Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Year

Control Engineering

Chapter 2

Lecture 7

Transfer functions

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2.7 Electric Circuit Analogs

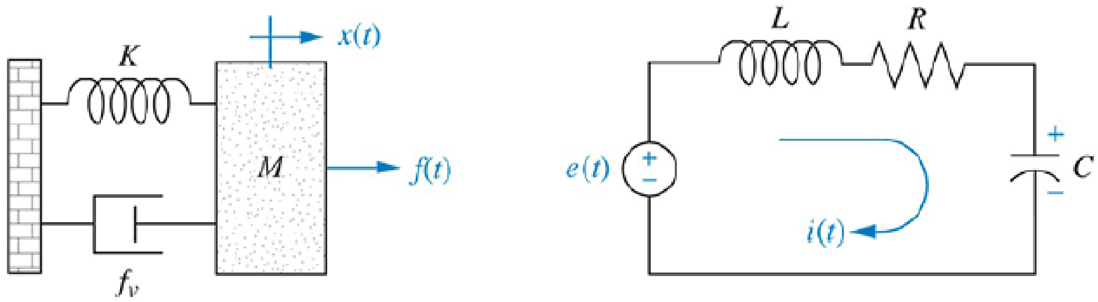
In this section, we show the commonality of systems from the various disciplines by demonstrating that the mechanical systems with which we worked can be represented by equivalent electric circuits. We have pointed out the similarity between the equations resulting from Kirchhoff's laws for electrical systems and the equations of motion of mechanical systems. We now show this commonality even more convincingly by producing electric circuit equivalents for mechanical systems. The variables of the electric circuits behave exactly as the analogous variables of the mechanical systems. In fact, converting mechanical systems to electrical networks before writing the describing equations is a problem-solving approach that you may want to pursue.

An electric circuit that is analogous to a system from another discipline is called an electric circuit analog. Analogs can be obtained by comparing the describing equations, such as the equations of motion of a mechanical system, with either electrical mesh or nodal equations. When compared with mesh equations, the resulting electrical circuit is called a series analog. When compared with nodal equations, the resulting electrical circuit is called a parallel analog.

Series Analog

Consider the translational mechanical system shown in Figure (a), whose equation of motion is

$$(Ms^2 + fvs + K)X(s) = F(s) \qquad \textbf{2.101}$$



(a) (b)

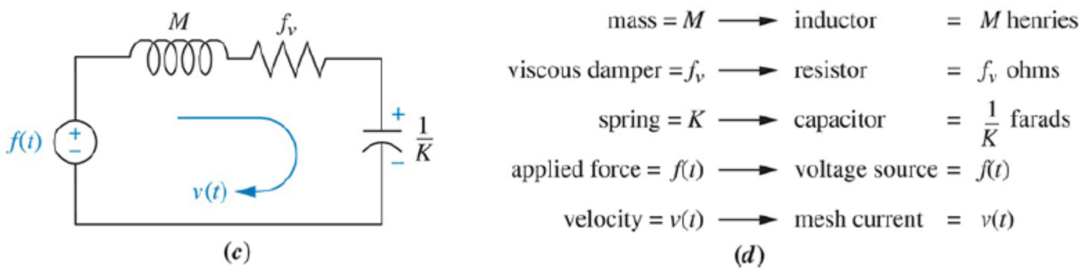


FIGURE Development of series analog: a. mechanical system; b. desired electrical representation; c. series analog; d. parameters for series analog

Kirchhoff's mesh equation for the simple series RLC network shown in Figure (b) is

$$(Ls + R + 1/Cs)\, I(s) = E(s) \qquad \textbf{2.102}$$

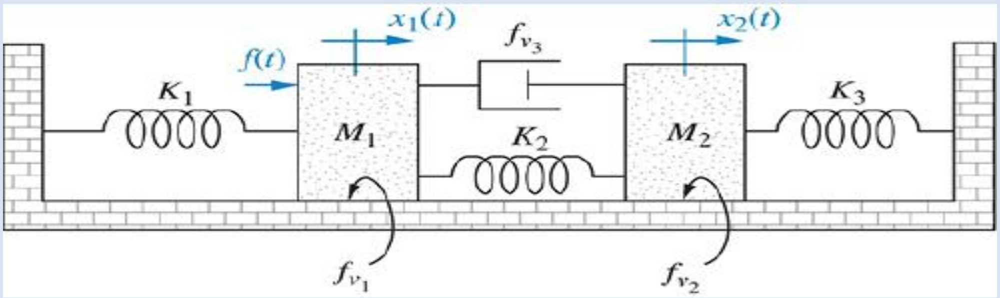
As we previously pointed out, Eq. (2.101) is not directly analogous to Eq. (2.102) because displacement and current are not analogous. We can create a direct analogy by operating on Eq. (2.101) to convert displacement to velocity by dividing and multiplying the left-hand side by s, yielding

$$\frac{Ms^2 + f_v s + K}{s} sX(s) = \left(Ms + f_v + \frac{K}{s}\right) V(s) = F(s) \qquad \textbf{2.103}$$

Comparing Eqs. (2.102) and (2.103), we recognize the sum of impedances and draw the circuit shown in Figure (c). The conversions are summarized in Figure (d). When we have more than one degree of freedom, the impedances associated with a motion appear as series electrical elements in a mesh, but the impedances between adjacent motions are drawn as series electrical impedances between the two corresponding meshes. We demonstrate with an example.

Example 2.18 Converting a Mechanical System to a Series Analog

PROBLEM: Draw a series analog for the mechanical system of Figure below



SOLUTION:

Equations (2.60) and (2.61) are analogous to electrical mesh equations after conversion to velocity. Thus,

$$\left[M_1 s+\left(f_{v_1}+f_{v_3}\right)+\frac{\left(K_1+K_2\right)}{s}\right] V_1(s)-\left(f_{v_3}+\frac{K_2}{s}\right) V_2(s)=F(s) \quad \textbf{2.104a}$$

$$-\left(f_{v_3}+\frac{K_2}{s}\right) V_1(s)+\left[M_2 s+\left(f_{v_2}+f_{v_3}\right)+\frac{\left(K_2+K_3\right)}{s}\right] V_2(s)=0 \quad \textbf{2.104b}$$

Coefficients represent sums of electrical impedance. Mechanical impedances associated with M1 form the first mesh, where impedances between the two masses are common to the two loops. Impedances associated with M2 form the second mesh. The result is shown in Figure below, where v1(t) and v2(t) are the velocities of M1 and M2, respectively

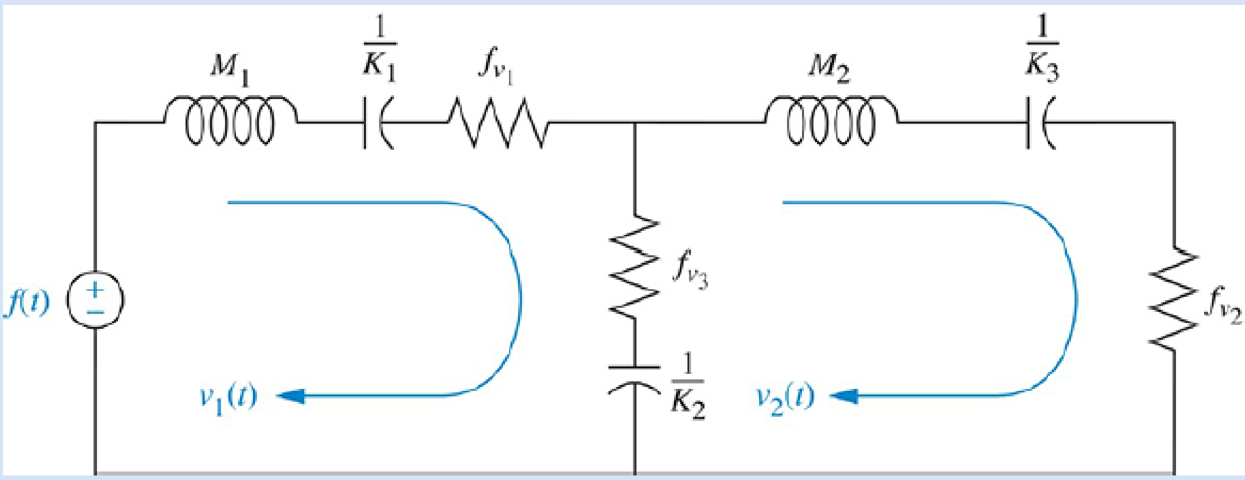


FIGURE Series analog of mechanical system

Parallel Analog

A system can also be converted to an equivalent parallel analog. Consider the translational mechanical system shown in Figure (a), whose equation of motion is given by Eq. (2.103). Kirchhoff's nodal equation for the simple parallel RLC network shown in Figure (b) is

$$\left(Cs+\frac{1}{R}+\frac{1}{Ls}\right)E(s)=I(s) \qquad \textbf{2.105}$$

Comparing Eqs. (2.103) and (2.105), we identify the sum of admittances and draw the circuit shown in Figure (c). The conversions are summarized in Figure (d).

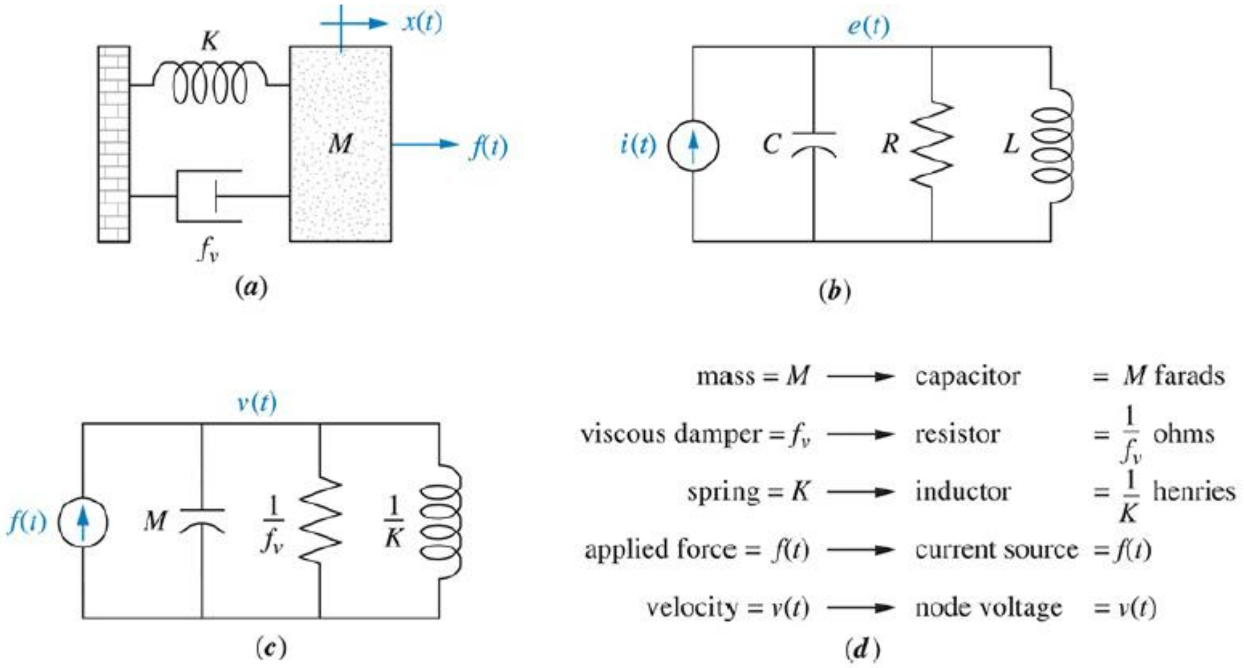


FIGURE Development of parallel analog: a. mechanical system; b. desired electrical representation; c. parallel analog; d. parameters for parallel analog

When we have more than one degree of freedom, the components associated with a motion appear as parallel electrical elements connected to a node. The components of adjacent motions are drawn as parallel electrical elements between two corresponding nodes. We demonstrate with an example.

Example 2.19 Converting a Mechanical System to a Parallel Analog

PROBLEM: Draw a parallel analog for the mechanical system of Figure example 2.18.

SOLUTION:

Equation (2.105) is also analogous to electrical node equations. Coefficients represent sums of electrical admittances. Admittances associated with M1 form the elements connected to the first node, where mechanical admittances between the two masses are common to the two nodes. Mechanical admittances associated with M2 form the elements connected to the second node. The result is shown in Figure below, where v1(t) and v2(t) are the velocities of M1 and M2, respectively.

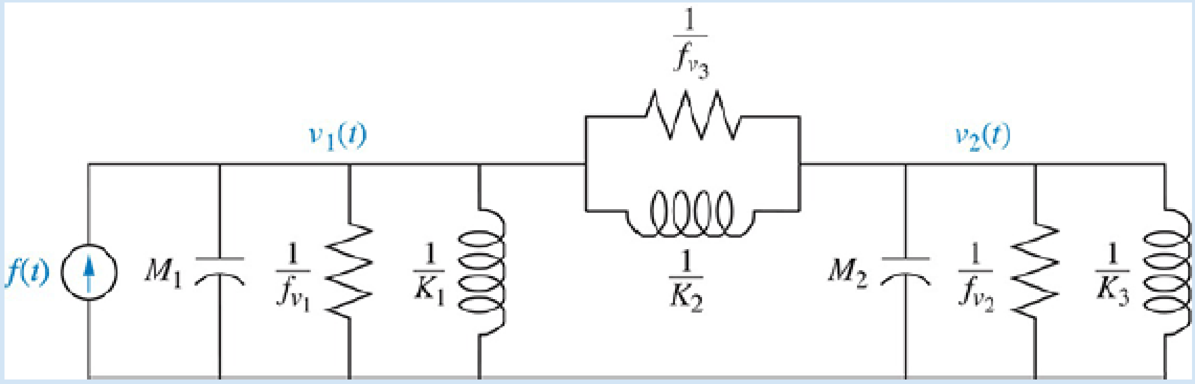


FIGURE Parallel analog of mechanical system