Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Class

Control Engineering

Chapter 4

Lecture 9

Signal-Flow Graphs

Prepared by

Asst Lecturer. Ahmed Saad Names

4. Signal-Flow Graphs

If the set of equations describing a system is known, it is possible to represent the system in another pictorial form which is known as signal-flow graph. It may be regarded as a simplified version of a block diagram. Signal-flow graph is a pictorial representation of a system which displays graphically the transmission of signals in it. S.J. Mason introduced the signal-flow graph for representing the cause and effect of linear systems which are modelled by algebraic equations. Although there is difference between physical appearances of the signal-flow graph (SFG) and block diagram, the signal-flow graph is constrained by more rigid mathematical rules whereas the usage of block diagram's notation is more liberal. In a signal-flow graph, all dependent and independent variables are represented by the nodes, and the lines joining the nodes are known as branches. A branch is associated with transmission gain and an arrow. Figure 4.1 shows the pictorial representation of a system having two variables where $x_2 = A\, x_1$

Figure 4.1 SFG of a system

In Fig. 4.1, ***A*** is the transmission gain and the arrow shows the flow of signal. The branches of any SFG are always unidirectional. All the branches of a signal-flow graph represent the cause-and-effect relationship existing between the various variables. The aim of this chapter is to introduce SFG in detail and its application to find the transfer function of the complicated systems.

4.1 BASIC DEFINITIONS IN SFG

The terminology used in SFG is given below:

Input ox source node: The node having only outgoing branches is called input or source node. In Fig.4.2, $\boldsymbol{x_1}$ is the input or source node.

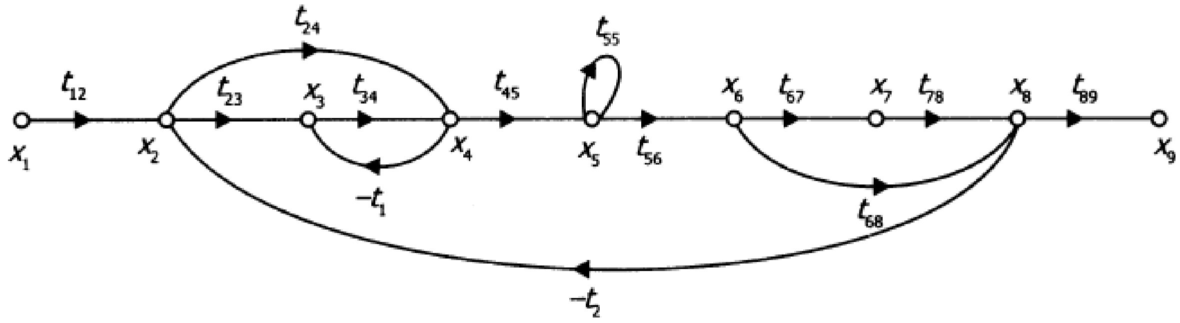


Figure 4.2 Signal-flow graph

Sink node: The node having only incoming branches is called sink node or output node. In Fig. 4.2, x9 is the sink or output node.

Chain node: The node having both incoming and outgoing branches is know as chain node. In Fig. 4.2,

x_2, x_3, x_4, x_5, x_6, x_7 and x_8 are all chain nodes.

Forward path: A path from input to output is known as forward path.

In Figure 4.2, there are four forward paths as follows:

x_1 -x_2 -x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9: First forward path

x_1 -x_2 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9: Second forward path

x_1 -x_2 -x_3 - x_4 - x_5 - x_6 - x_8 - x_9: Third forward path

x_1 -x_2 - x_4 - x_5 - x_6 - x_8 - x_9: Fourth forward path

In determining forward path, any node should not be traded twice.

Feedback Loop/Feedback Path: If a loop originates and terminates at the same node, it is known as feedback loop. In Fig. 4.2, there are five feedback loops or feedback paths as follows:

x_2 -x_3 - x_2: First feedback loop x_2 - x_4 - x_5 - x_6 - x_7 - x_8 - x_2***:*** Second feedback loop, x_2 - x_4 - x_5 - x_6 - x_7 - x_8 - x_2Third feedback loop, x_2 -x_3 - x_4 - x_5 - x_6 - x_8 - x_2 fourth feedback loop

x_2 - x_4 - x_5 - x_6 - x_8 - x_2 fifth feedback loop.

Self-Loop: A loop that consists of only one node is known as self-loop. In determining forward path or feedback path, the self-loop should not be taken into account. In Fig. 4.2, t_{55}, at x_5**,** is the self-loop;

Path Gain: The product of gains going through a forward path is called path gain. In Fig. 4.2., the path gain for the first forward path is

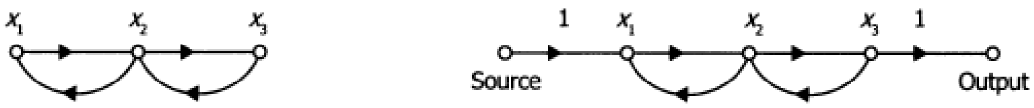
$P1= t_{12}\, t_{23}\, t_{34}\, t_{45}\, t_{56}\, t_{67}\, t_{78}\, t_{89}$

Similarly, the path gains for the second, third and fourth paths are

$P2= t_{12}\, t_{24}\, t_{45}\, t_{56}\, t_{67}\, t_{78}\, t_{89}$

$P3= t_{12}\, t_{23}\, t_{34}\, t_{45}\, t_{56}\, t_{68}\, t_{89}$. And $P4 = t_{12}\, t_{24}\, t_{45}\, t_{56}\, t_{68}\, t_{89}$ respectively.

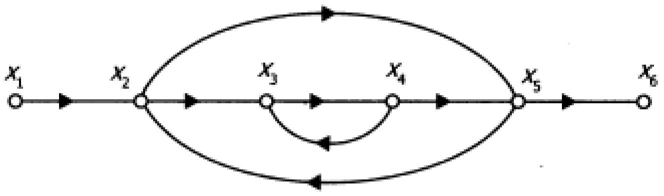
Dummy Node: If the incoming as well as outgoing branches exist at the first and the last node representing input and output variables, these nodes cannot be taken as input and output nodes. In such cases, separate input and output nodes are created by adding branches with gain 1. These nodes are known as dummy nodes. Fig. 4.3. shows the dummy nodes.



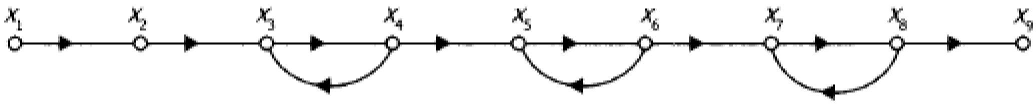
(a) Without input and output nodes (b) Dummy nodes

Figure 4.3 Dummy nodes

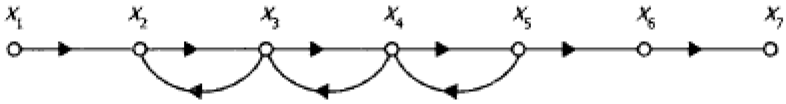
Non-touching Loops:



(a) Two non-touching loops



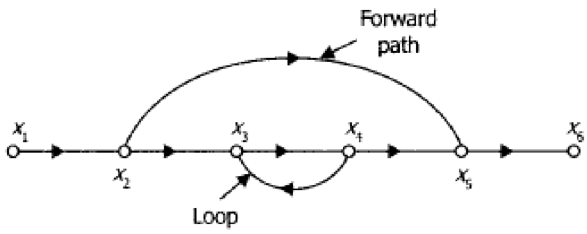
(b) Three non-touching loops



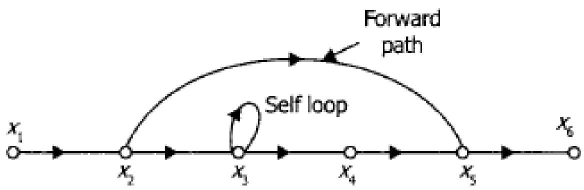
(c) No non-touching loops

Figure. 4.4 Existence of non-touching loop

If there is no node common between two or more loops, these loops are known as non-touching loops. In Figs. 4.4(a) and 4.4(b) there are two and three non-touching loops, respectively. In Fig. 4.4(c), there is no non-touching loop. Similarly, if a loop does not touch to a forward path, i.e., no node is common between a forward path and a feedback loop, this loop is non-touching to that forward path. Figure 4.5 shows non-touching loops to the forward paths.



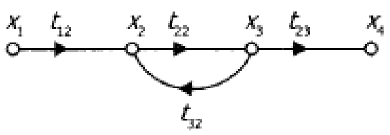
(a) Loop non-touching to forward path



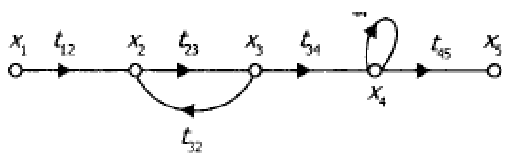
(b) Self loop non-touching to the forward path

Figure. 4.5 non-touching loop to a forward path

Loop Gain: It is defined as the product of all gains of the branches forming a loop. Figure 4.6 illustrates the loop gain.



(a) One loop



(b) Two loops

Figure. 4.6 Loop gain

In Fig. 4.6(a), the loop gain is C $t_{23}t_{32}$ In Fig. 7.6(b), the loop gains are L1, = $t_{23}t_{32}$ and L2 = t_{44} (self-loop).

4.2 RULES FOR SIGNAL-FLOW GRAPH

4.2.1 Addition Rule

The algebraic sum of all signals entering at a node gives the value of the variable of that node. This is known as additional rule. From Fig. 4.7, we can write

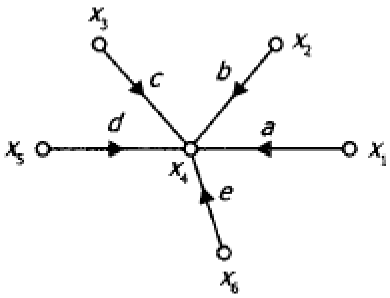


Figure. 4.7 Addition rule

X4. = ax1+ bx2 + cx3 + dx5, + ex6

4.2.2 Transmission Rule

The value of a variable at a node in signal-flow graph is transmitted on every branch that is leaving that node. This is known as transmission rule. Figure 4.8. demonstrates the transmission rule.

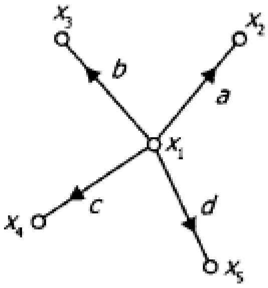


Figure. 4.8 Transmission rule

Using transmission rule, it can be written as

$x_2=ax_1$, $x_3=bx_1$, $x_4=cx_1$, $x_5=dx_1$

7.2.3 Multiplication Rule

If the branches are in series, the gain is multiplied. Figure 4.9 shows multiplication rules.

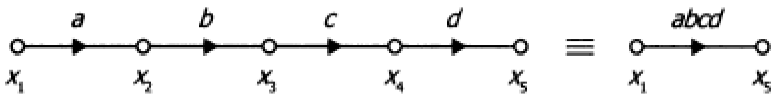


Figure. 4 .9 Multiplication rule

7.4 MASON'S GAIN FORMULA

To find transfer function of a complicated system of block diagram, reduction technique is a cumbersome process because step-by-step reduction of block diagram is required. On the other hand, it is possible to obtain the transfer function very easily by using Mason's gain formula given below:

$$\text{Transfer function} = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} p_i \Delta_i}{\Delta} \qquad 4.1$$

where N = total number of forward paths

P_i = gain of the ith forward path

D = 1 - ($\sum$ all individual feedback loop gain including self-loops)

+ ($\sum$ gain product of all possible combinations of two non-touching loops)

- ($\sum$ gain product o f all possible combinations of three non-touching loops)

+ •••

Δ_i= value of Δ after eliminating all loops that touch its forward path.

Example 4.1 Find the transfer function of the system shown in Fig. E4.1 using Mason's gain formula

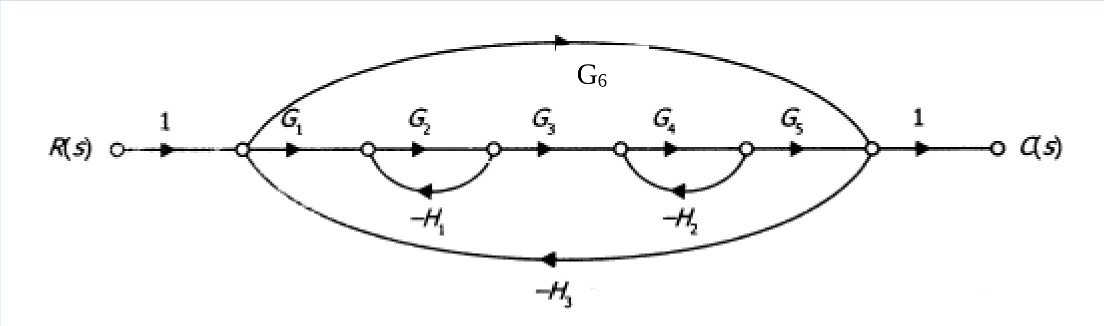


Figure. E4.1 SFG

Solution

Step 1: There are two forward paths as follows:

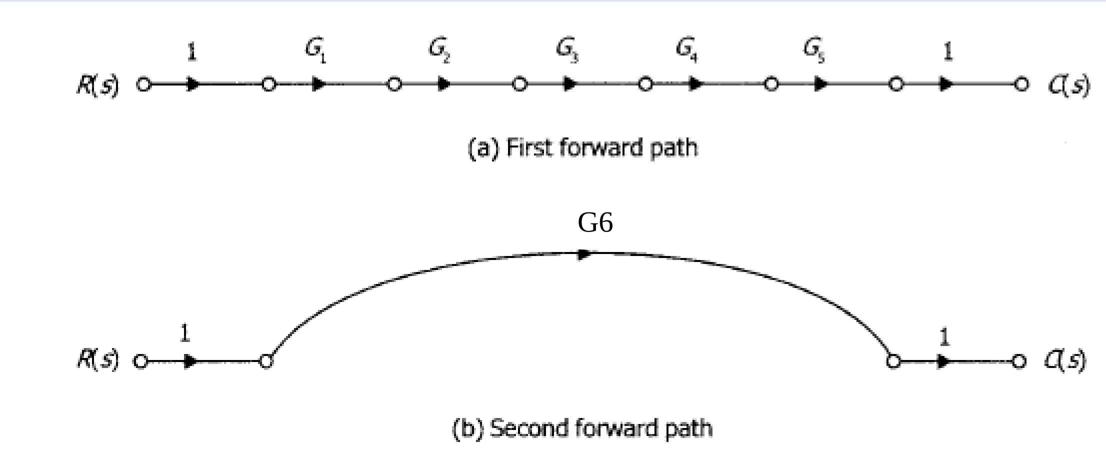


Figure. E4.2 Forward path of Fig. E7.1

Gain of first forward path (*P 1*) = *G 1G 2G G3 4G G5*

Gain of second forward path *(P 2) = G 6*

Step 2: There are four loops as shown in Fig. E4.3

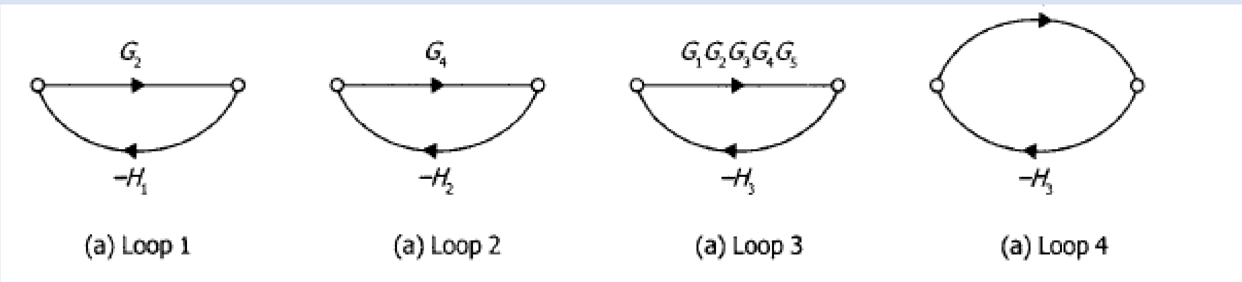


Figure. E4.3

Loop gain (L_1) of the loop shown in Fig. E7.3(a) = $-G_2H_1$
Loop gain (L_2) of the loop shown in Fig. E7.3(b) = $-G_4H_2$
Loop gain (L_3) of the loop shown in Fig. E7.3(c) = $-G_1G_2G_3G_4G_5H_3$
Loop gain (L_4) of the loop shown in Fig. E7.3(d) = $-G_6H_3$

Step 3: Out of four loops, Loop 1, Loop 2, and Loop 4 are non-touching. The combinations of two non-touching loops are

(i) Loop 1, Loop 2: Loop gain $L_{12} = G_2G_4H_1H_2$
(ii) Loop 1, Loop 4: Loop gain $L_{22} = G_2G_6H_1H_3$
(iii) Loop 2, Loop 4: Loop gain $L_{32} = G_4G_6H_2H_3$

L_{ij} indicates ith of j non-touching loops.

Step 4: Out of these four loops, Loop 1, Loop 2 and Loop 4 are possible combinations of three non-touching loops:

$\therefore\ L_{13} = -G_2G_4G_6H_1H_2H_3$

Step 5: There is no higher order non-touching loops.

Step 6:

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_{12} + L_{22} + L_{32}) - L_{13}$$
$$= 1 + (G_2H_1 + G_4H_2 + G_1G_2G_3G_4G_5H_3 + G_6H_3) + (G_2G_4H_1H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3) + G_2G_4G_6H_1H_2H_3$$

Step 7:

(i) Considering P_1, Loops 1, 2, 3, 4 touch it.

$\therefore\quad \Delta_1 = 1 - (0) = 1$

(ii) Considering P_2, Loops 1, 2 do not touch it.

$$\therefore\quad \Delta_2 = 1 - (-G_2H_1 - G_4H_2) + G_2G_4H_1H_2$$
$$= 1 + G_2H_1 + G_4H_2 + G_2G_4H_1H_2$$

Step 8:

$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4G_5 + G_6(1 + G_2H_1 + G_4H_2 + G_2G_4H_1H_2)}{1 + (G_2H_1 + G_4H_2 + G_1G_2G_3G_4G_5H_3 + G_6H_3) + (G_2G_4H_1H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3) + G_2G_4G_6H_1H_2H_3}$$