Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Class- Semester-2

Control Engineering

Chapter 11

Lecture 6

Design of Control Systems

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11-1 Design by Root-Locus Method.

The design by the root-locus method is based on reshaping the root locus of the system by adding poles and zeros to the system's open-loop transfer function and forcing the root loci to pass through desired closed-loop poles in the s plane. The characteristic of the root-locus design is its being based on the assumption that the closed-loop system has a pair of dominant closed-loop poles. This means that the effects of zeros and additional poles do not affect the response characteristics very much.

In essence, in the design by the root locus method, the root loci of the system are reshaped through the use of a compensator so that a pair of dominant closed-loop poles can be placed at the desired location.

Series Compensation and Parallel (or Feedback) Compensation. Figures 11–1(a) and (b) show compensation schemes commonly used for feedback control systems. Figure 11–1(a) shows the configuration where the compensator Gc(s) is placed in series with the plant. This scheme is called series compensation. An alternative to series compensation is to feed back the signal(s) from some element(s) and place a compensator in the resulting inner feedback path, as shown in Figure 11–1(b). Such compensation is called parallel compensation or feedback compensation.

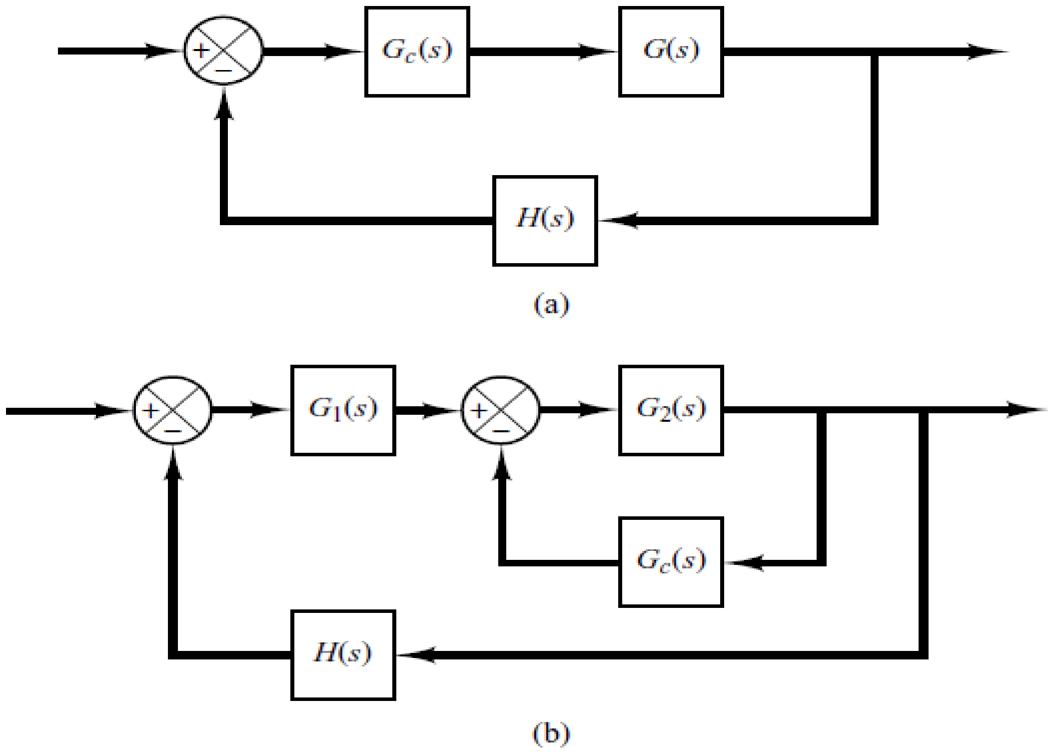

Figure 11–1 (a) Series compensation; (b) parallel or feedback compensation

In general, series compensation may be simpler than parallel compensation; however, series compensation frequently requires additional amplifiers to increase the gain and/or to provide isolation. (To avoid power dissipation, the series compensator is inserted at the lowest energy point in the feedforward path.) Note that, in general, the number of components required in parallel compensation will be less than the number of components in series compensation, provided a suitable signal is available, because the energy transfer is from a higher power level to a lower level. (This means that additional amplifiers may not be necessary.)

Commonly Used Compensators. If a sinusoidal input is applied to the input of a network, and the steady-state output (which is also sinusoidal) has a phase lead, then the network is called a lead network. (The amount of phase lead angle is a function of the input frequency.) If the steady-state output has a phase lag, then the network is called a lag network. In a lag–lead network, both phase lag and phase lead occur in the output but in different frequency regions; phase lag occurs in the low-frequency region and phase lead occurs in the high-frequency region.A compensator having a characteristic of a lead network, lag network, or lag–lead network is called a lead compensator, lag compensator, or lag–lead compensator. Frequently used series compensators in control systems are lead, lag, and lag–lead compensators. PID controllers which are frequently used in industrial control systems will be discussed in later.

11.1.1 Effects of the Addition of Poles.

The addition of a pole to the open-loop transfer function has the effect of pulling the root locus to the right, tending to lower the system's relative stability and to slow down the settling of the response. (Remember that the addition of integral control adds a pole at the origin, thus making the system less stable.) Figure 11–2 shows examples of root loci illustrating the effects of the addition of a pole to a single-pole system and the addition of two poles to a single-pole system.

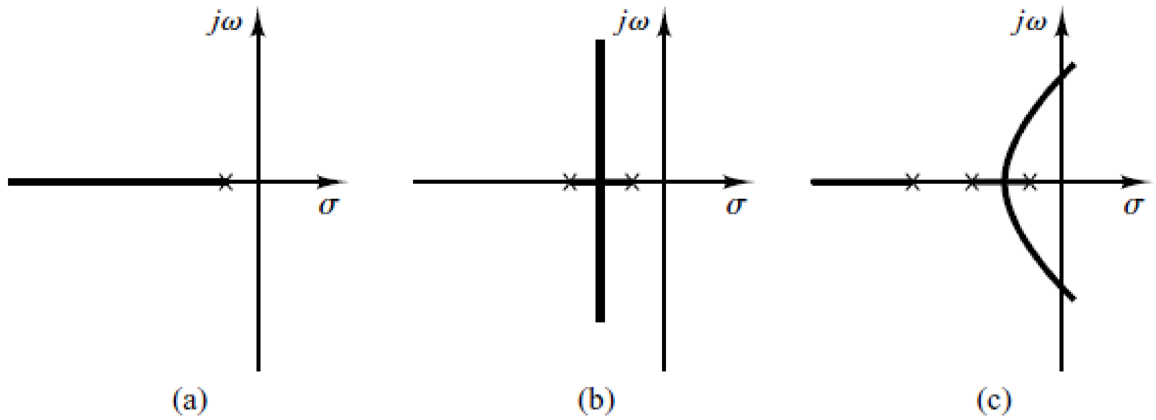

Figure 11–2 (a) Root-locus plot of a single-pole system; (b) root-locus plot of a two-pole system; (c) root-locus plot of a three-pole system.

11.1.2 Effects of the Addition of Zeros.

The addition of a zero to the open-loop transfer function has the effect of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response. (Physically, the addition of a zero in the feedforward transfer function means the addition of derivative control to the system. The effect of such control is to introduce a degree of anticipation into the system and speed up the transient response.) Figure 11–3(a) shows the root loci for a system

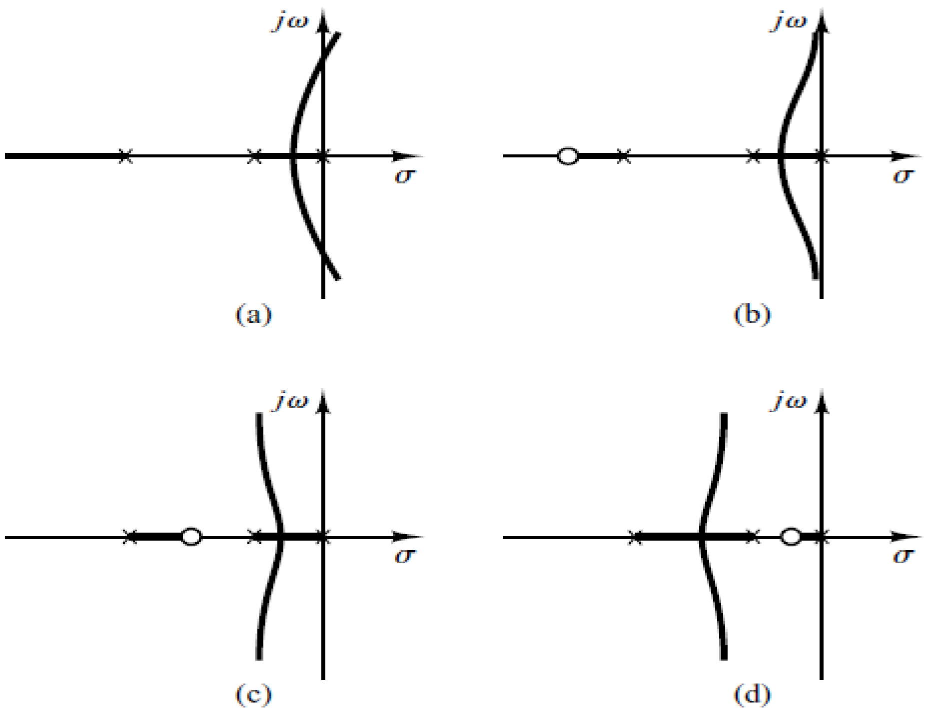

Figure 11–3 (a) Root-locus plot of a three-pole system; (b), (c), and (d) root-locus plots showing effects of addition of a zero to the three-pole system.

that is stable for small gain but unstable for large gain. Figures 11–3 (b), (c), and (d) show root-locus plots for the system when a zero is added to the open-loop transfer function. Notice that when a zero is added to the system of Figure 11–3 (a), it becomes stable for all values of gain.

11–2 Lead Compensation

In Section 11–1 we presented an introduction to compensation of control systems and discussed preliminary materials for the root-locus approach to control-systems design and compensation. In this section we shall present control-systems design by use of the lead compensation technique. In carrying out a control-system design, we place compensator in series with the unalterable transfer function G(s) to obtain desirable behaviour. The main problem then involves the judicious choice of the pole(s) and zero(s) of the compensator Gc(s) to have the dominant closed-loop poles at the desired location in the s plane so that the performance specifications will be met.

Lead Compensators and Lag Compensators. There are many ways to realize lead compensators and lag compensators, such as electronic networks using operational amplifiers, electrical RC networks, and mechanical spring-dashpot systems. Figure 11–4 shows an electronic circuit using operational amplifiers. The transfer function for this circuit was obtained in Chapter 2 as follows

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 R_4}{R_1 R_3}\frac{R_1 C_1 s + 1}{R_2 C_2 s + 1} = \frac{R_4 C_1}{R_3 C_2}\frac{s + \frac{1}{R_1 C_1}}{s + \frac{1}{R_2 C_2}}$$

$$= K_c \alpha \frac{Ts + 1}{\alpha T s + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \qquad \textbf{11.1}$$

$$T = R_1 C_1, \qquad \alpha T = R_2 C_2, \qquad K_c = \frac{R_4 C_1}{R_3 C_2}$$

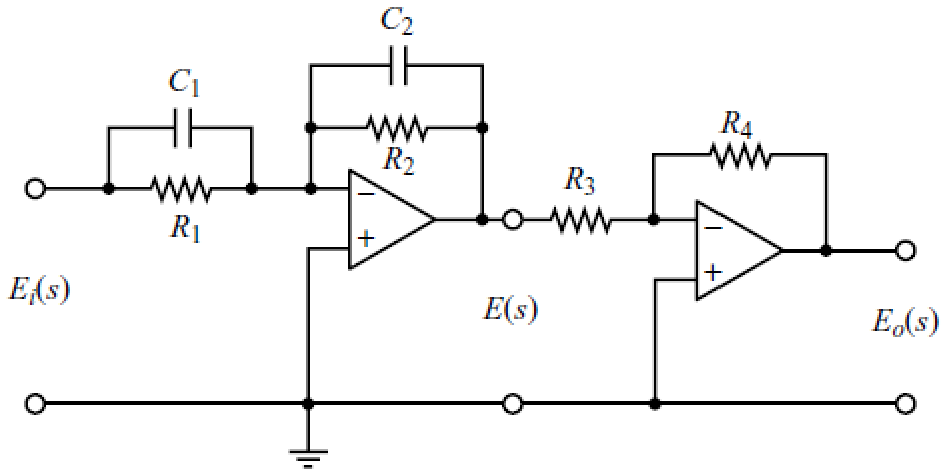

Figure 11–4 Electronic circuit that is a lead network if R1C1 >R2C2. and a lag network if R1C1 < R2C2

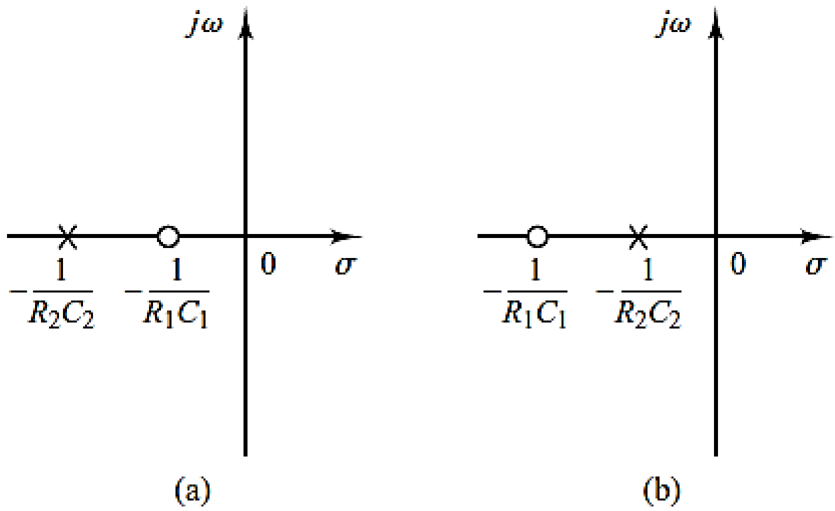

Figure 11–5 Pole-zero configurations: (a) lead network; (b) lag network

Notice that

$$K_c\alpha = \frac{R_4C_1}{R_3C_2}\frac{R_2C_2}{R_1C_1} = \frac{R_2R_4}{R_1R_3}, \qquad \alpha = \frac{R_2C_2}{R_1C_1}$$

This network has a dc gain of Kcα= R2 R4/R1R3.

From Equation (11–1), we see that this network is a lead network if

R1C1 > R2 C2 or α < 1. It is a lag network if R1C1< R2C2 The pole-zero configurations of this network when. R1 C1 > R2 C2. R1 C1 < R2 C2. are shown in Figure 11=5 (a) and (b), respectively

11.2.1 Lead Compensation Techniques Based on the Root-Locus Approach

The root-locus approach to design is very powerful when the specifications are given in terms of time-domain quantities, such as the damping ratio and undamped natural frequency of the desired dominant closed-loop poles, maximum overshoot, rise time, and settling time. Consider a design problem in which the original system either is unstable for all values of gain or is stable but has undesirable transient-response characteristics. In such a case, the reshaping of the root locus is necessary in the broad neighbourhood of the jw axis and the origin in order that the dominant closed-loop poles be at desired locations in the complex plane. This problem may be solved by inserting an appropriate lead compensator in cascade with the feedforward transfer function. The procedures for designing a lead compensator for the system shown in Figure 11–6 by the root-locus method may be stated as follows:

1. From the performance specifications, determine the desired location for the dominant closed-loop poles.

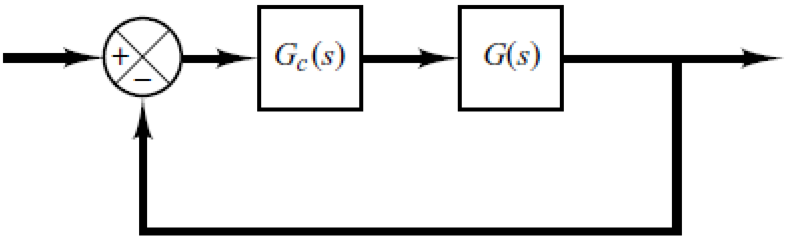

Figure 11–6 Control system.

2. By drawing the root-locus plot of the uncompensated system (original system), ascertain whether or not the gain adjustment alone can yield the desired closed loop poles. If not, calculate the angle deficiency ϕ. This angle must be contributed by the lead compensator if the new root locus is to pass through the desired locations for the dominant closed-loop poles.

3. Assume the lead compensator *G*c(s) to be

$$G_c(s) = K_c \alpha \frac{Ts + 1}{\alpha T s + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}, \qquad (0 < \alpha < 1)$$

where α and T are determined from the angle deficiency. Kc is determined from the requirement of the open-loop gain.

4. If static error constants are not specified, determine the location of the pole and zero of the lead compensator so that the lead compensator will contribute the necessary angle f. If no other requirements are imposed on the system, try to make the value of a as large as possible. A larger value of a generally results in a larger value of *K*v, which is desirable. Note that

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = K_c\alpha \lim_{s\to 0} sG_c(s)$$

5. Determine the value of Kc of the lead compensator from the magnitude condition.

EXAMPLE 11–1 Consider the position control system shown in Figure 11–7(a). The feedforward transfer function is

$$G(s) = \frac{10}{s(s+1)}$$

Design system to get Assume Mp <0.17

Solution

The root-locus plot for this system is shown in Figure 11–7(b).The closed-loop transfer function for the system is

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + s + 10} = \frac{10}{(s + 0.5 + j3.1225)(s + 0.5 - j3.1225)}$$

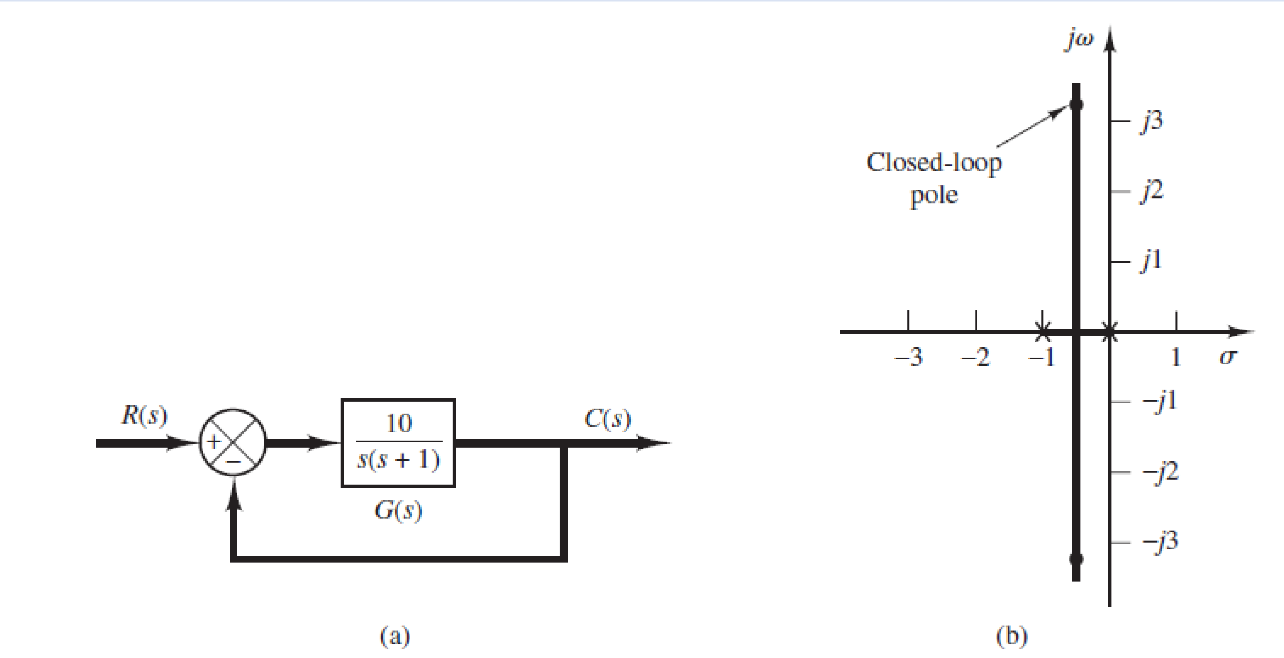

Figure 11–7 (a) Control system; (b) root-locus plot.

The closed-loop poles are located at s = -0.5 ± j3.1225. The damping ratio of the closed-loop poles is ζ =(1/2)/(√10) = 0.1581. The undamped natural frequency of the closed-loop poles wn=(√10) is = 3.1623 rad/sec. Because the damping ratio is small, this system will have a large overshoot in the step response and is not desirable.MP $= e^{-(\sigma/\omega_d)\pi} = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$

MP=0.604

To reduce MP to <0.17 It is desired to design a lead compensator Gc(s) as shown in Figure 11-8 (a) so that the dominant closed-loop poles have the damping ratio ζ= 0.5 and the undamped natural frequency wn = 3 rad/sec. & MP= 0.163. The desired location of the dominant closed-loop poles can be determined from

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 3s + 9$$
$$= (s + 1.5 + j2.5981)(s + 1.5 - j2.5981)$$
$$s = -1.5 \pm j2.5981$$

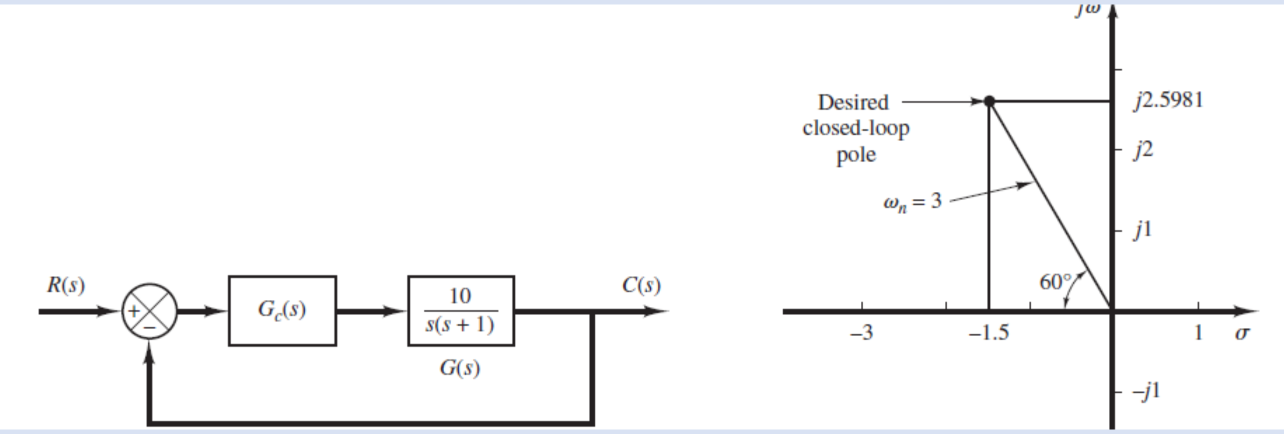

Figure 11–8 (a) Compensated system; (b) desired closed-loop pole location.

[See Figure 11-8 (b).] In some cases, after the root loci of the original system have been obtained, the dominant closed-loop poles may be moved to the desired location by simple gain adjustment. This is, however, not the case for the present system. Therefore, we shall insert a lead compensator in the feedforward path. A general procedure for determining the lead compensator is as follows: First, find the sum of the angles at the desired location of one of the dominant closed-loop poles with the open-loop poles and zeros of the original system, and determine the necessary angle ϕ to be added so that the total sum of the angles is equal to ;180°(2k + 1). The lead compensator must contribute this angle ϕ. (If the angle f is quite large, then two or more lead networks may be needed rather than a single one.)

Assume that the lead compensator Gc(s) has the transfer function as follows:

$$G_c(s) = K_c\alpha \frac{Ts + 1}{\alpha Ts + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}, \qquad (0 < \alpha < 1)$$

The angle from the pole at the origin to the desired dominant closed-loop pole at s = –1.5+j2.5981 is 120°.The angle from the pole at s=–1 to the desired closed-loop pole is 100.894°. Hence, the angle deficiency is Angle deficiency=180° – 120° – 100.894°=

- 40.894° or -ϕ-180=

-Φ= 180-220.894=- 40.894°

$$\left. \angle \frac{10}{s(s + 1)} \right|_{s=-1.5+j2.5981} = -220.894°$$

Deficit angle (ϕ) 40.894° must be contributed by a lead compensator.

then the locations of the zero and pole are found as follows: zero at s= –1.9432

$\left. \angle \frac{(s+1.9423)}{(s+pc)} \right|_{s=-1.5+j2.5981} = 40.894$ pole at s= –4.6458

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = K_c \frac{s + 1.9432}{s + 4.6458}$$

(For this compensator the value of α is $\alpha = 1.9432/4.6458 = 0.418$.)

The value of K_c can be determined by use of the magnitude condition.

$$\left| K_c \frac{s + 1.9432}{s + 4.6458} \frac{10}{s(s + 1)} \right|_{s=-1.5+j2.5981} = 1$$

or

$$K_c = \left| \frac{(s + 4.6458)s(s + 1)}{10(s + 1.9432)} \right|_{s=-1.5+j2.5981} = 1.2287$$

Hence, the lead compensator $G_c(s)$ just designed is given by

$$G_c(s) = 1.2287 \frac{s + 1.9432}{s + 4.6458}$$

Then, the open-loop transfer function of the designed system becomes

$$G_c(s)G(s) = 1.2287 \left(\frac{s + 1.9432}{s + 4.6458} \right) \frac{10}{s(s + 1)}$$

and the closed-loop transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{12.287(s + 1.9432)}{s(s + 1)(s + 4.6458) + 12.287(s + 1.9432)}$$

$$= \frac{12.287s + 23.876}{s^3 + 5.646s^2 + 16.933s + 23.876}$$

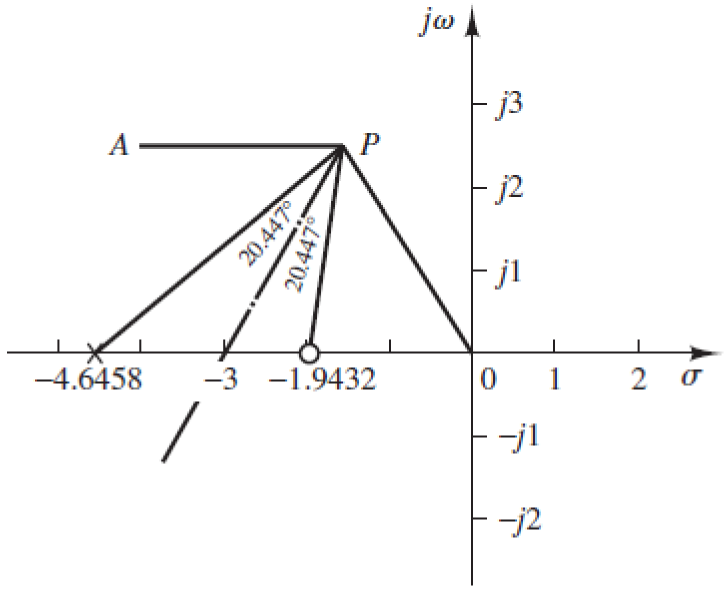

Figure 11–9 Determination of the pole and zero of the lead compensator

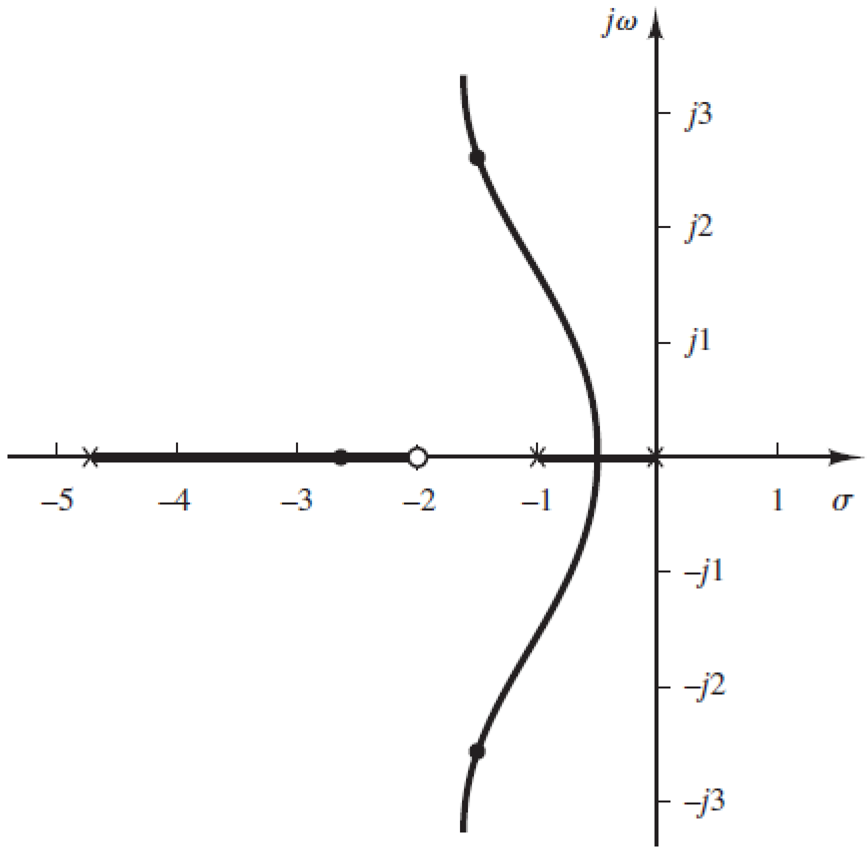

Figure 11–10 Root-locus plot of the designed system.

Figure 11–10 shows the root-locus plot for the designed system. It is worthwhile to check the static velocity error constant Kv for the system just designed.

$$K_v = \lim_{s\to 0} sG_c(s)G(s)$$
$$= \lim_{s\to 0} s\left[1.2287 \frac{s + 1.9432}{s + 4.6458} \frac{10}{s(s + 1)}\right]$$
$$= 5.139$$

Note that the third closed-loop pole of the designed system is found by dividing the characteristic equation by the known factors as follows:

$$s^3 + 5.646s^2 + 16.933s + 23.875 = (s + 1.5 + j2.5981)(s + 1.5 - j2.5981)(s + 2.65)$$

11–3 Lag Compensation

Electronic Lag Compensator Using Operational Amplifiers. The configuration of the electronic lag compensator using operational amplifiers is the same as that for the lead compensator shown in Figure 11–4. If we choose $R_2C_2 > R_1C_1$ in the circuit shown in Figure 6–36, it becomes a lag compensator. Referring to Figure 11–4, the transfer function of the lag compensator is given by

$$\frac{E_o(s)}{E_i(s)} = \hat{K}_c\beta \frac{Ts + 1}{\beta Ts + 1} = \hat{K}_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$$

where

$$T = R_1C_1, \qquad \beta T = R_2C_2, \qquad \beta = \frac{R_2C_2}{R_1C_1} > 1, \qquad \hat{K}_c = \frac{R_4C_1}{R_3C_2}$$

Note that we use β instead of α in the above expressions. [In the lead compensator we used a to indicate the ratio R2C2>R1C1, which was less than 1, or $0<\alpha<1$. We always assume that $0<\alpha<1$and $\beta>1$.

11.3.1 Lag Compensation Techniques Based on the Root-Locus Approach.

Consider the problem of finding a suitable compensation network for the case where the system exhibits satisfactory transient-response characteristics but unsatisfactory steady-state characteristics. Compensation in this case essentially consists of increasing the open loop gain without appreciably changing the transient-response characteristics. This means that the root locus in the neighbourhood of the dominant closed-loop poles should not be changed appreciably, but the open-loop gain should be increased as much as needed. This can be accomplished if a lag compensator is put in cascade with the given feedforward transfer function. To avoid an appreciable change in the root loci, the angle contribution of the lag network should be limited to a small amount, say less than 5°.To assure this, we place the pole and zero of the lag network relatively close together and near the origin of the s plane. Then the closed-loop poles of the compensated system will be shifted only slightly from their original locations. Hence, the transient-response characteristics will be changed only slightly.

Consider a lag compensator Gc(s), where

$$G_c(s) = \hat{K}_c\beta \frac{Ts+1}{\beta Ts+1} = \hat{K}_c \frac{s+\dfrac{1}{T}}{s+\dfrac{1}{\beta T}} \qquad \textbf{11.2}$$

If we place the zero and pole of the lag compensator very close to each other, then at $s = s_1$, where s_1 is one of the dominant closed-loop poles, the magnitudes $s_1 + (1/T)$ and $s_1 + 1/(\beta T)$ are almost equal, or

$$|G_c(s_1)| = \left| \hat{K}_c \frac{s_1+\dfrac{1}{T}}{s_1+\dfrac{1}{\beta T}} \right| \doteq \hat{K}_c$$

To make the angle contribution of the lag portion of the compensator small, we require

$$-5^\circ < \angle \frac{s_1+\dfrac{1}{T}}{s_1+\dfrac{1}{\beta T}} < 0^\circ$$

This implies that if gain $\check{K}_c$ of the lag compensator is set equal to 1, the alteration in the transient-response characteristics will be very small, despite the fact that the overall gain of the open-loop transfer function is increased by a factor of β, where β>1. If the pole and zero are placed very close to the origin, then the value of b can be made large. (A large value of β may be used, provided physical realization of the lag compensator is possible.) It is noted that the value of T must be large, but its exact value is not critical. However, it should not be too large in order to avoid difficulties in realizing the phase-lag compensator by physical components. An increase in the gain means an increase in the static error constants. If the open loop transfer function of the uncompensated system is G(s), then the static velocity error constant Kv of the uncompensated system is

$$K_v=\lim_{s\to 0} sG(s)$$

If the compensator is chosen as given by Equation (11–2), then for the compensated system with the open-loop transfer function Gc(s)G(s) the static velocity error constant Ǩv becomes $\hat{K}_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} G_c(s)K_v = \hat{K}_c\beta K_v$ where Kv is the static velocity error constant of the uncompensated system. Thus, if the compensator is given by Equation (11-2), then the static velocity error constant is increased by a factor of Ǩc β, where Ǩc approximately unity.

11.3.2 **Design Procedures for Lag Compensation by the Root-Locus Method.**

The procedure for designing lag compensators for the system shown in Figure 11-11 by the root-locus method may be stated as follows (we assume that the uncompensated system meets the transient-response specifications by simple gain adjustment;

1. Draw the root-locus plot for the uncompensated system whose open-loop transfer function is G(s). Based on the transient-response specifications, locate the dominant closed-loop poles on the root locus.

2. Assume the transfer function of the lag compensator to be given by Equation (11-2):

$$G_c(s) = \hat{K}_c\beta\frac{Ts+1}{\beta Ts+1} = \hat{K}_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

Then the open-loop transfer function of the compensated system becomes Gc(s)G(s).

3. Evaluate the particular static error constant specified in the problem.

4. Determine the amount of increase in the static error constant necessary to satisfy the specifications.

5. Determine the pole and zero of the lag compensators that produce the necessary increase in the particular static error constant without appreciably altering the original root loci. (Note that the ratio of the value of gain required in the specifications and the gain found in the uncompensated system is the required ratio between the distance of the zero from the origin and that of the pole from the origin.)

6. Draw a new root-locus plot for the compensated system. Locate the desired dominant closed-loop poles on the root locus. (If the angle contribution of the lag network is very small—that is, a few degrees—then the original and new root loci are almost identical. Otherwise, there will be a slight discrepancy between them. Then locate, on the new root locus, the desired dominant closed-loop poles based on the transient-response specifications.)

7. Adjust gain $\check{K}c$ of the compensator from the magnitude condition so that the dominant closed-loop poles lie at the desired location. $\check{K}c$ will be approximately 1.

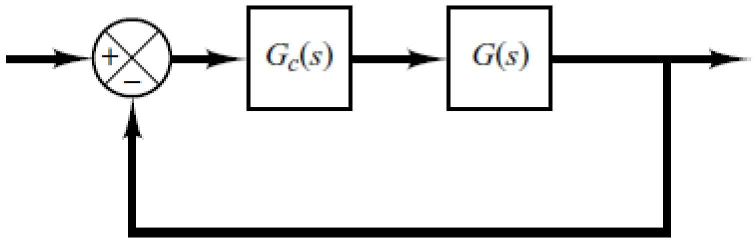

Figure 11–11 Control system