Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Class- Semester-2

Control Engineering

Chapter 12

Lecture 11

State Space Analysis

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12. Introduction

The conventional control theory which is applicable to linear time-invariant systems having single input and single output has already been discussed. The conventional control theory cannot be applied for time varying and/or non-linear systems. In the conventional control theory, the main theme is to formulate the transfer function putting all initial conditions to zero. On the other hand, state variable analysis, i.e., the modem control theory automatically, takes care of initial conditions and it is also possible to analyse time-varying or time-invariant, linear or non-linear, single or multiple input-output systems. The main purpose of this chapter is to introduce state variable analysis.

12.1 Advantages and Disadvantages of Modern Control Theory

Advantages

• It is possible to analyse time-variant or time-invariant, linear or non-linear, single or multiple input output systems.

• It is possible to confirm the state of the system parameters also and not merely input-output relations.

• It is possible to optimise the systems useful for optimal design.

• It is possible to include initial conditions.

Disadvantages

• Techniques are complex.

• Many computations are required.

12.2 Basic Concepts

State. The state of a dynamic system is the smallest set of variables (called state variables) such that knowledge of these variables at $t=t0$, together with knowledge of the input for $t \geq t0$, completely determines the behaviour of the system for any time $t \geq t0$.

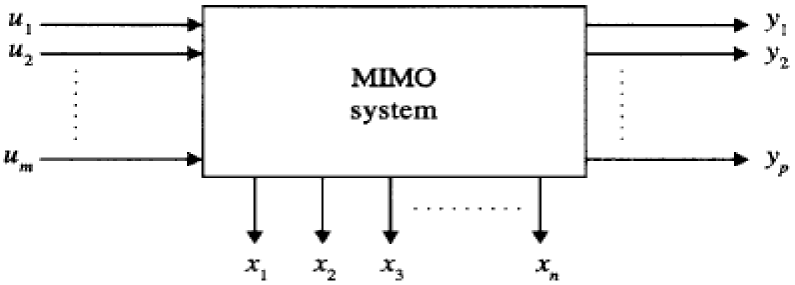
State Variables. The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system. The knowledge of capacitor, voltage at $t = 0$, i.e., the initial voltage of the capacitor is a history dependent term and it forms a state variable. Similarly, initial current in an inductor is treated as a state variable.

State vector: The n state variables that completely describe the behaviour of a given system are said to be n components of a vector.

State Space. The n-dimensional space whose coordinate axes consist of the x1 axis, x2 axis, p , xn axis, where x1, x2, p , xn are state variables, is called a *state space*.Any state can be represented by a point in the state space.

12.3 State Model

Figure 12.1 shows an nth-order system having multiple input and output.



State-Space Equations. In state-space analysis we are concerned with three types of variables that are involved in the modeling of dynamic systems: input variables, output variables, and state variables.

Assume that a multiple-input, multiple-output system involves n integrators. Assume also that there are r inputs u1(t), u2(t), , , ur(t) and m outputs y1(t), y2(t), , , ym(t). Define n outputs of the integrators as state variables: x1(t), x2(t),,,, xn(t) Then the system may be described by

$$
\begin{aligned}
\dot{x}_1(t) &= f_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\
\dot{x}_2(t) &= f_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\
&\;\;\vdots \\
\dot{x}_n(t) &= f_n(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)
\end{aligned}
\qquad 12.1
$$

The outputs y1(t), y2(t),,, ym(t) of the system may be given by

$$
\begin{aligned}
y_1(t) &= g_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\
y_2(t) &= g_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\
&\;\;\vdots \\
y_m(t) &= g_m(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)
\end{aligned}
\qquad 12.2
$$

then Equations (2–8) and (2–9) become the following linearized state equation and output equation:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) \qquad 12.3$$

$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t) \qquad 12.4$$

where **A**(t) is called the state matrix, **B**(t) the input matrix, **C**(t) the output matrix, and **D**(t) the direct transmission matrix. A block diagram representation of Equations (12–3) and (12-4) is shown in Figure 12–2.

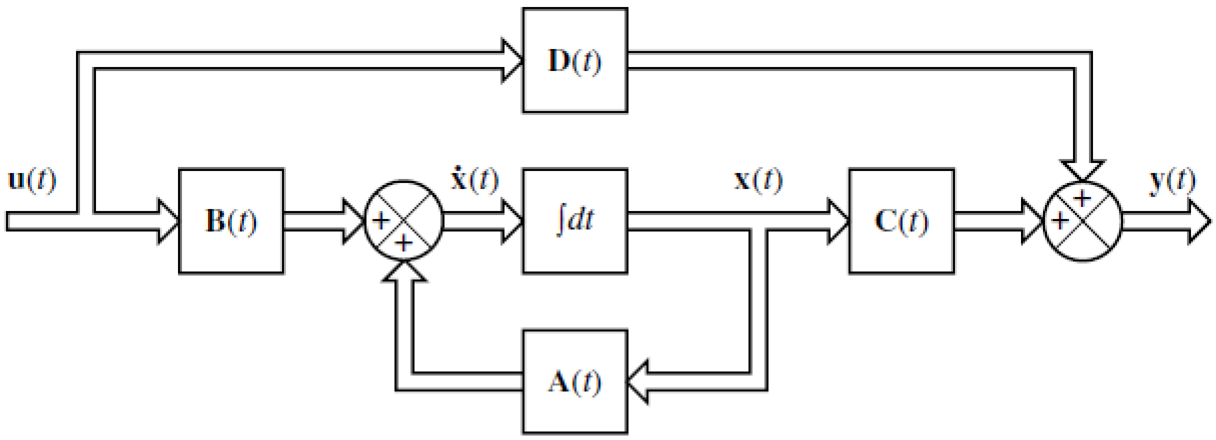


Fig 12–2Block diagram of the linear, continuous time control system represented in state space.

If vector functions **f** and **g** do not involve time t explicitly then the system is called a time-invariant system. In this case, Equations (12-3) and (12-4) can be simplified to

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \qquad 12.5$$

$$\dot{\mathbf{y}}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \qquad 12.6$$

The order of the above matrices is given below:

A —> ***n x n*** known as evolution matrix

B —» ***n x m*** known as control matrix

C —>***p x n*** known as observation matrix

D —> ***p x m*** known as direct transmission matrix

This is known as the state model of the given system.

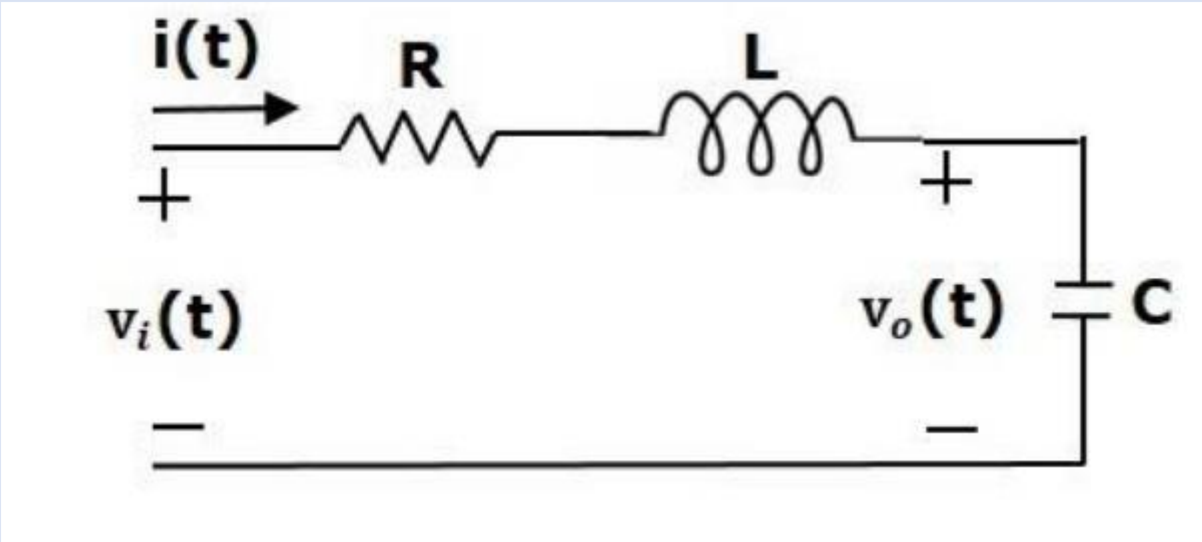
12.4 Selection of state variables

1. For a physical system, the number of state variables needed to represent the system must be equal to the number of energy storing elements present in the system

2. If a system is represented by a linear constant coefficient differential equation, then the number of state variables needed to represent the system must be equal to the order of the differential equation

3. If a system is represented by a transfer function, then the number of sate variables needed to represent the system must be equal to the highest power of s in the denominator of the transfer function.

Example 12.1 Consider the following series of the RLC circuit. It is having an input voltage vi (t) and the current flowing through the circuit is i(t).



- There are two storage elements (inductor and capacitor) in this circuit. So, the number of the state variables is equal to two.
- These state variables are the current flowing through the inductor, i(t) and the voltage across capacitor, vc (t).
- From the circuit, the output voltage, v0 (t) is equal to the voltage across capacitor, vc (t).

$Y(t) = v_0\,(t) = vc\,(t)$ Apply KVL around the loop, Vi (t) = R i(t) + L di(t)/dt + vc (t)

$$\dot{i(t)} = \frac{di(t)}{dt} = -\frac{R}{L}\, i(t) - \frac{1}{L}\, v_c(t) + \frac{1}{L}\, V_i(t)$$

1

The voltage across the capacitor is

$$v_c(t) = \frac{1}{c} \int i(t)dt$$

Differentiate the equation with respect to time,

$$\dot{v_c(t)} = \frac{dv_c(t)}{dt} = \frac{i(t)}{c}$$

2

State vector, $X = \begin{bmatrix} i(t) \\ v_c(t) \end{bmatrix}$; Differential state vector, $\dot{X} = \begin{bmatrix} \frac{di(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix}$

Arrange the differential equations and output equation into standard form of state space model as,

$$\dot{X} = \begin{bmatrix} \frac{di(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} [v_i(t)]$$

$$Y = [0 \quad 1] \begin{bmatrix} i(t) \\ v_c(t) \end{bmatrix}$$

Here $A = \begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}$; $B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$; $C = [0 \quad 1]$; $D = [0]$

Example **12.2** Obtain the state model of the parallel RLC network as shown in Fig. 12.3

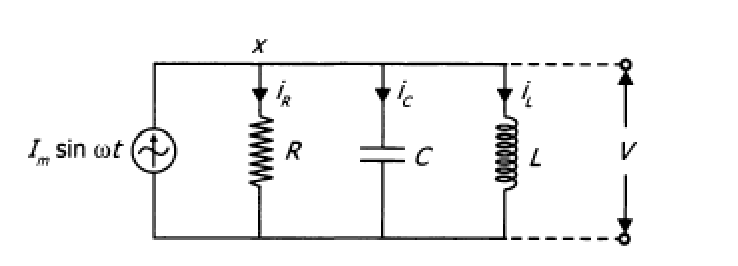


Fig 12.3

Solution

Applying KCL at node X, we get

Let $$i(t) = i_R + i_L + i_C$$

$$\therefore \quad i(t) = \frac{v(t)}{R} + \frac{1}{L}\int v(t)dt + C\frac{dv(t)}{dt}$$

$$\therefore \quad C\frac{d^2v(t)}{dt^2} + \frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t) = \frac{di(t)}{dt} = I_m\,\omega\cos\omega t$$

Let $$v(t) = x_1(t) \tag{1}$$

and $$\frac{dv(t)}{dt} = \check{X}_1(t) = X_2(t) \tag{2}$$

$$\therefore \quad \frac{d^2v(t)}{dt^2} = \check{X}_2(t) = -\frac{1}{LC}x_1(t) - \frac{1}{RC}x_2(t) + \frac{\omega I_m}{C}\cos\omega t \tag{3}$$

From Eqs. (2) and (3), the state equation can be written as

$$\begin{bmatrix} \check{X}_1(t) \\ \check{X}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\omega I_m}{C}\cos\omega t \end{bmatrix} [u]$$

From Eq. (1), the output equation can be written as

$$y(t) = v(t) = [1,0]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Here A= $\begin{bmatrix} 0 & 1 \\ \frac{-1}{LC} & \frac{-1}{RC} \end{bmatrix}$ B= $\begin{matrix} 0 \\ \frac{wI_m}{C}coswt \end{matrix}$ C= [1 0] D= [0]