Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Second Class

Electronic II

Chapter 5

Lec4

BJT AC Analysis

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19 The Hybrid Equivalent Model

The hybrid equivalent model was mentioned in the earlier sections of this chapter as one that was used in the early years before the popularity of the *re* model developed. Today there is a mix of usage depending on the level and direction of the investigation.

The re model has the advantage that the parameters are defined by the actual operating conditions,

whereas

the parameters of the hybrid equivalent circuit are defined in general terms for any operating conditions.

In other words, the hybrid parameters may not reflect the actual operating conditions but simply provide an indication of the level of each parameter to expect for general use. The *re* model suffers from the fact that parameters such as the output impedance and the feedback elements are not available, whereas the hybrid parameters provide the entire set on the specification sheet. In most cases, if the *re* model is employed, the investigator will simply examine the specification sheet to have some idea of what the additional elements might be.

This section will show how one can go from one model to the other and how the parameters are related. Because all specification sheets provide the hybrid parameters and the model is still extensively used, it is important to be aware of both models. The hybrid parameters as shown in Fig. 92 are derived from the specification sheet for the 2N4400 transistor. In addition, a range of values is provided for each parameter for guidance in the initial design or analysis of a system. One obvious advantage of the specification sheet listing is the immediate knowledge of typical levels for the parameters of the device as compared to other transistors.

		Min.	Max.	
Input impedance (I_C = 1 mA dc, V_{CE} = 10 V dc, f = 1 kHz)	h_{ie}	0.5	7.5	kΩ
Voltage feedback ratio (I_C = 1 mA dc, V_{CE} = 10 V dc, f = 1 kHz)	h_{re}	0.1	8.0	$\times 10^{-4}$
Small-signal current gain (I_C = 1 mA dc, V_{CE} = 10 V dc, f = 1 kHz)	h_{fe}	20	250	—
Output admittance (I_C = 1 mA dc, V_{CE} = 10 V dc, f = 1 kHz)	h_{oe}	1.0	30	1μS

FIG. 92
Hybrid parameters for the 2N4400 transistor.

The description of the hybrid equivalent model will begin with the general two-port system of Fig. 93. The following set of equations (131) and (132) is only one of a number of ways in which the four variables of Fig. 93 can be related.

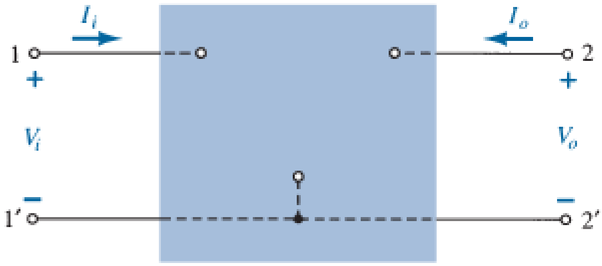

FIG. 93
Two-port system.

$$V_i = h_{11}I_i + h_{12}V_o \qquad (133)$$

$$I_o = h_{21}I_i + h_{22}V_o \qquad (134)$$

The parameters relating the four variables are called *h-parameters*, from the word "hybrid." The term *hybrid* was chosen because the mixture of variables (*V* and *I*) in each equation results in a "hybrid" set of units of measurement for the *h*-parameters. A clearer understanding of what the various *h*-parameters represent and how we can determine their magnitude can be developed by isolating each and examining the resulting relationship.

***h*11** If we arbitrarily set *Vo* = 0 (short circuit the output terminals) and solve for *h*11 in Eq. (133), we find

$$h_{11} = \left.\frac{V_i}{I_i}\right|_{V_o=0} \quad \text{ohms} \qquad (135)$$

The ratio indicates that the parameter *h*11 is an impedance parameter with the units of ohms. Because it is the ratio of the *input* voltage to the *input* current with the output terminals *shorted*, it is called the *short-circuit input-impedance parameter*. The subscript 11 of *h*11 refers to the fact that the parameter is determined by a ratio of quantities measured at the input terminals.

***h*12** If *Ii* is set equal to zero by opening the input leads, the following results for *h*12:

$$h_{12} = \left.\frac{V_i}{V_o}\right|_{I_i=0} \quad \text{unitless} \qquad (136)$$

It has no units because it is a ratio of voltage levels and is called the open-circuit reverse transfer voltage ratio parameter.

***h*21** If in Eq. (134) *Vo* is set equal to zero by again shorting the output terminals, the following results for $h21$:

$$h_{21} = \left.\frac{I_o}{I_i}\right|_{V_o=0} \quad \text{unitless} \qquad (137)$$

It is formally called the short circuit forward transfer current ratio parameter.

***h*22** The last parameter, $h22$, can be found by again opening the input leads to set $I1 = 0$ and solving for $h22$ in Eq. (134):

$$h_{22} = \left.\frac{I_o}{V_o}\right|_{I_i=0} \quad \text{siemens} \qquad (138)$$

Because it is the ratio of the output current to the output voltage, it is the output conductance parameter, and it is measured in siemens (S). It is called the ***open-circuit output admittance parameter.***

Because each term of Eq. (133) has the unit volt, let us apply Kirchhoff's voltage law "in reverse" to find a circuit that "fits" the equation. Performing this operation results in the circuit of Fig. 94. Because the parameter $h11$ has the unit ohm, it is represented by a resistor in Fig. 94. The quantity $h12$ is dimensionless and therefore simply appears as a multiplying factor of the "feedback" term in the input circuit. Because each term of Eq. (134) has the units of current, let us now apply Kirchhoff's current law "in reverse" to obtain the circuit of Fig. 95. Because $h22$ has the units of admittance, which for the transistor model is conductance, it is represented by the resistor symbol. Keep in mind, however, that the resistance in ohms of this resistor is equal to the reciprocal of conductance ($1/h22$).

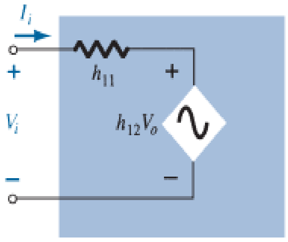

FIG. 94
Hybrid input equivalent circuit.

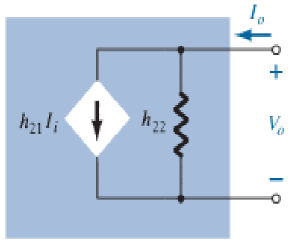

FIG. 95
Hybrid output equivalent circuit.

The complete "ac" equivalent circuit for the basic three-terminal linear device is indicated in Fig. 96 with a new set of subscripts for the *h*-parameters. The notation of Fig. 96 is of a more practical nature because it relates the *h*-parameters to the resulting ratio obtained in the last few paragraphs. The choice of letters is obvious from the following listing:

*h*11 →*i*nput resistance →*hi* *h*12 →*r*everse transfer voltage ratio →*hr*

*h*21→ *f*orward transfer current ratio →*hf* *h*22 →*o*utput conductance →*ho*

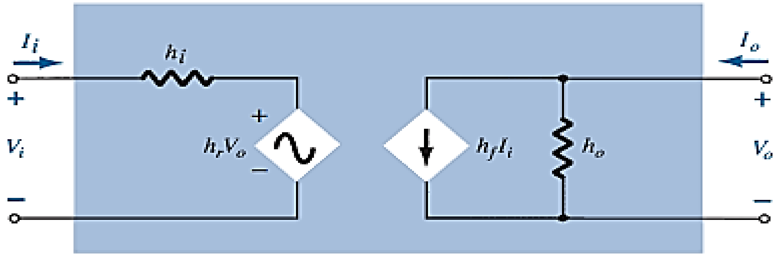

FIG. 96
Complete hybrid equivalent circuit.

In each case, the bottom of the input and output sections of the network of Fig. 96 can be connected as shown in Fig. 97 because the potential level is the same. Essentially, therefore, the transistor model is a three-terminal two-port system. The *h*-parameters, however, will change with each configuration.

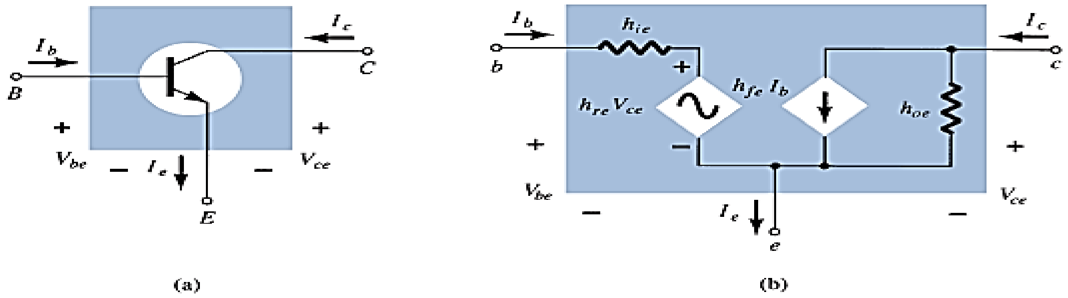

FIG. 97
Common-emitter configuration: (a) graphical symbol; (b) hybrid equivalent circuit.

For the common-base configuration, the lowercase letter *b* was added, whereas for the common-emitter and common-collector configurations, the letters *e* and *c* were added, respectively. The hybrid equivalent network for the common-emitter configuration appears with the standard notation in Fig. 97. Note that *Ii* = *Ib*, *Io* = *Ic*, and, through an application of Kirchhoff's current law, *Ie* = *Ib* + *Ic*. The input voltage is now *Vbe*, with the output voltage *Vce*.

For the common-base configuration of Fig. 98, *Ii* = *Ie*, *Io* = *Ic* with *Veb* = *Vi* and *Vcb* = *Vo*. The networks of Figs. 97 and 98 are applicable for *pnp* or *npn* transistors.

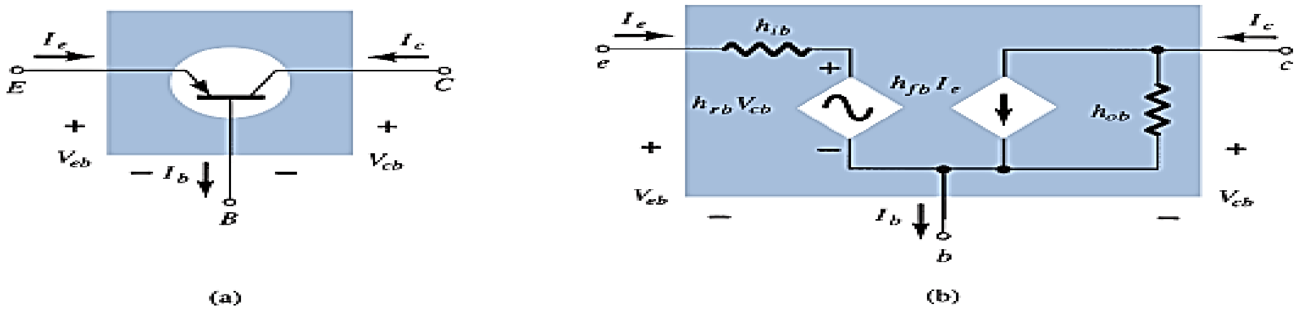

FIG. 98
Common-base configuration: (a) graphical symbol; (b) hybrid equivalent circuit.

For the common-emitter and common-base configurations, the magnitude of *hr* and *ho* is often such that the results obtained for the important parameters such as *Zi*, *Zo*, *Av*, and *Ai* are only slightly affected if *hr* and *ho* are not included in the model. Because *hr* is normally a relatively small quantity, its removal is approximated by *hr* = 0 and *hrVo* = 0, resulting in a short-circuit equivalent for the feedback element as shown in Fig. 99.

The resistance determined by 1/*ho* is often large enough to be ignored in comparison to a parallel load, permitting its replacement by an open-circuit equivalent for the CE and CB models, as shown in Fig. 99. The resulting equivalent of Fig. 100 is quite similar to the general structure of the common-base and common-emitter equivalent circuits obtained with the *re* model.

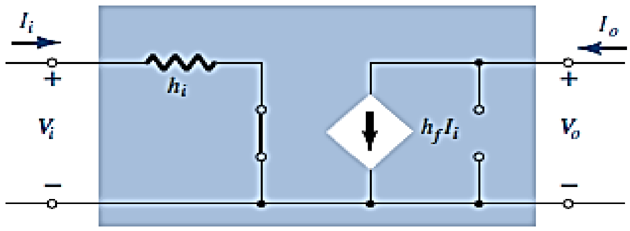

FIG. 99
Effect of removing h_{re} and h_{oe} from the hybird equivalent circuit.

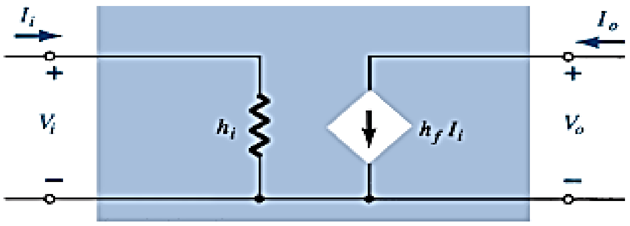

FIG. 100
Approximate hybrid equivalent model.

In fact, the hybrid equivalent and the re models for each configuration are repeated in Fig. 101 for comparison. It should be reasonably clear from Fig. 101a that

$$h_{ie} = \beta r_e \quad (139)$$

$$h_{fe} = \beta_{ac} \quad (140)$$

From Fig. 101b,

$$h_{ib} = r_e \quad (141)$$

$$h_{fb} = -\alpha \cong -1 \quad (142)$$

In particular, note that the minus sign in Eq. (142) accounts for the fact that the current source of the standard hybrid equivalent circuit is pointing down rather than in the actual direction as shown in the *re* model of Fig. 101b.

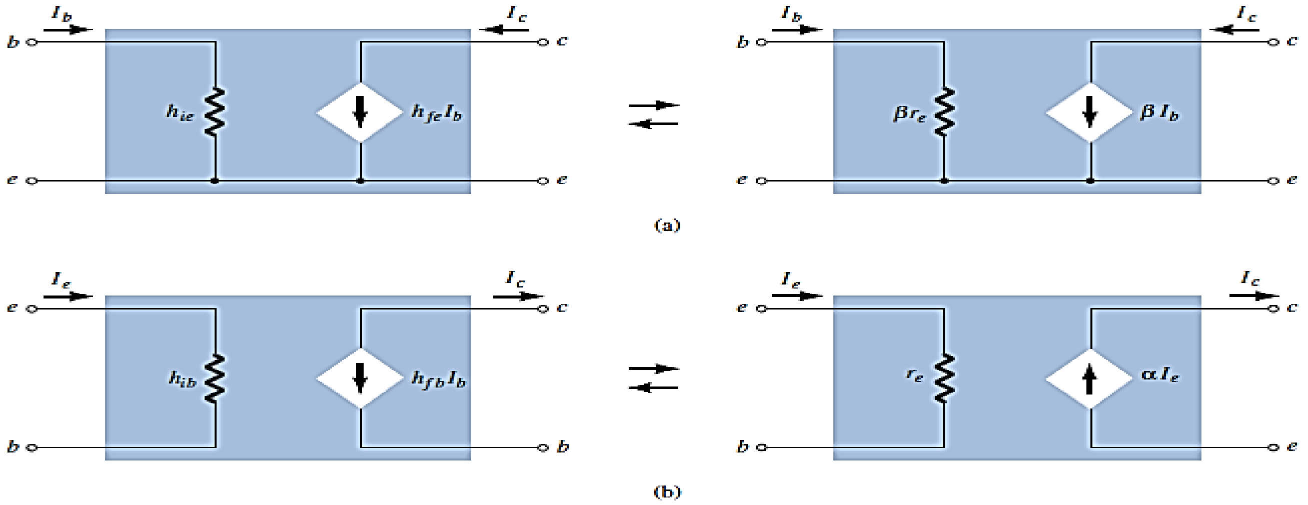

FIG. 101
Hybrid versus r_e model: (a) common-emitter configuration; (b) common-base configuration.

EXAMPLE 19 Given $I_E = 2.5$ mA, $h_{fe} = 140$, $h_{oe} = 20\ \mu S$ (μmho), and $h_{ob} = 0.5\ \mu S$, determine:

a. The common-emitter hybrid equivalent circuit.
b. The common-base r_e model.

Solution:

a. $$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{2.5\text{ mA}} = \mathbf{10.4\ \Omega}$$

$$h_{ie} = \beta r_e = (140)(10.4\ \Omega) = \mathbf{1.456\ k\Omega}$$

$$r_o = \frac{1}{h_{oe}} = \frac{1}{20\ \mu S} = \mathbf{50\ k\Omega}$$

Note Fig. 102.

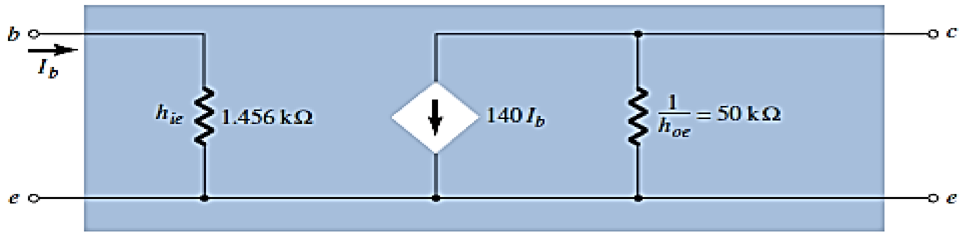

FIG. 102
Common-emitter hybrid equivalent circuit for the parameters of Example 19.

b. $r_e = \mathbf{10.4\ \Omega}$

$$\alpha \cong \mathbf{1}, \qquad r_o = \frac{1}{h_{ob}} = \frac{1}{0.5\ \mu S} = \mathbf{2\ M\Omega}$$

Note Fig. 103.

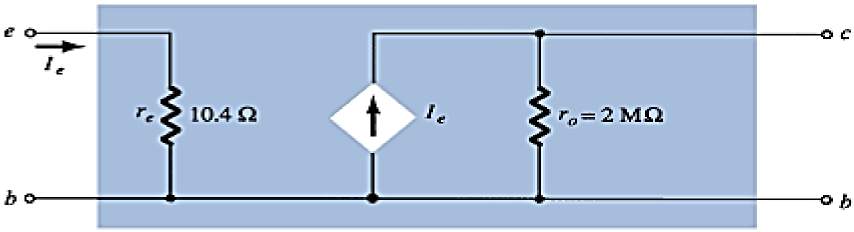

FIG. 103
Common-base r_e model for the parameters of Example 19.

20 Approximate Hybrid Equivalent Circuit

The analysis using the approximate hybrid equivalent circuit of Fig. 104 for the common emitter configuration and of Fig. 105 for the common-base configuration is very similar to that just performed using the *re* model. A brief overview of some of the most important configurations will be included in this section to demonstrate the similarities in approach and the resulting equations.

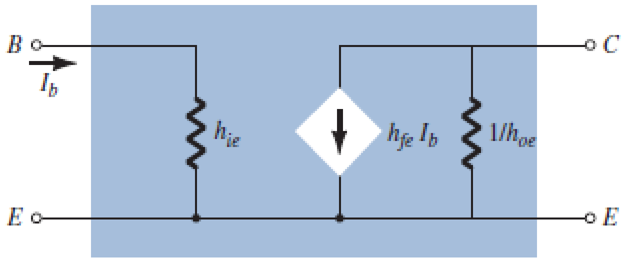

FIG. 104
Approximate common-emitter hybrid equivalent circuit.

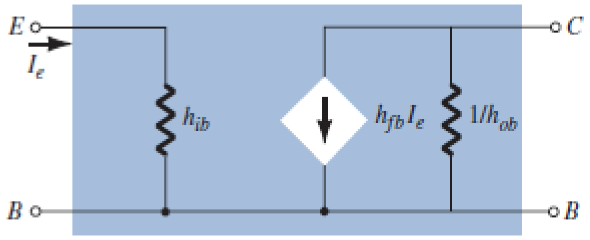

FIG. 105
Approximate common-base hybrid equivalent circuit.

Because the various parameters of the hybrid model are specified by a data sheet or experimental analysis, the dc analysis associated with use of the *re* model is not an integral part of the use of the hybrid parameters. In other words, when the problem is presented, the parameters such as *hie*, *hfe*, *hib*, and so on, are specified. Keep in mind, however, that the hybrid parameters and components of the *re* model are related by the following equations, as discussed earlier in this chapter: *hie* = *βre*, *hfe* = *β*, *hoe* = 1/*ro*, *hfb* = -α, and *hib* = *re*.

20.1 Fixed-Bias Configuration

For the fixed-bias configuration of Fig. 106, the small-signal ac equivalent network will appear as shown in Fig. 107 using the approximate common-emitter hybrid equivalent model. Compare the similarities in appearance with Fig. 22 and the *re* model analysis. The similarities suggest that the analyses will be quite similar, and the results of one can be directly related to the other.

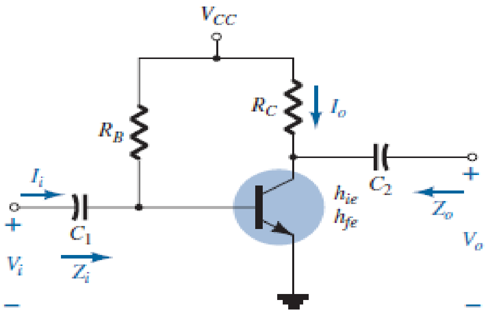

FIG. 106
Fixed-bias configuration.

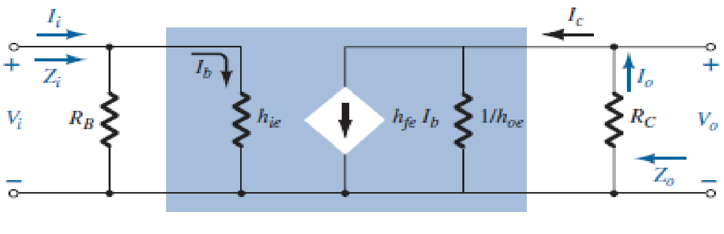

FIG. 107
Substituting the approximate hybrid equivalent circuit into the ac equivalent network of Fig. 106.

Z_i From Fig. 107,

$$Z_i = R_B \| h_{ie} \tag{143}$$

Z_o From Fig. 107,

$$Z_o = R_C \| 1/h_{oe} \tag{144}$$

A_v Using $R' = 1/h_{oe} \| R_C$, we obtain

$$V_o = -I_o R' = -I_C R' = -h_{fe} I_b R'$$

and

$$I_b = \frac{V_i}{h_{ie}}$$

with

$$V_o = -h_{fe} \frac{V_i}{h_{ie}} R'$$

so that

$$A_v = \frac{V_o}{V_i} = -\frac{h_{fe}\ (R_C \| 1/h_{oe})}{h_{ie}} \tag{145}$$

A_i Assuming that $R_B \gg h_{ie}$ and $1/h_{oe} \geq 10R_C$, we find $I_b \cong I_i$ and $I_o = I_c = h_{fe} I_b = h_{fe} I_i$, and so

$$A_i = \frac{I_o}{I_i} \cong h_{fe} \tag{146}$$

EXAMPLE 20 For the network of Fig. 108, determine:

a. Z_i.
b. Z_o.
c. A_v.
d. A_i.

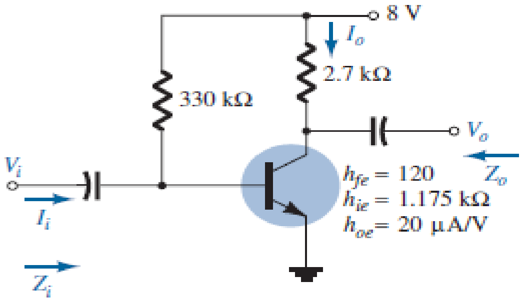

FIG. 108
Example 20.

Solution:

a. $Z_i = R_B \| h_{ie} = 330\ \text{k}\Omega \| 1.175\ \text{k}\Omega$
$\cong h_{ie} = \mathbf{1.171\ k\Omega}$

b. $r_o = \frac{1}{h_{oe}} = \frac{1}{20\ \mu\text{A/V}} = 50\ \text{k}\Omega$

$Z_o = \frac{1}{h_{oe}} \| R_C = 50\ \text{k}\Omega \| 2.7\ \text{k}\Omega = \mathbf{2.56\ k\Omega} \cong R_C$

c. $A_v = -\frac{h_{fe}(R_C \| 1/h_{oe})}{h_{ie}} = -\frac{(120)(2.7\ \text{k}\Omega \| 50\ \text{k}\Omega)}{1.171\ \text{k}\Omega} = \mathbf{-262.34}$

d. $A_i \cong h_{fe} = \mathbf{120}$

20.2 Voltage-Divider Configuration

For the voltage-divider bias configuration of Fig. 109, the resulting small-signal ac equivalent network will have the same appearance as Fig. 107, with *RB* replaced by $R' = R1 \| R2$.

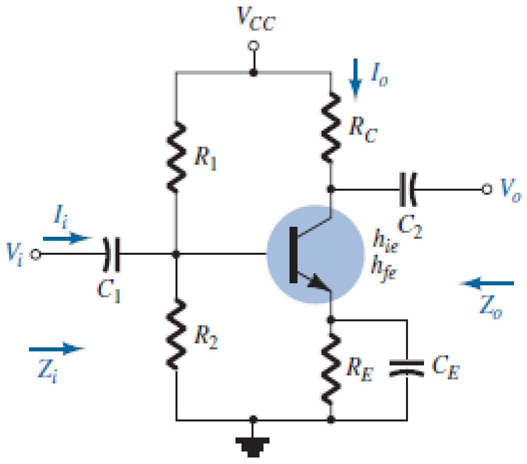

FIG. 109
Voltage-divider bias configuration.

Z_i From Fig. 107 with $R_B = R'$,

$$Z_i = R_1 \| R_2 \| h_{ie} \tag{147}$$

Z_o From Fig. 107,

$$Z_o \cong R_C \tag{148}$$

A_v

$$A_v = -\frac{h_{fe}(R_C \| 1/h_{oe})}{h_{ie}} \tag{149}$$

A_i

$$A_i = \frac{h_{fe}(R_1 \| R_2)}{R_1 \| R_2 + h_{ie}} \tag{150}$$

20.3 Unbypassed Emitter-Bias Configuration

For the CE unbypassed emitter-bias configuration of Fig. 110, the small-signal ac model will be the same as Fig. 30, with *βre* replaced by *hie* and *βIb* by *hfeIb*. The analysis will proceed in the same manner.

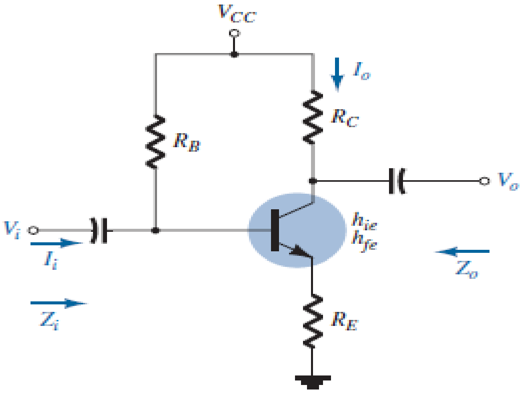

FIG. 110
CE unbypassed emitter-bias configuration.

Z_i

$$Z_b \cong h_{fe} R_E \tag{151}$$

and

$$Z_i = R_B \| Z_b \tag{152}$$

Z_o

$$Z_o = R_C \tag{153}$$

A_v

$$A_v = -\frac{h_{fe}R_C}{Z_b} \cong -\frac{h_{fe}R_C}{h_{fe}R_E}$$

and

$$A_v \cong -\frac{R_C}{R_E} \tag{154}$$

A_i

$$A_i = -\frac{h_{fe}R_B}{R_B + Z_b} \tag{155}$$

or

$$A_i = -A_v \frac{Z_i}{R_C} \tag{156}$$

20.4 Emitter-Follower Configuration

For the emitter-follower of Fig. 38, the small-signal ac model will match that of Fig. 111, with β*re* = *hie* and β = *hfe*. The resulting equations will therefore be quite similar.

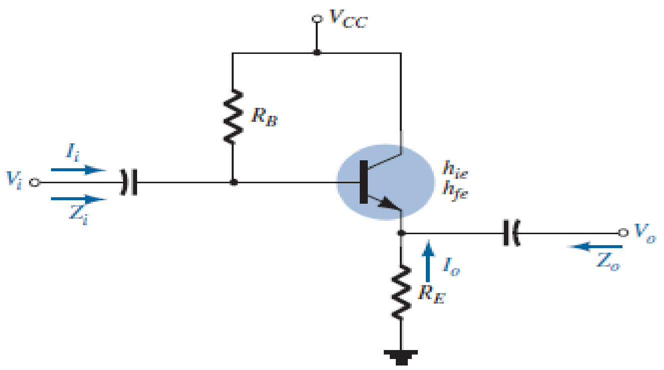

FIG. 111
Emitter-follower configuration.

Z_i

$$Z_b \cong h_{fe} R_E \quad (157)$$

$$Z_i = R_B \| Z_b \quad (158)$$

Zo For *Zo*, the output network defined by the resulting equations will appear as shown in Fig. 112. Review the development of the equations in Section 8 and

$$Z_o = R_E \| \frac{h_{ie}}{1 + h_{fe}}$$

or, because $1 + h_{fe} \cong h_{fe}$,

$$Z_o \cong R_E \| \frac{h_{ie}}{h_{fe}} \quad (159)$$

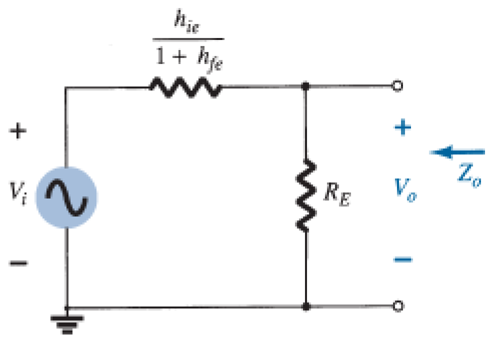

FIG. 112
Defining Z_o for the emitter-follower configuration.

A_V For the voltage gain, the voltage-divider rule can be applied to Fig. 112 as follows:

$$V_o = \frac{R_E(V_i)}{R_E + h_{ie}/(1 + h_{fe})}$$

but, since $1 + h_{fe} \cong h_{fe}$,

$$A_v = \frac{V_o}{V_i} \cong \frac{R_E}{R_E + h_{ie}/h_{fe}} \quad (160)$$

A_i

$$A_i = \frac{h_{fe} R_B}{R_B + Z_b} \quad (161)$$

or

$$A_i = -A_v \frac{Z_i}{R_E} \quad (162)$$

20.5 Common-base Configuration

The last configuration to be examined with the approximate hybrid equivalent circuit will be the common-base amplifier of Fig. 113. Substituting the approximate common-base hybrid equivalent model results in the network of Fig. 114, which is very similar to Fig. 44.

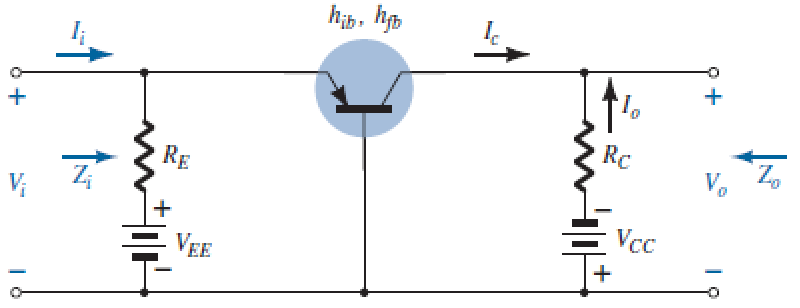

FIG. 113
Common-base configuration.

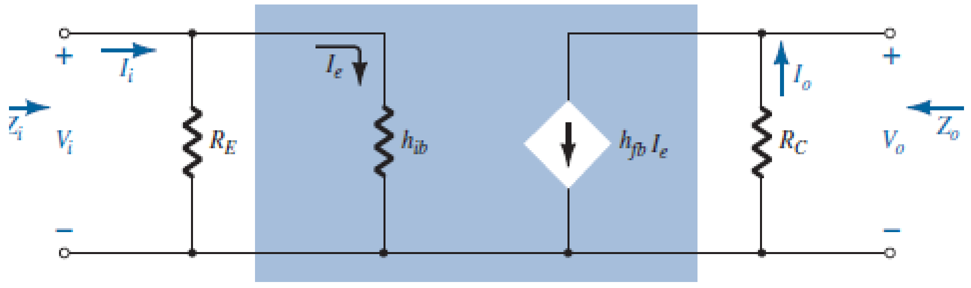

FIG. 114
Substituting the approximate hybrid equivalent circuit into the ac equivalent network of Fig. 113.

We have the following results from Fig. 114.

Z_i

$$Z_i = R_E \| h_{ib} \tag{163}$$

Z_o

$$Z_o = R_C \tag{164}$$

A_v

$$V_o = -I_o R_C = -(h_{fb} I_e) R_C$$

with $I_e = \dfrac{V_i}{h_{ib}}$ and $V_o = -h_{fb}\dfrac{V_i}{h_{ib}} R_C$

so that

$$A_v = \frac{V_o}{V_i} = -\frac{h_{fb} R_C}{h_{ib}} \tag{165}$$

A_i

$$A_i = \frac{I_o}{I_i} = h_{fb} \cong -1 \tag{166}$$

EXAMPLE 21 For the network of Fig. 115, determine:

a. Z_i.
b. Z_o.
c. A_v.
d. A_i.

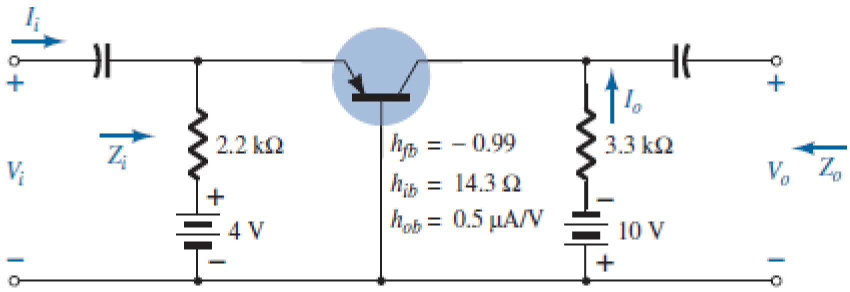

FIG. 115
Example 21.

Solution:

a. $Z_i = R_E \| h_{ib} = 2.2\text{ k}\Omega \| 14.3\ \Omega = \mathbf{14.21\ \Omega} \cong h_{ib}$

b. $r_o = \dfrac{1}{h_{ob}} = \dfrac{1}{0.5\ \mu\text{A/V}} = \mathbf{2\ M\Omega}$

$Z_o = \dfrac{1}{h_{ob}} \| R_C \cong R_C = \mathbf{3.3\ k\Omega}$

c. $A_v = -\dfrac{h_{fb} R_C}{h_{ib}} = -\dfrac{(-0.99)(3.3\text{ k}\Omega)}{14.21} = \mathbf{229.91}$

d. $A_i \cong h_{fb} = \mathbf{-1}$