Continuous-Time and Discrete-Time Signals:

A discrete-time signal ***x[n]*** can be defined in two ways:

1- We can specify a rule for calculating the n^{th} value of the sequence. For example,

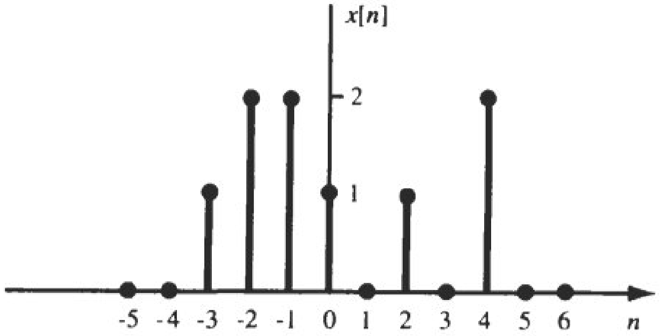
$$x[n] = x_n = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$\{x_n\} = \left\{1, \tfrac{1}{2}, \tfrac{1}{4}, \ldots, \left(\tfrac{1}{2}\right)^n, \ldots\right\}$$

2 - We can also explicitly list the values of the sequence. For example, the sequence shown in Figure below can be written as

$$\{x_n\} = \{\ldots, 0, 0, 1, 2, 2, \underset{\uparrow}{1}, 0, 1, 0, 2, 0, 0, \ldots\}$$

$$\{x_n\} = \{1, 2, 2, \underset{\uparrow}{1}, 0, 1, 0, 2\}$$



The arrow denotes the $\boldsymbol{n} = 0$ term. If no arrow is indicated, then the first term corresponds to $\boldsymbol{n} = 0$ and all the values of the sequence are zero for $\boldsymbol{n} < 0$.

The sum and product of two sequences are defined as follows:

$$\{c_n\} = \{a_n\} + \{b_n\} \longrightarrow c_n = a_n + b_n$$

$$\{c_n\} = \{a_n\}\{b_n\} \longrightarrow c_n = a_n b_n$$

$$\{c_n\} = \alpha\{a_n\} \longrightarrow c_n = \alpha a_n \qquad \alpha = \text{constant}$$

Analog and Digital Signals:

The concept of continuous-time is often confused with that of analog. The two are not the same. The same is true of the concepts of discrete-time and digital. A signal whose amplitude can take on any value in a continuous range is an analog signal. This means that an analog signal amplitude can take on an infinite number of values. A digital signal, on the other hand, is one whose amplitude can take on only a finite number of values. Signals associated with typical digital devices take on only two values (binary signals). The terms “continuous-time” and “discrete-time” qualify the nature of a signal along the time (horizontal) axis. The terms “analog” and “digital,” on the other hand, qualify the nature of the signal amplitude (vertical axis). Fig. 1.3 demonstrates the various differences.

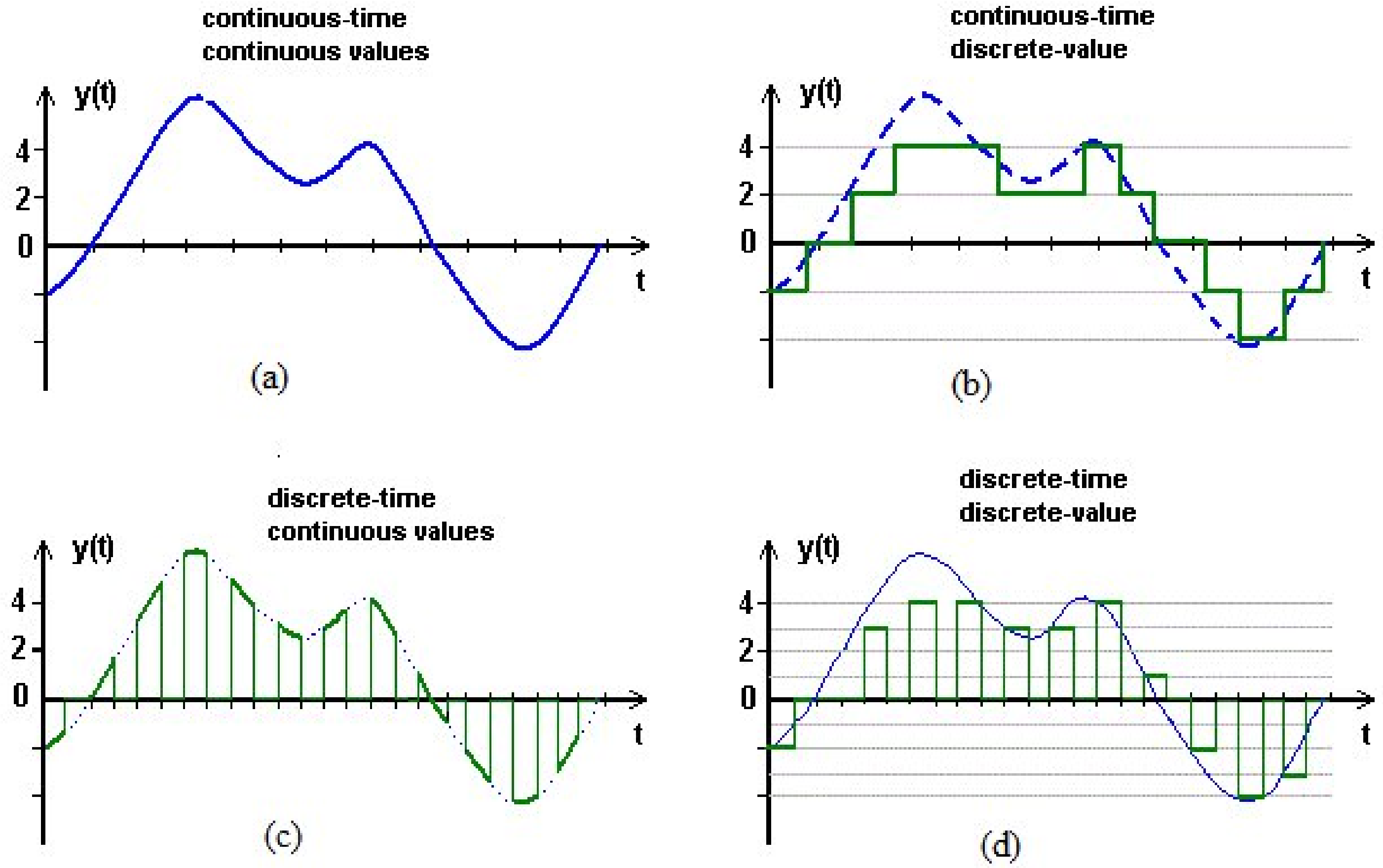


Fig. 1.3
(a) Analog continuous
(b) Digital Continuous
(c) Analog discrete
(d) Digital discrete

Real and Complex Signals:

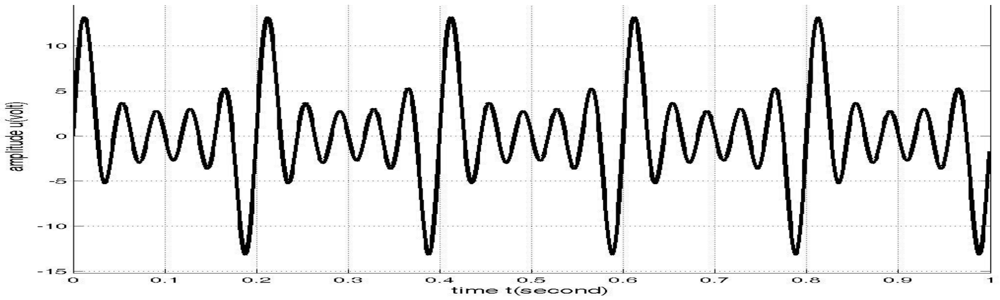
A signal $x(t)$ is a real signal if its value is a real number, and a signal $x(t)$ is a complex signal if its value is a complex number. A general complex signal $x(t)$ is a function of the form

$$x(t) = x_1(t) + j\,x_2(t)$$

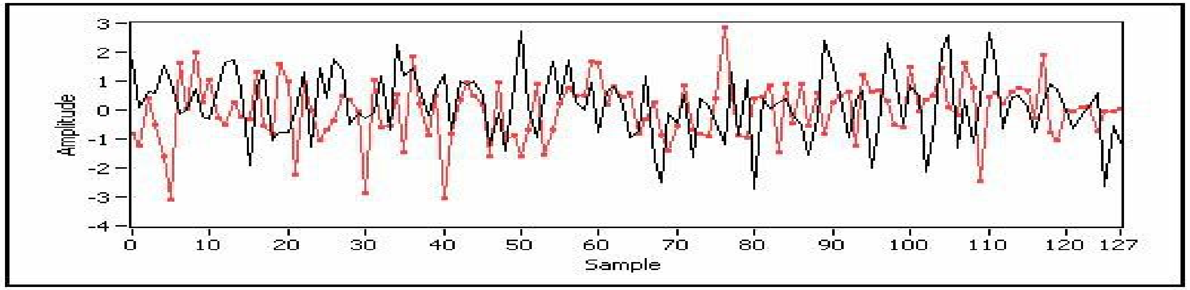
where $x_1(t)$ and $x_2(t)$ are real signals and $j = \sqrt{-1}$. Note that $x_1(t)$ and $x_2(t)$ can represent either a continuous or a discrete variable.

Deterministic and Random Signals:

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time t . Random signals are those signals that take random values at any given time and must be characterized statistically. Random signals cannot be characterized by a simple, well-defined mathematical equation and their future values cannot be predicted and called non-deterministic signals.



Deterministic signal example
Electrocardiography (ECG) signal



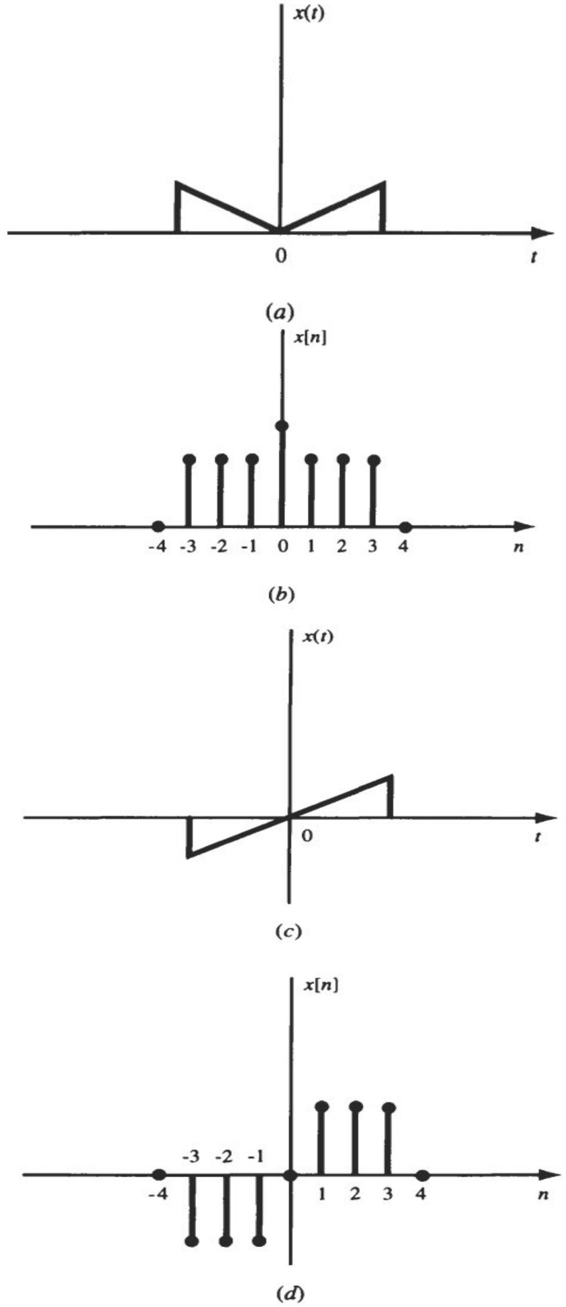
Random signal example noise signal

Even and Odd Signals:

A signal ***x(t)*** or ***x[n]*** is referred to as an even signal if

$$x(-t) = x(t)$$
$$x[-n] = x[n]$$

An even signal is symmetric about the origin.



A signal ***x(t)*** or ***x [n]*** is referred to as an odd signal if

$$x(-t) = -x(t)$$
$$x[-n] = -x[n]$$

An odd signal is antisymmetric about the origin. $x(t) = -x(-t)$.

Examples of even and odd signals are shown in Fig. 1-4

Fig. 1-4 Examples of even signals(a and b) and odd signals(c and d).

Even and Odd Signals:

Note that the product of two even signals or of two odd signals is an even signal and that the product of an even signal and an odd signal is an odd signal.

Example :

Show that the product of two even signals or of two odd signals is an even signal and that the product of an even and an odd signal is an odd signal.

Let $x(t) = x_1(t)x_2(t)$. If $x_1(t)$ and $x_2(t)$ are both even, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ and $x_2(t)$ are both odd, then

$$x(-t) = x_1(-t)x_2(-t) = -x_1(t)[-x_2(t)] = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ is even and $x_2(t)$ is odd, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)[-x_2(t)] = -x_1(t)x_2(t) = -x(t)$$

and $x(t)$ is odd. Note that in the above proof, variable t represents either a continuous or a discrete variable.

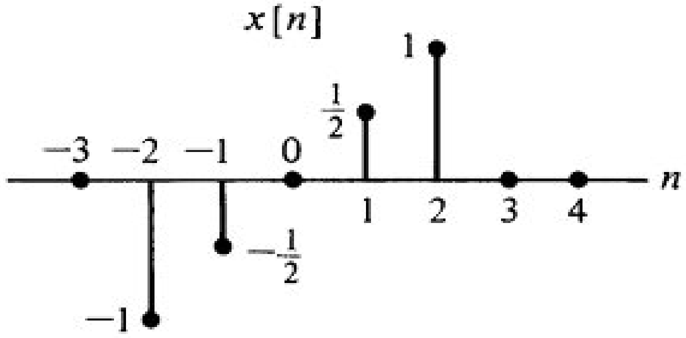
Example : consider the signal $x(t) = \begin{cases} \sin\left(\frac{\pi t}{T}\right), & T \le t \le T \\ 0, & otherwise \end{cases}$ Is the signal $x(t)$ an even or an odd function of time t

Solution:

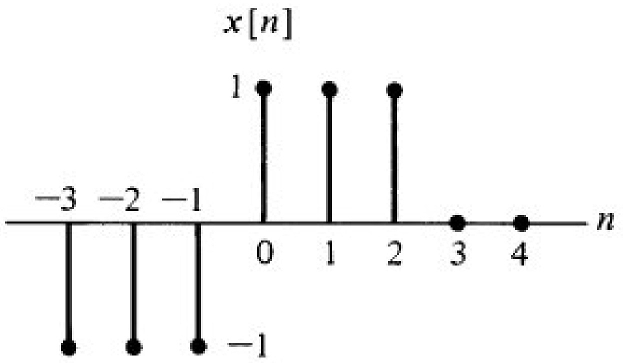
Replacing t with –t we get

$$x(-t) = \begin{cases} \sin\left(-\frac{\pi t}{T}\right), & T \le t \le T \\ 0, & otherwise \end{cases} = \begin{cases} -\sin\left(\frac{\pi t}{T}\right), & T \le t \le T \\ 0, & otherwise \end{cases} = -x(t) \quad \textit{Hence, } x(t) \textit{ is an odd signal.}$$

Example: For the following signal, determine whether it is even, odd, or neither.



(a)



(b)

Solution:

a) By definition a signal is even if and only if *x(t)* **=** *x(-t)* or *x[n] = x[-n]*, while a signal is odd if and only if *x(t)* **=** *-x(- t)* or *x[n]* **=** *-x[-n]*. Hence x(t) is odd.

b)

Here $x[n]$ seems like an odd signal at first glance. However, note that $x[n] = -x[-n]$ evaluated at $n = 0$ implies that $x[0] = -x[0]$ or $x[0] = 0$. The analogous result applies to continuous-time signals. The signal is therefore neither even nor odd.

A continuous-time signal ***x(t)*** is said to be periodic with period ***T*** if there is a positive nonzero value of ***T*** for which (See figures below)

$$x(t+T) = x(t) \qquad \text{for all } t$$

$$x(t+mT) = x(t) \qquad \text{for all } t \text{ and any integer } m$$

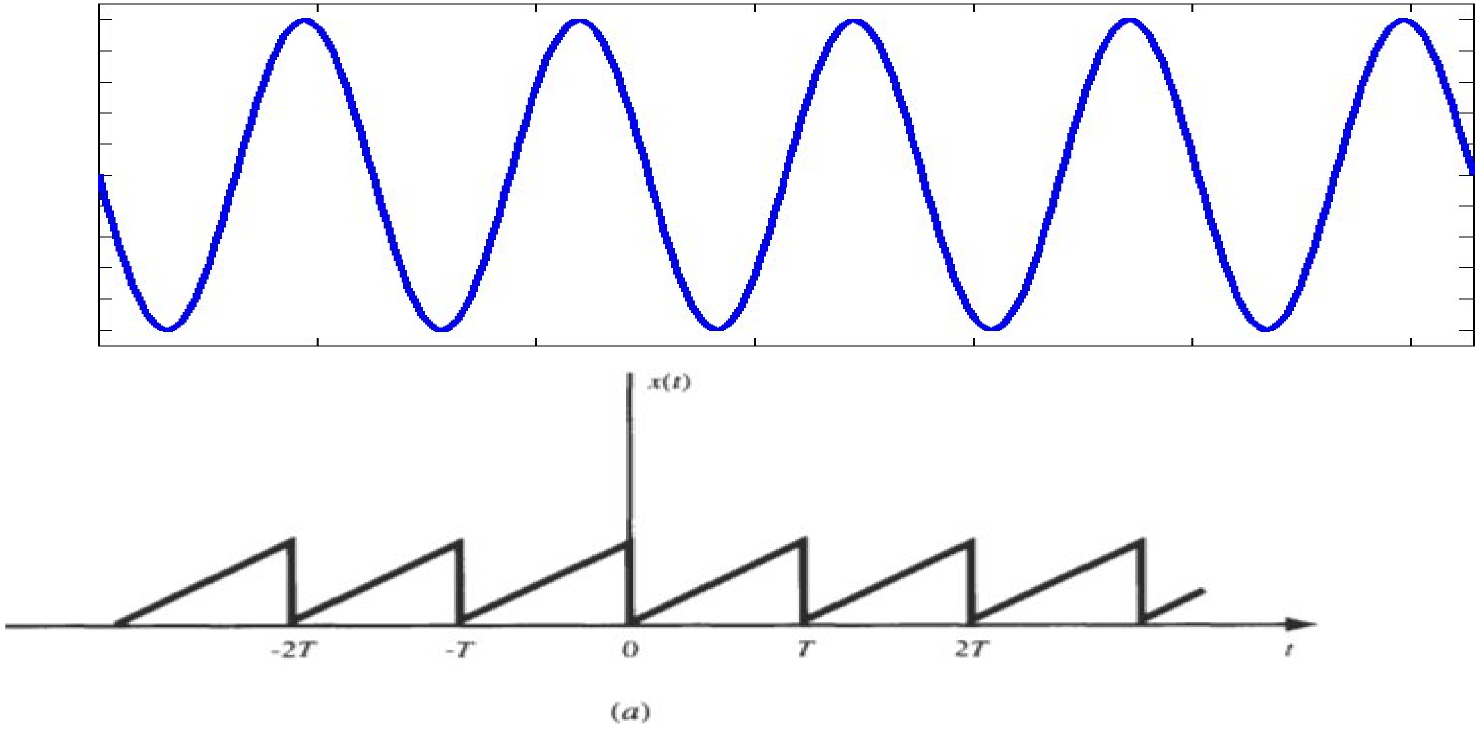


Fig. Examples of CT periodic signals

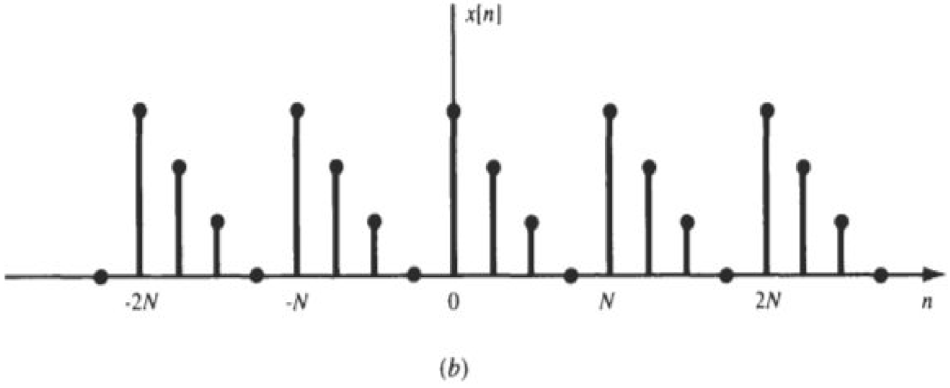
Periodic discrete-time signals are defined analogously. A sequence (discrete-time signal) $x[n]$ is periodic with period N if there is a positive integer N for which

$$x[n+N] = x[n] \qquad \text{for all } n$$

An example of such a sequence is given in Fig. 1-5(b). Also, it follows that

$$x[n+mN] = x[n] \text{ for all } n \text{ and any integer } m$$

Fig. Examples of DT periodic signals



Any continuous-time signal, which is not periodic, is called a nonperiodic (or aperiodic) signal. Also, any sequence which is not periodic is called a nonperiodic or aperiodic sequence. Note that a sequence obtained by uniform sampling of a periodic continuous-time signal may not be periodic. Note also that the sum of two continuous-time periodic signals may not be periodic but that the sum of two periodic sequences is always periodic.

Energy and Power Signals:

Consider ***v(t)*** to be the voltage across a resistor ***R*** producing a current ***i(t).*** The instantaneous power ***p(t)*** per ohm is defined as

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t)$$

Total energy ***E*** and average power ***P*** on a per-ohm basis are

$$E = \int_{-\infty}^{\infty} i^2(t)\,dt \quad \text{joules}$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} i^2(t)\,dt \quad \text{watts}$$

For an arbitrary continuous-time signal ***x(t),*** the normalized energy content ***E*** of ***x(t)*** is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

The normalized average power ***P*** of ***x(t)*** is defined as

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt$$

Similarly, for a discrete-time signal ***x[n],*** the normalized energy content ***E*** of ***x[n]*** is defined as

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

The normalized average power ***P*** of ***x[n]*** is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

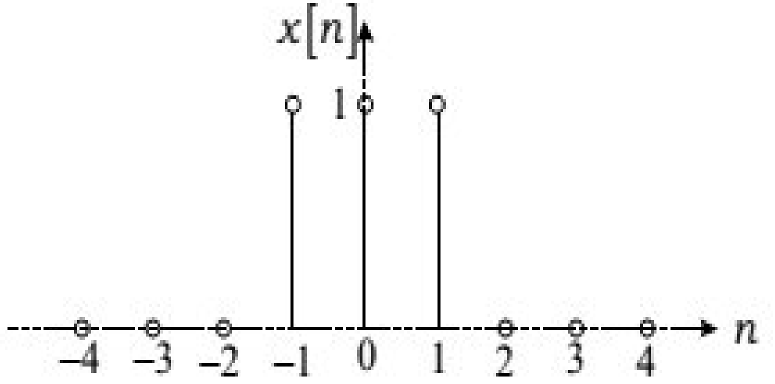
Based on definitions the following classes of signals are defined:

1. ***x(t)*** (or ***x[n]***) is said to be an energy signal (or sequence) if and only if $0 < E < \infty$, and so $P = 0$.
2. ***x(t) (or x[n])*** is said to be a power signal (or sequence) if and only if $0 < P < \infty$, thus implying that $E = \infty$.
3. Signals that satisfy neither property are referred to as neither energy signals nor power signals.

Note that a periodic signal is a power signal if its energy content per period is finite, and then the average power of this signal need only be calculated over a period.

Example: a sine wave of infinite length is a power signal.

Example : Determine the total energy of the discrete-time signal shown in Fig.

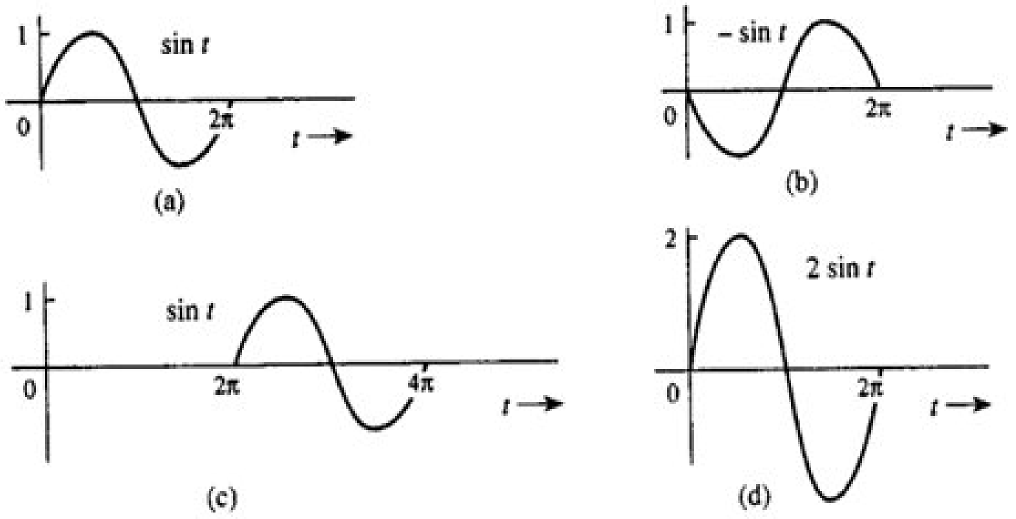


Solution:

$$E = \sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-1}^{1} x^2[n] = 1 + 1 + 1 = 3$$

Energy and Power Signals:

Example: Find the energies of the following signals. Comment on the effect on energy of sign change, time shifting or doubling of the signal?



Solution $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$

For parts (a) and (b)

$$E_f = \int_0^{2\pi} \sin^2 t\, dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\, dt = \pi + 0 = \pi$$

(c) $$E_f = \int_{2\pi}^{4\pi} \sin^2 t\, dt = \frac{1}{2}\int_{2\pi}^{4\pi} dt - \frac{1}{2}\int_{2\pi}^{4\pi} \cos 2t\, dt = \pi + 0 = \pi$$

(d) $$E_f = \int_0^{2\pi} (2\sin t)^2\, dt = 4\left[\frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\, dt\right] = 4[\pi + 0] = 4\pi$$

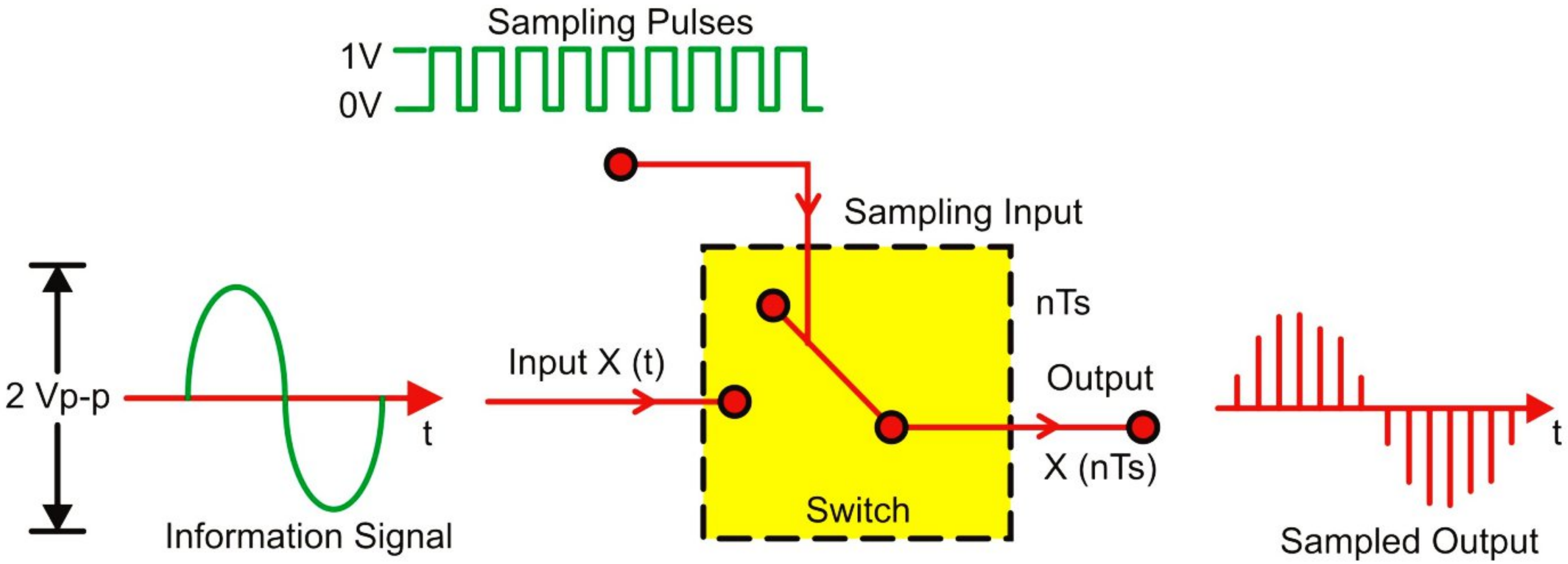
Hint:
$\sin^2(x) = 1/2 - 1/2 \cos(2x)$

No effect of sign change and shifting the signal on its energy. Multiplication signal by a constant **k** increases its energy by a factor **k^2**.

Sampling Theorem and Nyquist Rate

Sampling means taking several samples from the modulating signal (information or message signal which will be transmitted). From this signal, several samples will be taken to represent the waveform. Sampling is common in all pulse modulation techniques.

Consider an analogue signal ***x(t)*** that can be viewed as a continuous function of time, as shown in figure. We can represent this signal as a discrete time signal by using values of x(t) at intervals of nTs to form x(nTs) as shown in figure. We are "grabbing" points from the function x(t) at regular intervals of time, Ts, called the sampling period.



Basic Sampling Process

Sampling Theorem and Nyquist Rate

Analog signal is sampled every T_s secs. T_s is referred to as the sampling interval. $f_s = 1/T_s$ is called the **sampling rate** or **sampling frequency**. Based on Shannon sampling theorem, minimum sampling frequency f_s should be twice or higher than the highest frequency of modulating signal f_m.

$$f_s \,\mathbf{min} \geq 2\, f_m$$

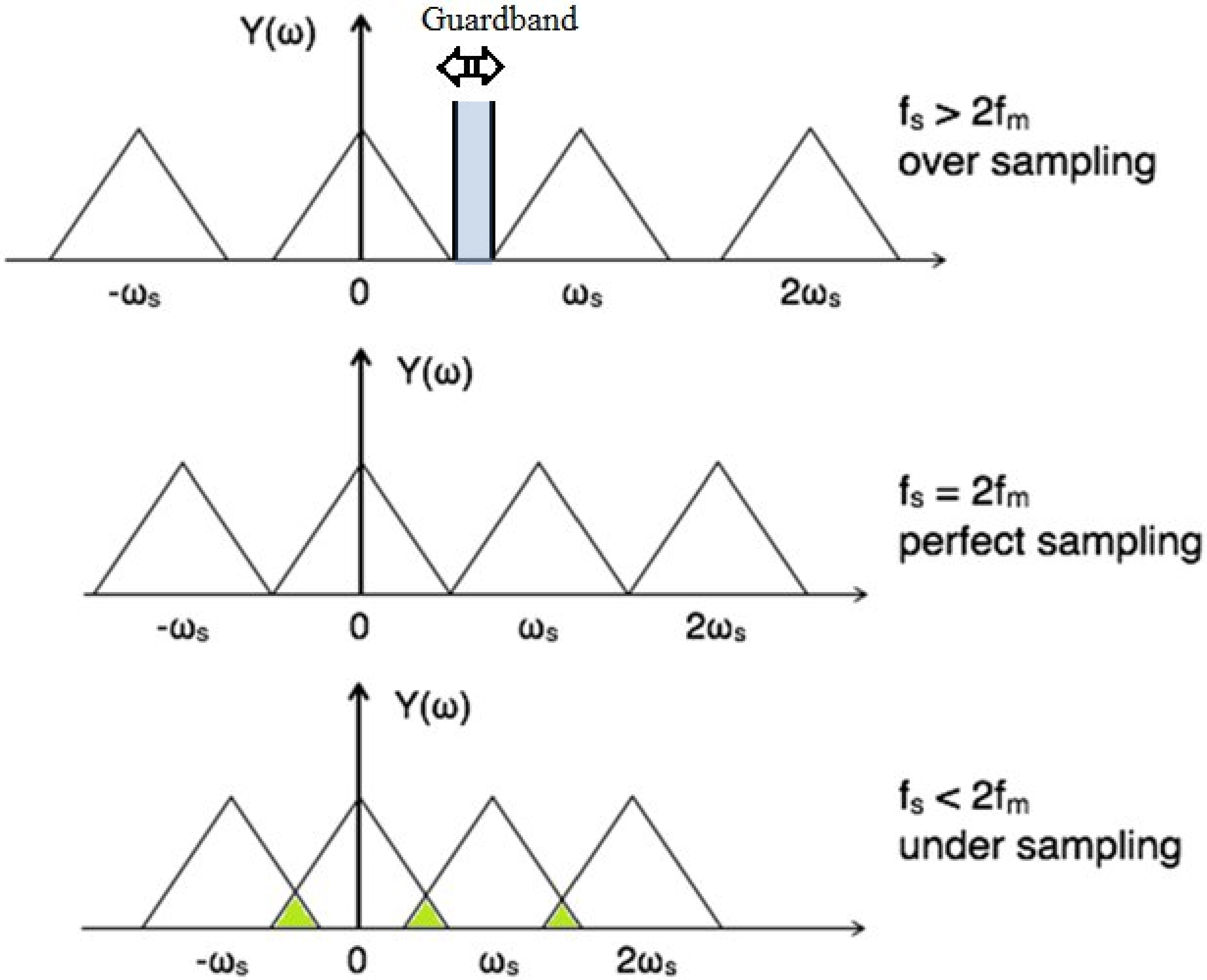
When $f_s = 2\, f_m$ the sampling rate is called **Nyquist rate**. Sampled spectrum is repeating periodically without overlapping.

Original spectrum is centred at $\omega = 0$ and having bandwidth of ω_m. Spectrum can be recovered by passing through low pass filter with cut off ω_m

When $f_s < 2\, f_m$ sampled spectrum will overlap and cannot be recovered back. This is called **aliasing or foldover distortion.**

Sampling Theorem

The sampling rate higher than highest frequency of modulating signal creates a **guard band** between sampled spectrum . Therefore, a more practical LPF can be used to restore the modulating signal.



The overlapped region in case of under sampling represents aliasing effect, which can be removed by considering $f_s > 2f_m$