seen in star-delta starters for motors, where the motor starts in the star configuration (low voltage) and then switches to delta (full voltage) for normal operation.

Wye-Delta Transformations

These are the wye (**Y**) or tee (**T**) network shown in Fig. 3.8 and the delta (Δ) or pi (**π**) network shown in Fig. 3.9

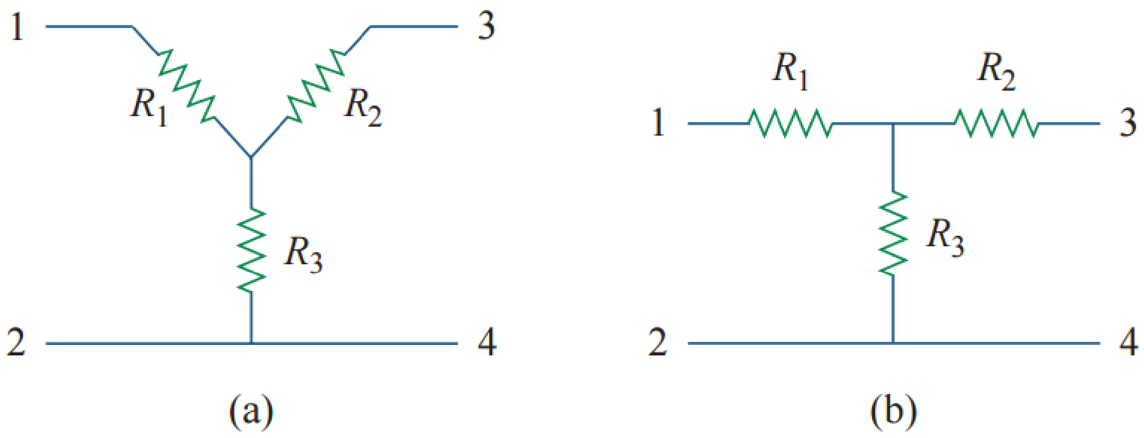


Figure 3-8 Two forms of the same network: (a) **Y** ,(b) . **T**.

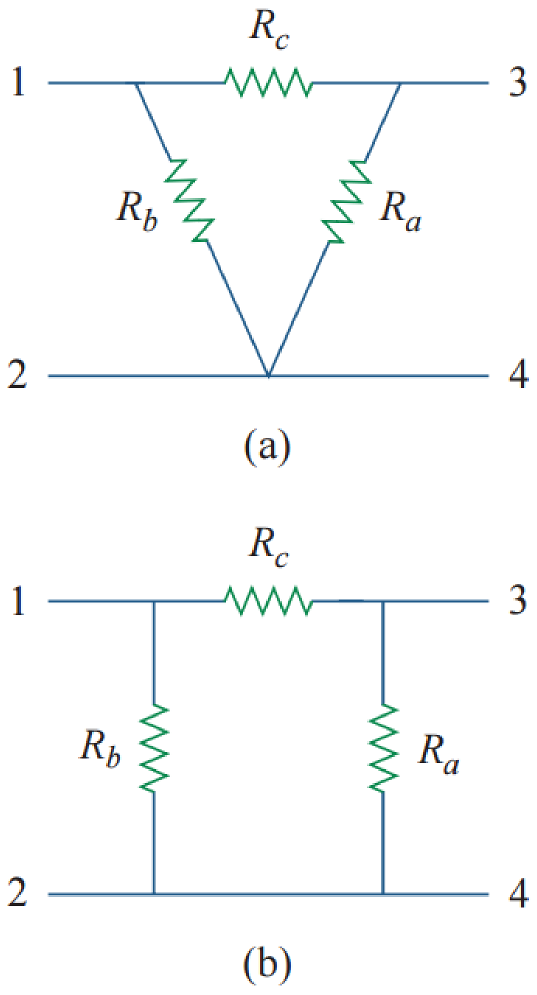


Figure 3-9 Two forms of the same network: (a) Δ ,(b) . **π**.

Delta to Wye Conversion

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

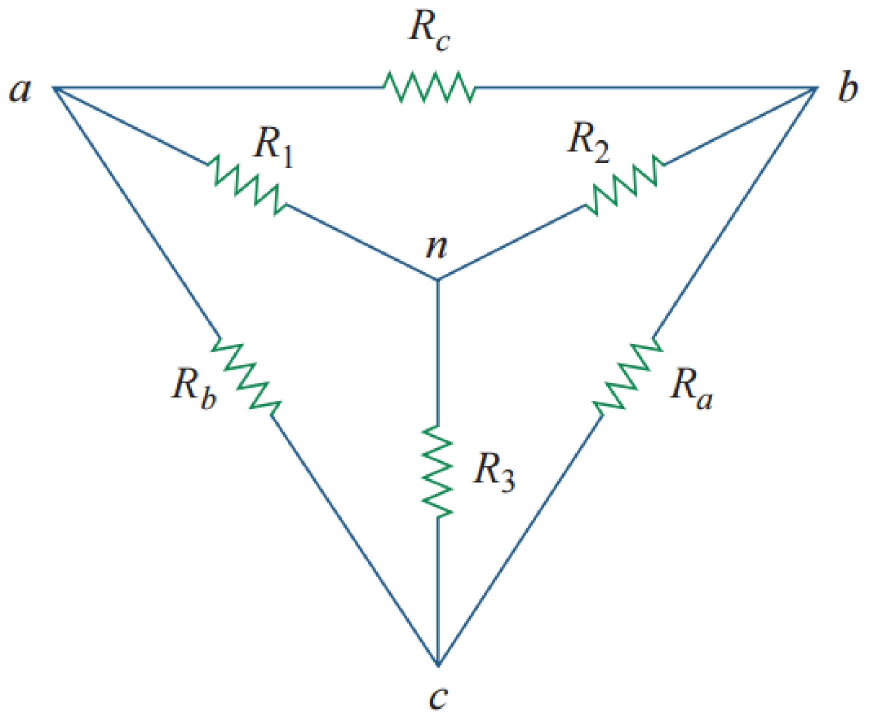


Figure 3-10 Superposition of Y and Δ networks as an aid in transforming one to the other

Wye to Delta Conversion

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

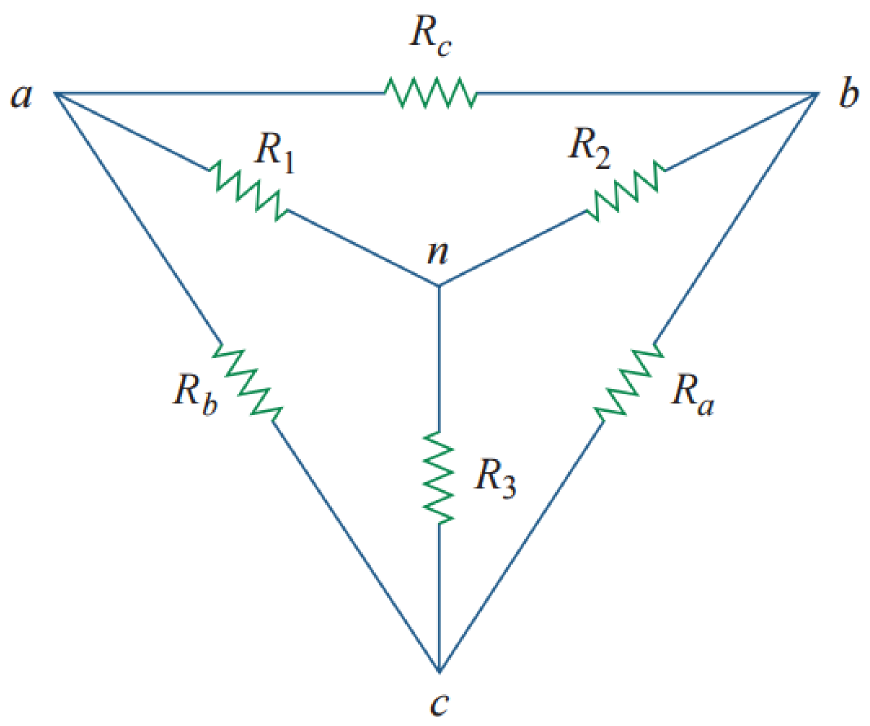


Figure 3-10 Superposition of Y and Δ networks as an aid in transforming one to the other

Wye-Delta Transformations

- **Note :-**

- The **Y** and Δ network are said to be **balanced** when :

R_1=R_2=R_3

And

R_a=R_b=R_c

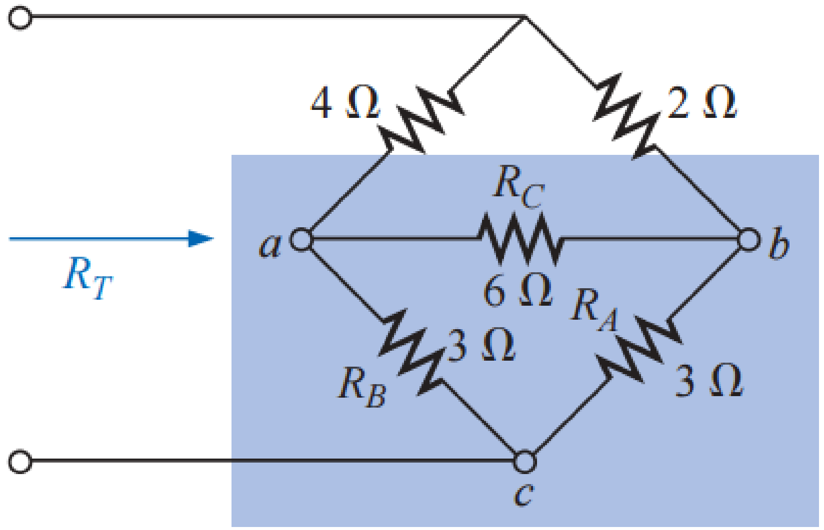
- Under balance condition, the convention equations become:

$$R_{\mathbf{Y}}=\frac{R_{\Delta}}{3}$$

$$V_{\mathbf{Y}}=\frac{V_{\Delta}}{\sqrt{3}}$$

$$R_{\Delta} = 3R_{\mathbf{Y}}$$

EX: Find the total resistance of the network of Figure below where R_A =3Ω , R_B =3 Ω, and R_C=6 Ω.



Solution:

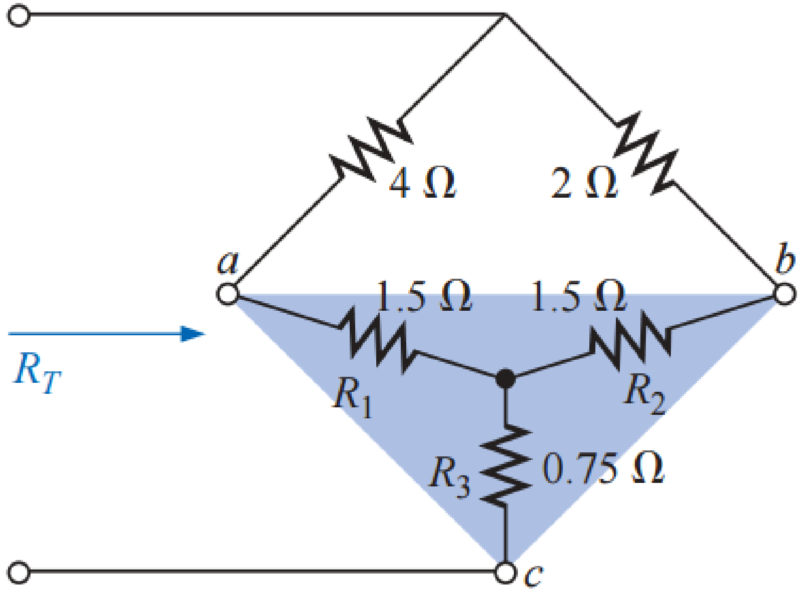
Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown in Fig.



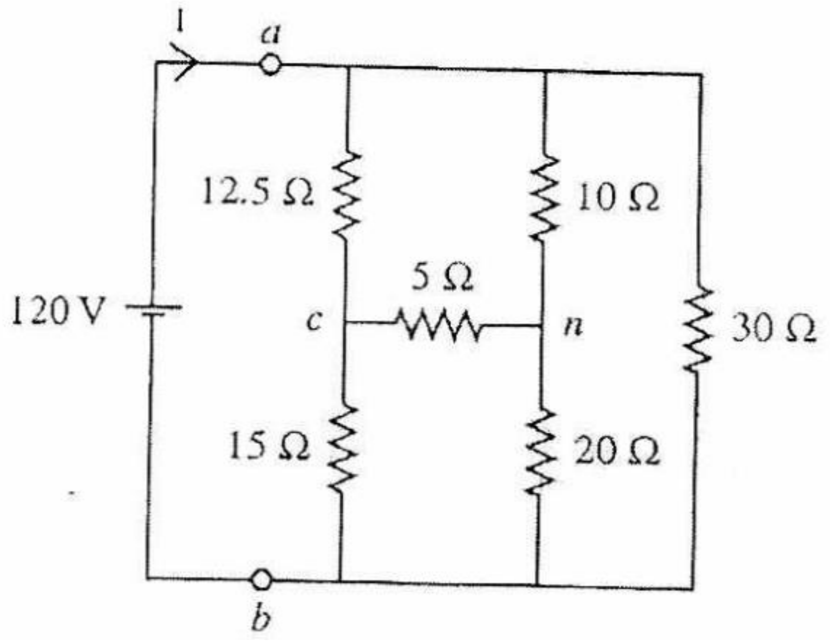
$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$

$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$

$$= 0.75\ \Omega + 2.139\ \Omega$$

$$R_T = \mathbf{2.889\ \Omega}$$

EX: Find the current (I) for the circuit shown in figure below.



Solution: $I = \frac{E}{R_{eq}}$

If we convert the (**Y**) network comprising (5 **Ω**, 10 **Ω** and 20 **Ω**) resistors, then:

$$R_a = \frac{5 * 10 + 10 * 20 + 20 * 5}{20} = \frac{350}{20} = 17.5\ \Omega$$

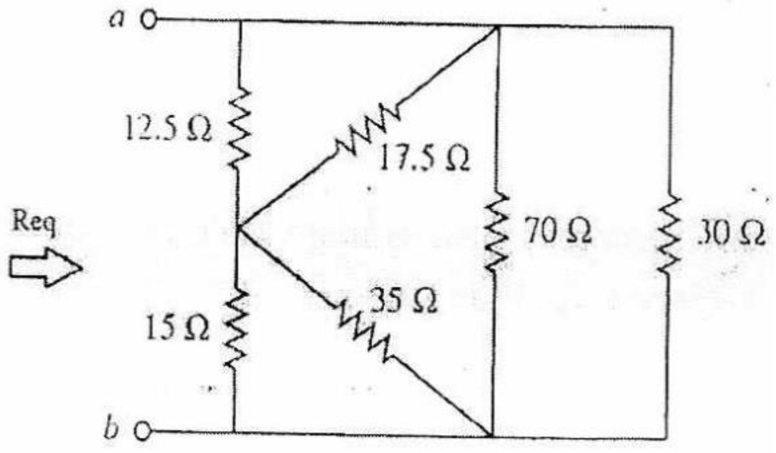
$$R_b = \frac{350}{5} = 70\ \Omega$$

$$R_c = \frac{350}{10} = 35\ \Omega$$

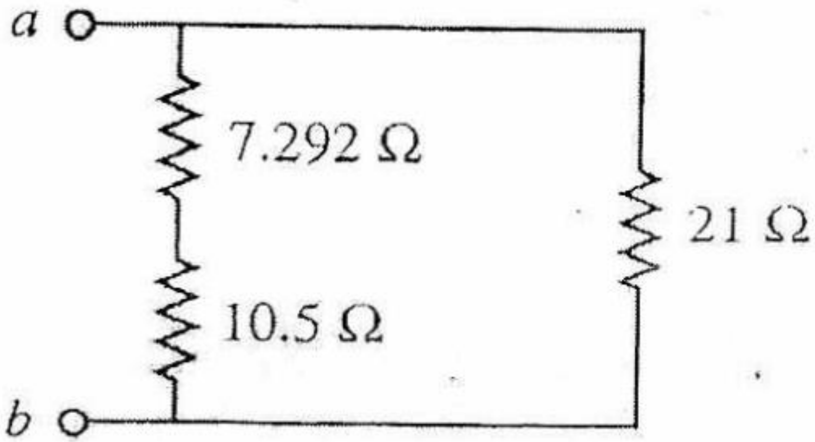
$$12.5 \,||\, 17.5 = \frac{12.5 * 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \,||\, 35 = \frac{15 * 35}{15 + 35} = 10.5\ \Omega$$

$$70 \,||\, 30 = \frac{70 * 30}{70 + 30} = 21\ \Omega$$



$$\therefore R_{eq} = (7.292 + 10.5) \,||\, 21 = \frac{17.792 * 21}{17.792 + 21} = 9.632\ \Omega$$



$$\therefore I = \frac{12}{9.632} = 12.458\ A$$