Tikrit university

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Department of Electrical Engineering

Fourth Class

Control Engineering

Chapter 6

Lecture 12

Stability of Control System

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Every system has to pass through a transient stage for a small period before reaching steady state. Naturally the question comes into play, "Will the system reach its steady state after passing through transients?" To get the answer of this question, study of stability is of utmost importance.

6.1 Effect of Location of Poles on Stability

Location of poles has a direct effect on stability. The entire S-plane is taken into account and it is divided as

follows:

(a) Left half plane (LHP)

(b) jw-axis

(c) Right half plane (RHP)

Let us consider the following cases:

(a) LHP poles

(i) On real axis and simple,

(ii) On real axis and multiple,

(iii) Complex conjugates and simple.

The system is stable for all of the above conditions.

(b) jw-axis

(i) On jw-axis and simple

(ii) On jw-axis and multiple

(iii) At origin

(iv) At origin and multiple

In this case, if the pole has zero real part and is not repeated, the system is called stable.

(c) RHP poles

(i) On real axis and simple

(ii) On real axis and multiple

(iii) Complex conjugates and simple

The system is unstable for all of the above conditions.

Figure 6.1 depicts all the above conditions

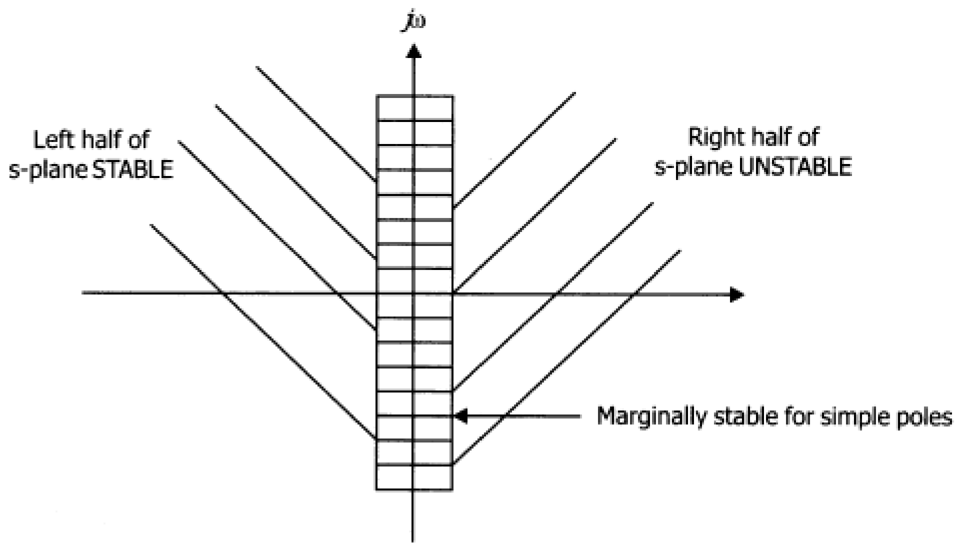

Fig.6.1

6.2 Routh–Hurwitz Criterion

In this section, we learn a method that yields stability information without the need to solve for the closed-loop system poles. Using this method, we can tell how many closed-loop system poles are in the left half-plane, in the right half plane, and on the $j\omega$-axis. (Notice that we say *how many*, not *where*.) We can find the number of poles in each section of the *s*-plane, but we cannot find their coordinates. The method is called the ***Routh–Hurwitz criterion*** for stability (*Routh*, *1905*).

The method requires two steps:

(1) Generate a data table called a Routh table and

(2) interpret the Routh table to tell how many closed-loop system poles are in the left half-plane, the right half-plane, and on the jω-axis

6.2.1 Generating a Basic Routh Table

Look at the equivalent closed-loop transfer function shown in Figure 6.2. Since we are interested in the system poles, we focus our attention on the denominator. We first create the Routh table shown in Table 6.1. Begin by labelling the rows with powers of *s* from the highest power of the denominator of the closed-loop transfer function to s^0. Next start with the coefficient of the highest power of *s* in the denominator and list, horizontally in the first row, every other coefficient. In the second row, list horizontally, starting with the next highest power of *s*, every coefficient that was skipped in the first row.

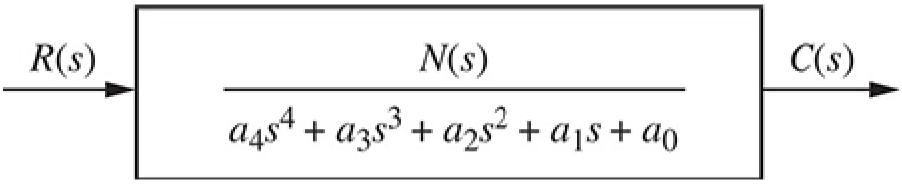

Fig. 6.2 Equivalent closed-loop transfer function

TABLE 6.1

Initial layout for Routh table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2			
s^1			
s^0			

The remaining entries are filled in as follows. Each entry is a negative determinant of entries in the previous two rows divided by the entry in the first column directly above the calculated row. The left-hand column of the determinant is always the first column of the previous two rows, and the righthand column is the elements of the column above and to the right. The table is complete when all of the rows are completed down to s^0. Table 6.2 is the completed Routh table. Let us look at an example.

TABLE 6.2

Completed Routh table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

Example 6.1 Examine the stability of the system in fig 6.3 using Routh's method

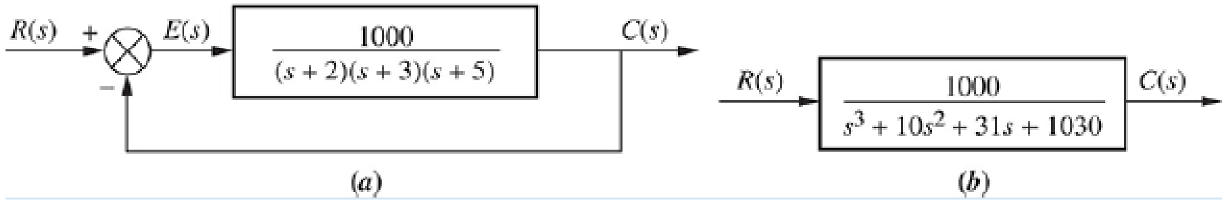

TABLE 6.3

Completed Routh table for Example 6.1

s^3	1	31	0
s^2	10 1 $\cancel{1030}$	103	0
s^1	$\dfrac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$
s^0	$\dfrac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$

Table 6.3 has two sign changes in the first column. The first sign change occurs from 1 in the s^2 row to −72 in the s^1 row. The second occurs from −72 in the s^1 row to 103 in the s^0 row. Thus, the system of Figure 6.3 is unstable since two poles exist in the right half-plane.

6.3 Routh–Hurwitz Criterion: Special Cases

Two special cases can occur: (1) The Routh table sometimes will have a zero only in the first column of a row, or (2) the Routh table sometimes will have an entire row that consists of zeros. Let us examine the first case.

6.3.1 Zero Only in the First Column

If the first element of a row is zero, division by zero would be required to form the next row. To avoid this phenomenon, an epsilon, ε, is assigned to replace the zero in the first column. The value ε is then allowed to approach zero from either the positive or the negative side, after which the signs of the entries in the first column can be determined. Let us look at an example.

Example 6.2 Determine the stability of the closed-loop transfer function

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

SOLUTION:

The solution is shown in Table 6.4. We form the Routh table using the denominator of Eq. (6.2). Begin by assembling the Routh table down to the row where a zero appears *only* in the first column (the *s*3 row). Next replace the zero by a small number, ε, and complete the table. To begin the interpretation, we must first assume a sign, positive or negative, for the quantity ε. Table 6.5 shows the first column of Table 6.4 along with the resulting signs for choices of ε positive and ε negative.

TABLE 6.4

Completed Routh table for Example 6.2

s^5	1	3	5
s^4	2	6	3
s^3	$\cancel{0}$ ε	$\frac{7}{2}$	0
s^2	$\frac{6\varepsilon-7}{\varepsilon}$	3	0
s^1	$\frac{42\varepsilon-49-6\varepsilon^2}{12\varepsilon-14}$	0	0
s^0	3	0	0

$$\lim_{\varepsilon\to+0} \frac{6\varepsilon-7}{\varepsilon} = -\infty = -$$

$$\lim_{\varepsilon\to+0} \frac{42\varepsilon-49-6\varepsilon}{12\varepsilon-14} = +\frac{-49}{-14} = +$$

TABLE 6.5

Determining signs in first column of a Routh table with zero as first element in a row

Label	First column	$\varepsilon = +$	$\varepsilon = -$
s^5	1	+	+
s^4	2	+	+
s^3	$\cancel{0}$ ε	+	−
s^2	$\frac{6\varepsilon-7}{\varepsilon}$	−	+
s^1	$\frac{42\varepsilon-49-6\varepsilon^2}{12\varepsilon-14}$	+	+
s^0	3	+	+

If ε is chosen positive, Table 6.5 will show a sign change from the $s3$ row to the $s2$ row, and there will be another sign change from the $s2$ row to the $s1$ row. Hence, the system is unstable and has two poles in the right half-plane.

Another method that can be used when a zero appears only in the first column of a row. This method is usually computationally easier than the epsilon method just described.

$$s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = 0$$

$$\left(\frac{1}{d}\right)^n + a_{n-1}\left(\frac{1}{d}\right)^{n-1} + \cdots + a_1\left(\frac{1}{d}\right) + a_0 = 0$$

$$\left(\frac{1}{d}\right)^n \left[1 + a_{n-1}\left(\frac{1}{d}\right)^{-1} + \cdots + a_1\left(\frac{1}{d}\right)^{(1-n)} + a_0\left(\frac{1}{d}\right)^{-n}\right] = 0$$

$$\left(\frac{1}{d}\right)^n \left[1 + a_{n-1}d + \cdots + a_1 d^{(n-1)} + a_0 d^n\right] = 0$$

Example 6.3 Determine the stability of the closed-loop transfer function

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

SOLUTION:

First write a polynomial that has the reciprocal roots of the denominator of above Eq.

$D(s) = 3s^5 + 5s^4 + 6s^3 + 3s^2 + 2s + 1$

We form the Routh table as shown in Table 6.6 using D(s)Eq. the system is unstable and has two poles in the right half-plane.

TABLE 6.6

Routh table for Example 6.3

s^5	3	6	2
s^4	5	3	1
s^3	4.2	1.4	
s^2	1.33	1	
s^1	−1.75		
s^0	1		

6.3.2 Entire Row is Zero

We now look at the second special case. Sometimes while making a Routh table, we find that an entire row consists of zeros because there is an even polynomial that is a factor of the original polynomial. This case must be handled differently from the case of a zero in only the first column of a row. Let us look at an example that demonstrates how to construct and interpret the Routh table when an entire row of zeros is present.

Example 6.4 Determine the number of right-half-plane poles in the closed-loop transfer function

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

SOLUTION:

Start by forming the Routh table for the denominator of T(s) Eq. (see Table 6.7). At the second row, we multiply through by 1/7 for convenience. We stop at the third row, since the entire row consists of zeros, and use the following procedure. First we return to the row immediately above the row of zeros and form an auxiliary polynomial.

$P(s) = s^4 + 6s^2 + 8$

$\frac{dp(s)}{ds} = 4s^3 + 12s + 0 \div 4 = s^3 + 3s$

TABLE 6.7

Routh table for Example 6.4

s^5	1	6	8
s^4	$\cancel{7}$ 1	42 6	56 8
s^3	$\cancel{0}$ $\cancel{4}$ 1	$\cancel{0}$ 12 3	$\cancel{0}$ $\cancel{0}$ 0
s^2	3	8	0
s^1	$\frac{1}{3}$	0	0
s^0	8	0	0

Hence, there are no right-half-plane poles.