Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Class- Semester-2

Control Engineering

Chapter 12

Lecture 9

Design of Control Systems

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Example 11.5 Ideal Derivative Compensator Design

PROBLEM:

Given the system of Figure 11.22, design an ideal derivative compensator to yield a 16% overshoot, with a threefold reduction in settling time.

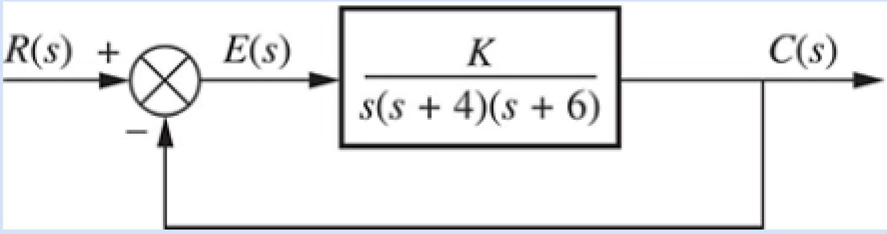


Fig 11.22

SOLUTION:

1- Adding a zero at S = -*KP*/*KD* to the closed-loop transfer function

$$G(s)G_{PD}(s)=\frac{K\times K_D\left(s+\frac{K_P}{K_D}\right)}{s(s+4)(s+6)}$$

Let us first evaluate the performance of the uncompensated system operating with 16% overshoot.

$$\%OS = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$$

Or

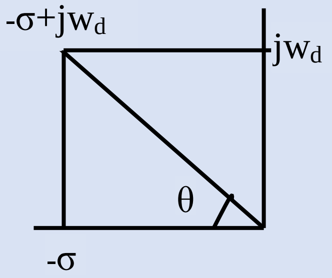
$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}}$$

The root locus for the uncompensated system is shown in Figure 11.23. Since 16% overshoot is equivalent to ζ = 0.504,

From root locus characteristics **1+kG(s)H(s)**=0

K +S(S+4) (S+6) =0

$K+S^3+10S^2+24S=0$ 11-18



$\theta = \cos^{-}(\zeta)= \cos^{-}(0.504)= 59.735$

$\frac{wd}{\sigma}$ =tan (59.735) → σ= wd/ tan (59.735) =wd/1.71369 → σ = 0.5835wd

When S= -σ+jwd = (-0.5835wd +jwd) =wd (-0.5835 +j1)

When Replacing S by (wd (-0.5835 +j1)) in Eq 11-18

$Wd^3 (-0.5835 +j1)^3 +10wd^2(-0.5835 +j1)^2 +24 wd (-0.5835 +j1) +K=0$

$Wd^3(1.55+j 0.023) -wd^2(6.59+j11.68)-14wd +j24wd+K=0$

$j0.023 wd^3-j11.68wd wd^2 +j24wd=0$ divide by 0.023wd

$wd^2 -507.8w +1043.5 =0$

wd= 2.064

$1.55wd^3 +-6.59wd^2 -14wd+K=0$

k=43.53

The root locus for the uncompensated system is shown in Figure 9.18. Since 16% overshoot is equivalent to ζ = 0.504, we search along that damping ratio line for an odd multiple of 180° and find that the dominant, second-order pair of poles is at −1.205 ±j2.064. Thus, the settling time of the uncompensated system is

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.205} = 3.320$$

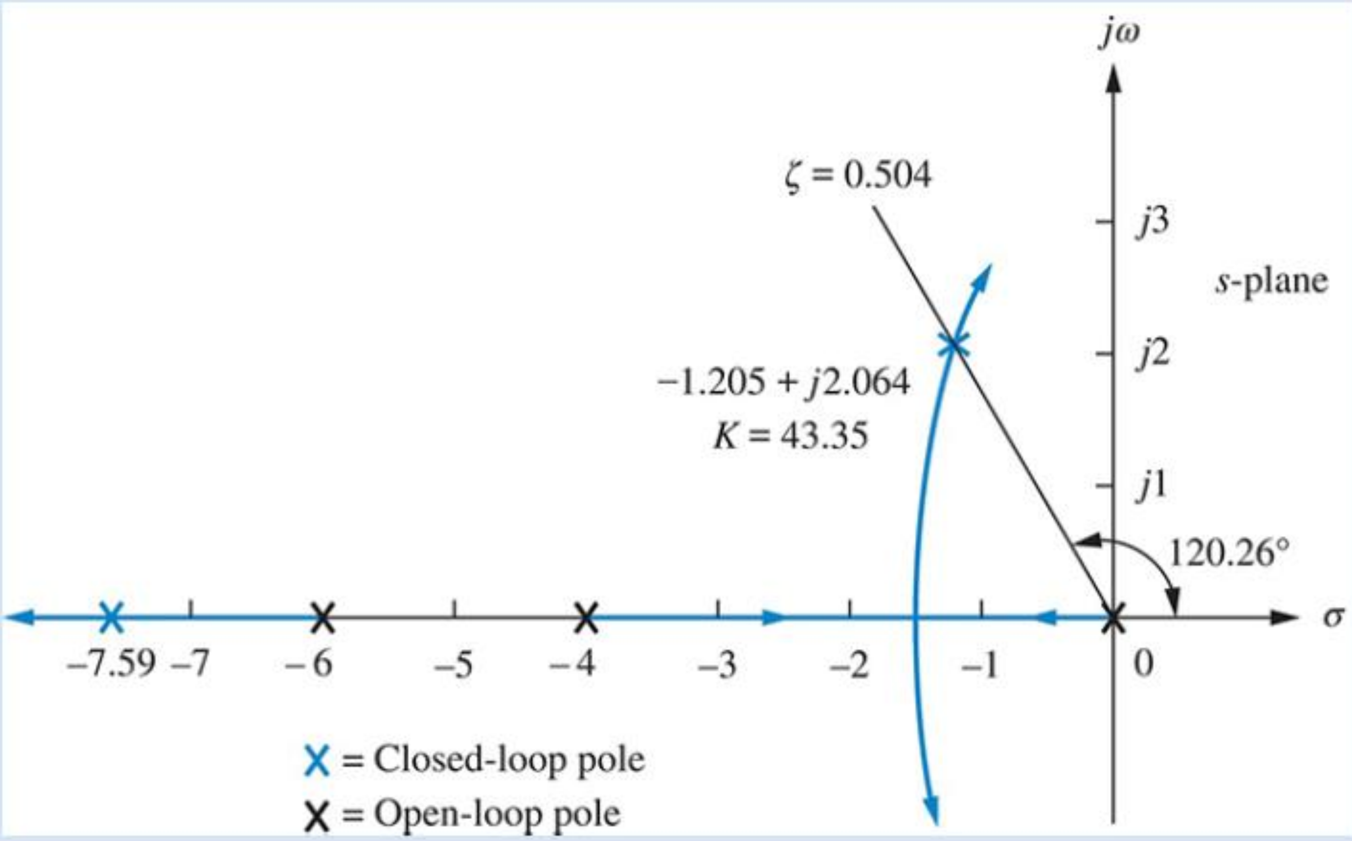


FIGURE 11.23 Root locus for uncompensated system shown in Figure 11.22

Since our evaluation of percent overshoot and settling time is based upon a second order approximation, we must check the assumption by finding the third pole and justifying the second-order approximation. Searching beyond −6 on the real axis for a gain equal to the gain of the dominant, second-order pair, 43.35, we find a third pole at −7.59, which is over six times as far from the jω-axis as the dominant, second-order pair. We conclude that our approximation is valid.

Now we proceed to compensate the system. First we find the location of the compensated system's dominant poles. In order to have a threefold reduction in the settling time, the compensated system's settling time will be one-third of Eq. The new settling time will be 1.107. Therefore, the real part of the compensated system's dominant, second-order pole is 3.320/3= 1.017

$$\sigma = \frac{4}{T_s} = \frac{4}{1.107} = 3.613$$

Figure 11.24 shows the designed dominant, second-order pole, with a real part equal to −3.613 and an imaginary part of

Wd = σ tan(θ)→ ωd = 3.613 tan (59.74°) = 6.193

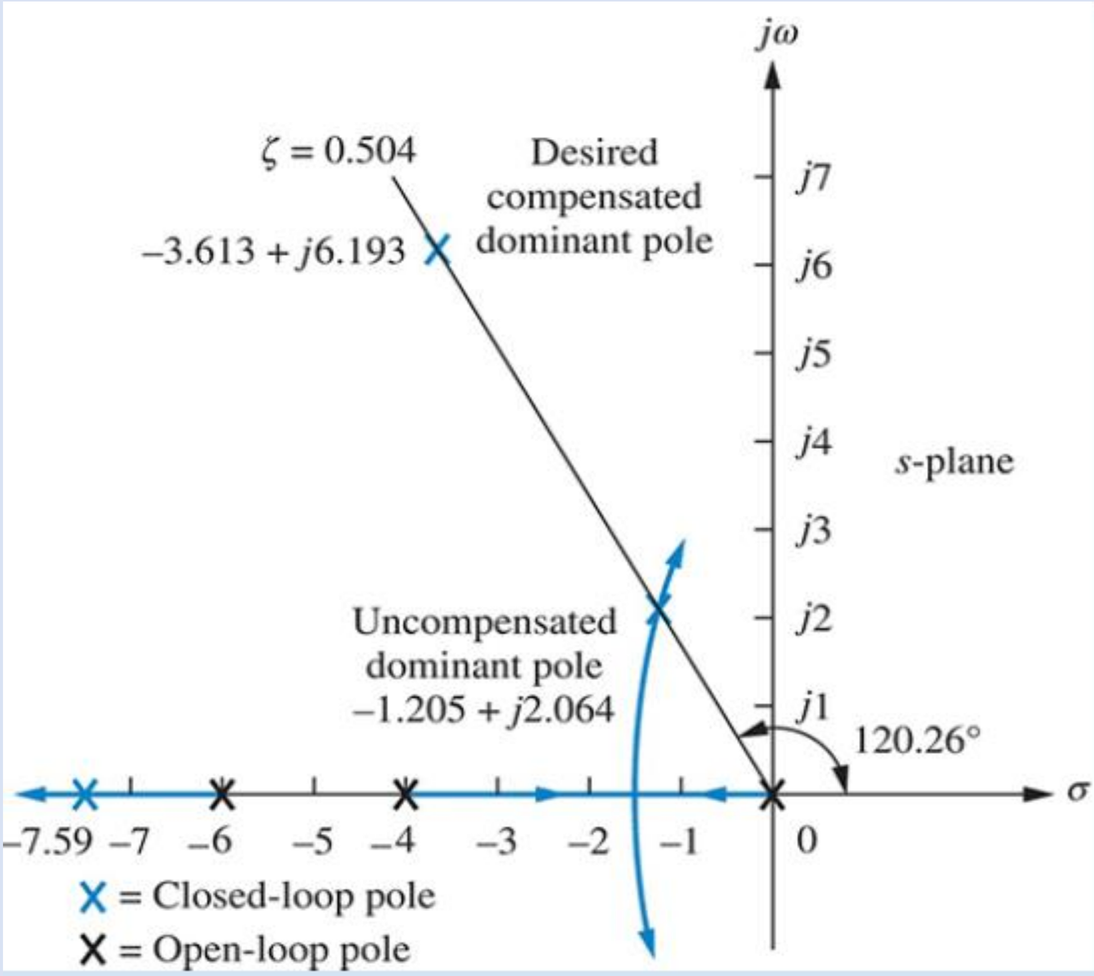


Fig 11.25 Compensated dominant pole superimposed over the uncompensated root locus for Example 11.5

Next we design the PD compensator.

The desired location of the dominant closed-loop = −3.613 ±j6.193 as a test point.

$$\left.\frac{K}{s(s+4)(s+6)}\right|_{s=-3.613+j6.193} = -275.6^\circ$$

the angle deficiency is Angle deficiency -Φ= (180- 275.6) = -95.6° →Φ= 95.6°

The geometry is shown in Figure 11.26, where we now must solve for $-\sigma$, the location of the compensator zero.

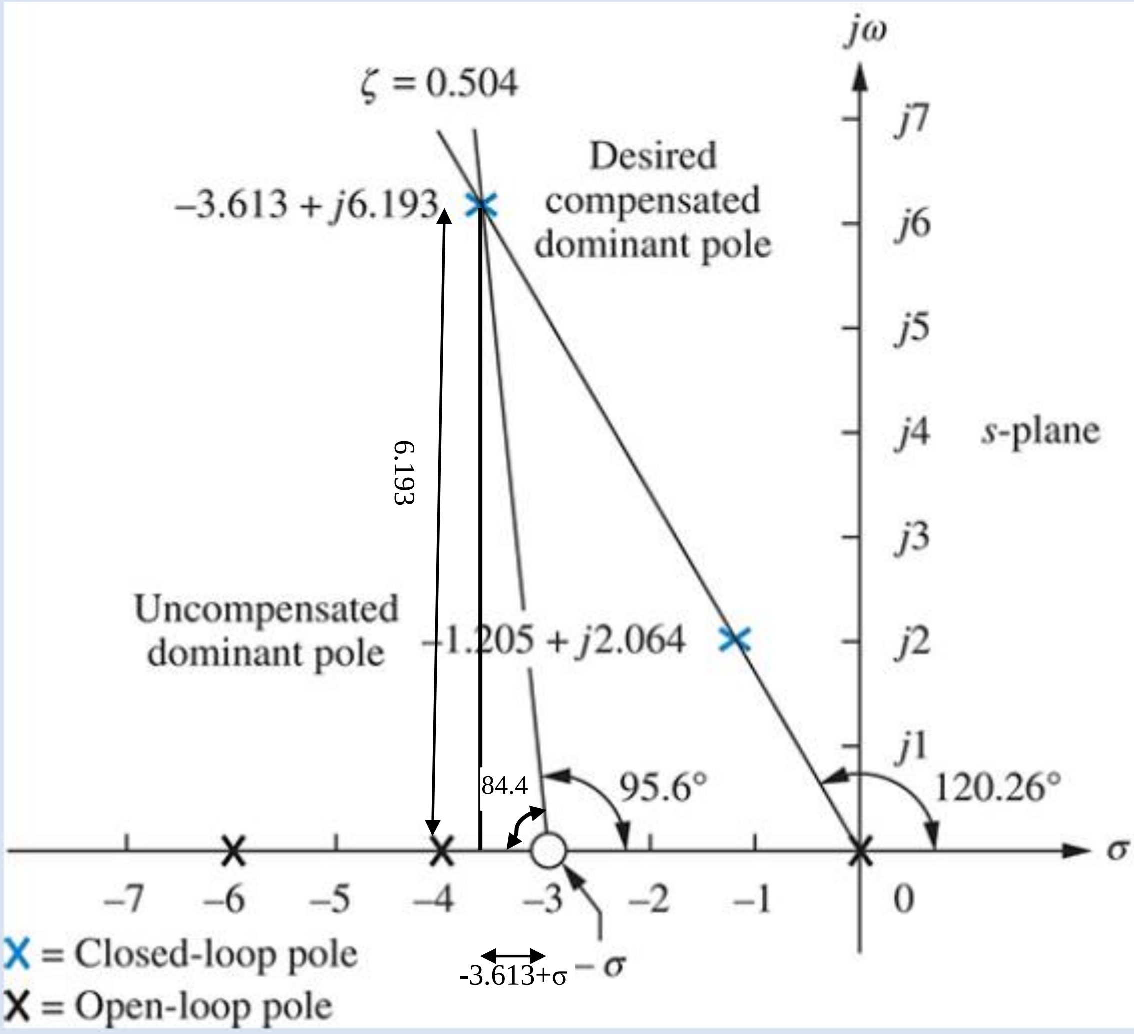


Fig 11.26 Evaluating the location of the PD compensating zero for Example 11.5

From the figure,

$$\frac{6.193}{3.613-\sigma} = \tan\left(180^\circ - 95.6^\circ\right)$$

Thus, σ = -3.006. The complete root locus for the compensated system is shown in

K_P/K_D= 3.006 from root locus characteristics → $\left|\frac{43.53\times K_D(s+3.006)}{s(s+4)(s+6)}\right|_{s=-3.613+j6.193} = 1$

$K_D = \left| \frac{s(s+4)(s+6)}{43.53\times(s+3.006)} \right|_{s=-3.613+j6.193} = 1.09$ Kp/Kd= 3.006 →KP= 1.09 x 3.006= 3.27

$G(s)\, G_{PD}(S) = \frac{47.45(s+3.006)}{s(s+4)(s+6)}$

Example 11.6

PROBLEM:

Given the system of Figure 11.27(*a*), operating with a damping ratio of 0.174, show that Design PI compensator to increase the static velocity error constant Kv to about 0.82 sec–1 without appreciably changing the location of the dominant closed-loop poles

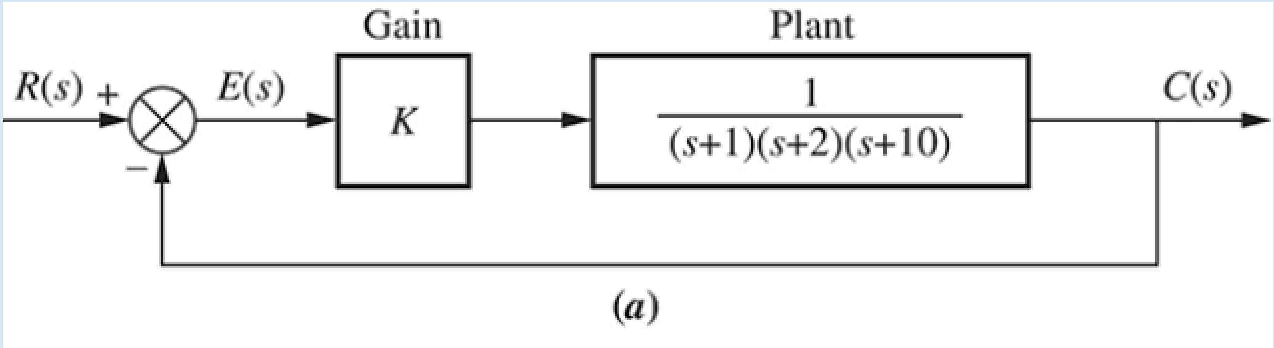


Fig 11.27a

SOLUTION:

First we find that the dominant

poles are -0.694 ±*j*3.926 for a gain, *K*, of 164.6.

the PI compensator is $\frac{Kp(s+\frac{kI}{KP})}{s}$

The transfer function of the system become

$$\frac{164.6Kp(s+\frac{kI}{KP})}{s(s+1)(s+2)(s+10)}$$

$$Kv = \lim_{s\to 0} s\,\frac{164.6Kp(s+\frac{kI}{KP})}{s(s+1)(s+2)(s+10)} = 0.82$$

$\frac{164.6KI}{20} = 0.82 \rightarrow KI = 0.1$ if $KP = 1$

the PI compensator is $\frac{1(s+0.1)}{s}$

$\angle \frac{1(s+0.1)}{s}$ s=-694+j3.926 =-1.42^{o} $\left| \frac{1(s+0.1)}{s} \right|_{s=-694+j3.926}$ =0.99