Frequency Modulation (FM):

$$f_i(t) = f_c + \frac{K_f}{2\pi} m(t)$$

$$\theta(t) = 2\pi f_c t + K_f \int_0^t m(\lambda) d\lambda$$

Where K_f is the deviation sensitivity for frequency or frequency deviation constant for FM

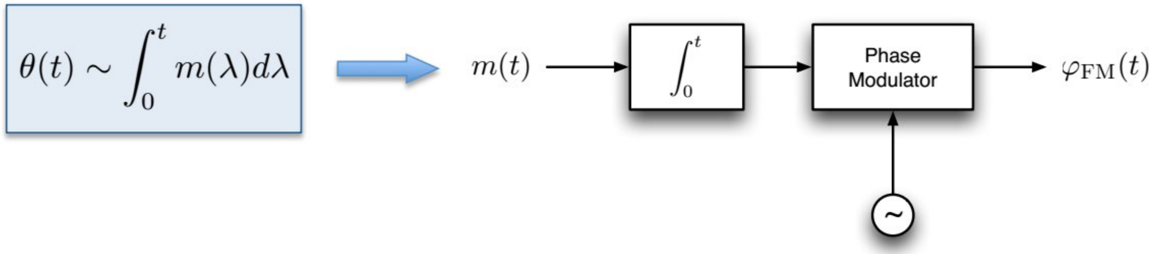
The modulated FM signal is given by

$$\varphi_{\text{FM}}(t) = A_c \cos\left(2\pi f_c t + K_f \int_0^t m(\lambda) d\lambda \right)$$

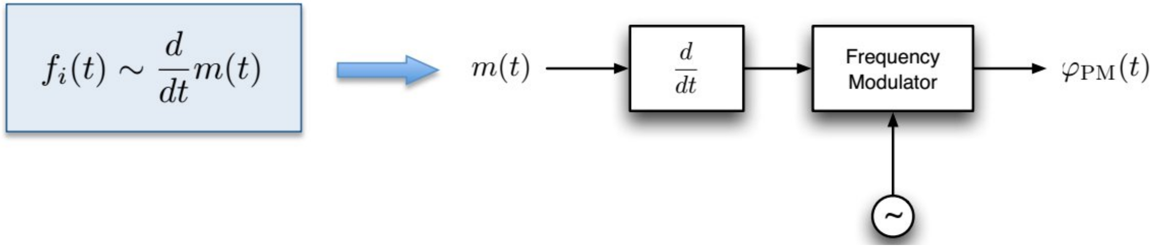
Information contained in $m(t)$ is embedded in the instantaneous frequency:

$f_i(t)$ is proportional to $m(t)$

PM and FM are tightly related to each other. We see from the phase and frequency relations for PM and FM given above that replacing *m(t)* in the PM signal with ∫m (λ) dλ gives an FM signal and replacing m(t) in the FM signal with $\frac{d\,m(t)}{dt}$ gives a PM signal. This is illustrated in the following block diagrams.



or we can consider PM as a special case of FM where:



Example : An angle modulated signal is given by $x(t) = \cos(2000t + 0.1\sin(20t))$

a) Find the frequency of the modulated carrier f_c

b) Instantaneous frequency $f_i(t)$

Solution

a) 2p f_c =2000, f_c=318.31Hz.

b) $\theta(t) = 2000t + 0.1\sin(20t))$

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = \frac{1}{2\pi}(2000 + 2\cos(20t)) = 318.31 + 0.32\cos(20t)$$

Assume ***m(t)*** is information signal.

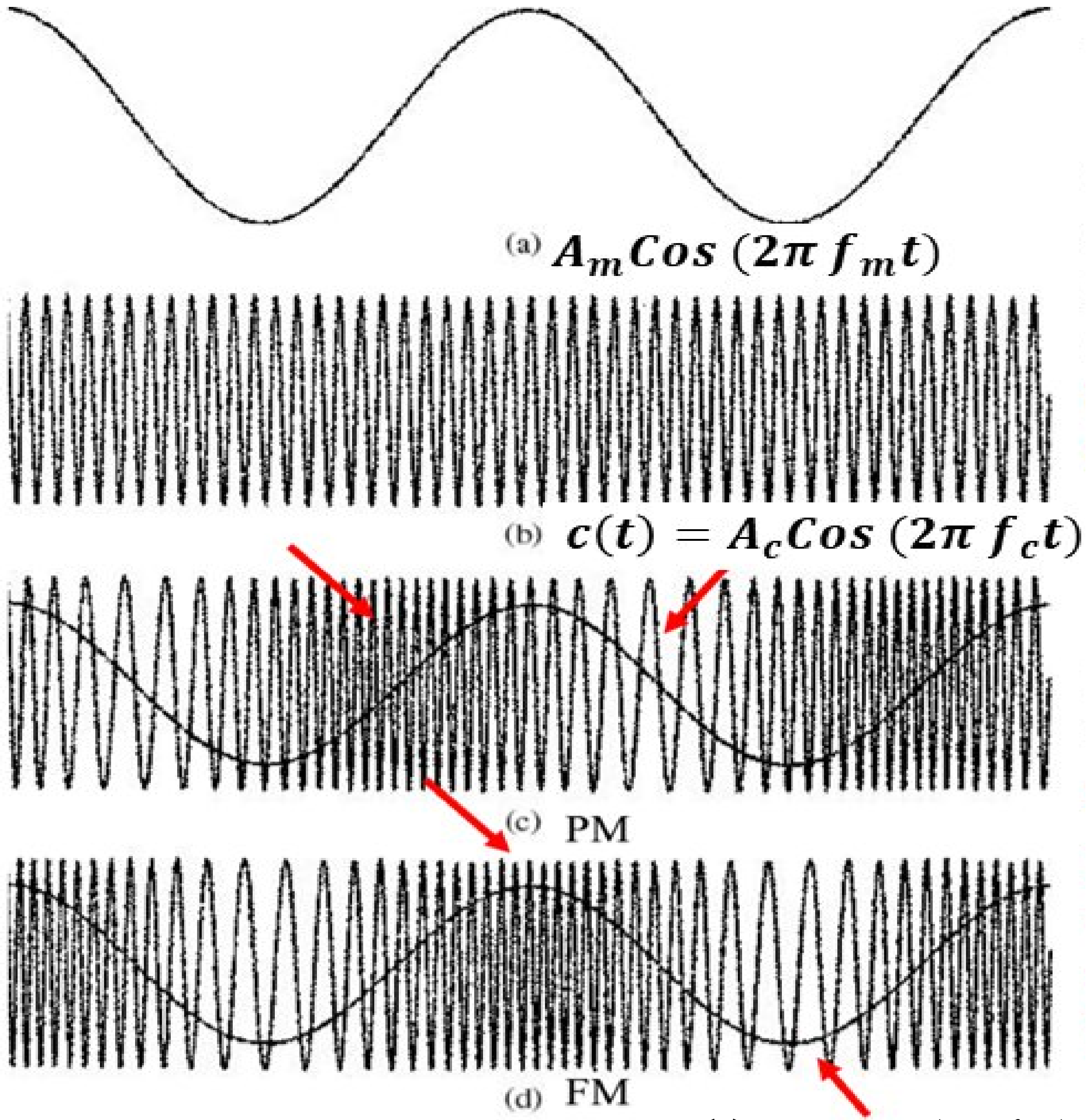
For PM

The phase deviation of modulator output is proportional to ***m(t).*** The frequency deviation is proportional to the **derivative** of the phase deviation, thus the $\boldsymbol{\omega_i}$ is maximum when the **slope** of ***m(t)*** is maximum and minimum when the **slope** of ***m(t)*** is minimum.

For FM

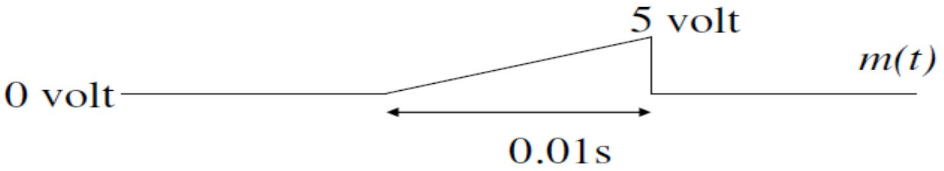
Frequency deviation is proportional to m(t), thus the $\boldsymbol{\omega_i}$ is max when the ***m(t)*** is maximum and it is minimum when ***m(t)*** is minimum.

Note that if ***m(t)*** is not shown along with the modulator o/p. It would not be possible to distinguish the PM and FM o/p



PM and FM modulators for a tone information signal $m(t) = A_m Cos\,(2\pi f_m t)$

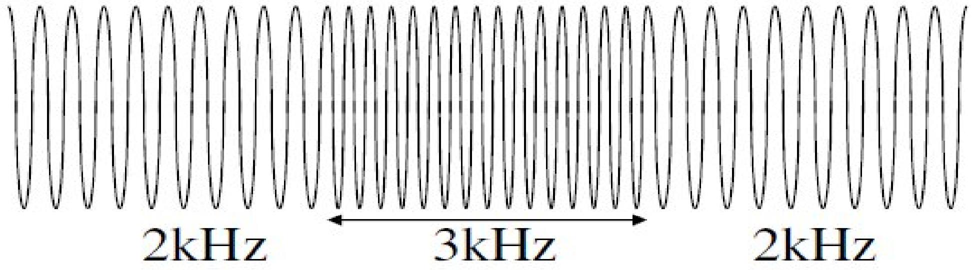
Example : A phase modulated signal has a carrier of frequency 2kHz. Draw the phase modulated signal given the phase sensitivity is 2 volt^{-1} for the below modulating signal m(t).



Solution

$$\varphi_{PM}(t) = A_c \cos(2\pi f_c t + K_p m(t))$$

$$\theta(t) = 2\pi f_c t + K_p m(t)$$



The instantaneous frequency is

- $f_i = f_c + 2(0) = 2kHz$, for $m(t)=0$.
- $f_i = f_c + 2(500) = 3kHz$, for $m(t)=500t$.

Example : A phase modulated signal has a carrier of 100 MHz and the modulating signal is 1 kHz sinusoid of amplitude 0.01 V. The proportionality constant, K_p, is 10^4 V^{-1}. Find the band of frequencies occupied by the PM waveform.

Solution: PM

$$\theta(t) = 2\pi f_c t + k_p \, m(t)$$

$$= 2\pi 10^8 t + 10^4 \times 0.01 \cos\left(2\pi \times 10^3 t\right)$$

- The instantaneous frequencies,

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta}{dt} = 10^8 - 2\pi \times 10^5 \sin\left(2\pi \times 10^3 t\right)$$

Example : Tone signal $m(t) = A_m \cos(\omega_m t)$ is given. Find FM and PM waveforms.

Solution

$$\varphi(t) = A_c \cos\theta(t)$$

A_c = constant : we need to find $\theta(t)$

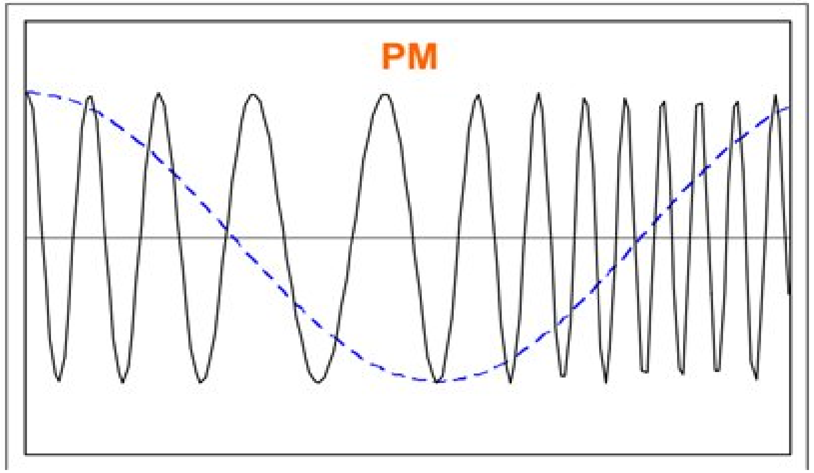
$$\theta(t) = 2\pi f_c t + K_p m(t) + \theta_0 \; = \; \omega_c t + k_p\, m(t) = \; \omega_c t + k_p A_m \cos(\omega_m t) \qquad \text{(PM)}$$

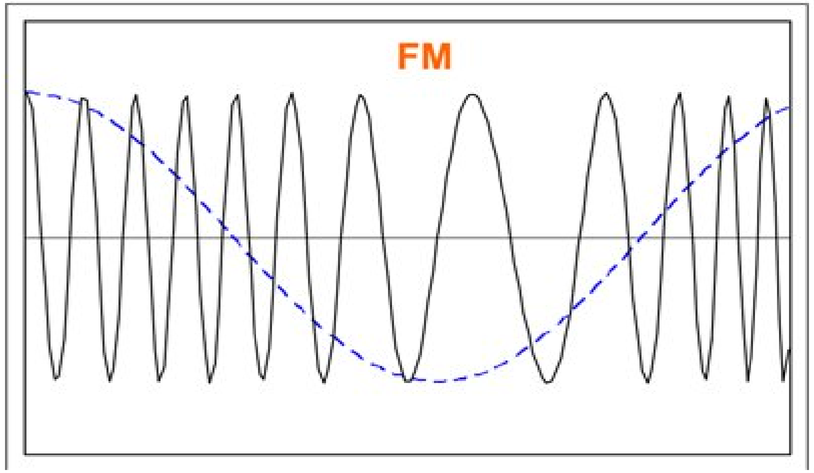
$$\theta(t) = 2\pi f_c t + k_f \int_{-\infty} m(\tau) d\tau = \; 2\pi f_c t + k_f \int_{-\infty} A_m \cos(\omega_m t)\, d\tau = \; \omega_c t + \frac{k_f A_m}{\omega_m} \sin(\omega_m t) \qquad \text{(FM)}$$

therefore

$$\varphi_{\mathrm{PM}}(t) = A_c \cos(\omega_c t + k_p A_m \cos(\omega_m t)) \qquad \text{(PM)}$$

$$\varphi_{\mathrm{FM}}(t) = A_c \cos(\omega_c t + \frac{k_f A_m}{\omega_m} \sin(\omega_m t)) \qquad \text{(FM)}$$





One primary difference between PM and FM is the way the modulation index $\boldsymbol{\mu}$ is defined

- **For PM**, $\boldsymbol{\mu}$ is proportional to the amplitude of the modulating signal (A_m) and independent of its frequency

$$\mu_p = K_p A_m$$ (unit in radians) (equal to Peak phase deviation $\boldsymbol{\Delta\theta}$)

- **For FM** , $\boldsymbol{\mu}$ is proportional to the amplitude of the modulating signal (A_m) and inversely proportional its frequency (f_m)

$$\mu_f = \frac{K_f A_m}{f_m}$$ (unitless)

Thus for tone signal the modulated signal can be rewritten as

$$\varphi_{\mathrm{PM}}(t) = A_c \cos(\omega_c t + \mu_p \cos(\omega_m t))$$ (PM)

$$\varphi_{\mathrm{FM}}(t) = A_c \cos(\omega_c t + \mu_f \sin(\omega_m t))$$ (FM)

Frequency Deviation – the change in frequency that occurs in the carrier when it is acted on by a modulating-signal frequency.

- Typically given as a peak frequency shift in hertz (Δ*f*).
- The peak-to-peak frequency deviation (2 Δ*f*) is called carrier swing.
- For FM, the deviation sensitivity, K_f is often given in hertz/volt.

Thus, the peak frequency deviation can be expressed mathematically as:

$$\Delta f = K_f A_m \quad \text{(Hz)}$$

Therefore the modulation index can be expressed as

$$\mu_f = \frac{\Delta f}{f_m} \quad \text{(unitless)}$$

Percent Modulation – the ratio of actual frequency deviation to the maximum frequency deviation allowed in percentage form.

$$\%\ modulation = \frac{\Delta f\ (actual)}{\Delta f\ (Max)} \times 100$$

Example Determine the peak frequency deviation (Δf) and modulation index μ for an FM modulator with a deviation sensitivity, K_f = 10kHz/V and a modulating signal
***m(t)* = 5 cos(2π×2500*t*).**

Solution

1. $\Delta f = K_f A_m = \frac{10\ KHz}{V}\ (5V) = 50\ KHz$
2. $\mu_f = \frac{\Delta f}{f_m} = \frac{50KHz}{2500} = 20$

Example Determine the peak phase deviation ($\Delta\theta$) for a PM modulator with a deviation sensitivity K_p = 2.5rad/V and a modulating signal ***m(t)* = 2 cos(2π(2000)*t*).**

Solution

$\Delta\theta = K_p A_m = 2.5\ (\frac{rad}{V}) \times 2V = 5$ radians

Example *The carrier frequency in an FM modulator is 1000 kHz. If the modulating frequency is 15 kHz, what are the first three upper sideband and lower sideband frequencies?*

Solution.

Carrier frequency, f_c = 1000 kHz

Modulating frequency, f_m = 15 kHz

Upper sideband frequencies

$f_c + f_m$	$f_c + 2f_m$	$f_c + 3f_m$
1000 + 15	1000 + 2 × 15	1000 + 3 × 15
1015 kHz	**1030 kHz**	**1045 kHz**

Lower sideband frequencies

$f_c - f_m$	$f_c - 2f_m$	$f_c - 3f_m$
1000 – 15	1000 – 2 × 15	1000 – 3 × 15
985 kHz	**970 kHz**	**955 kHz**

- Theoretically, the generation and transmission of FM requires infinite bandwidth. Practically, FM system have finite bandwidth and they perform well.
- The value of **modulation index** determine the number of sidebands that have the significant relative amplitudes
- If $\boldsymbol{n}$ is the number of sideband pairs, and line of frequency spectrum are spaced by $\boldsymbol{f_m}$, thus, the bandwidth is:

$$B_{FM} = 2n f_m \qquad \text{For } n \geq 1$$

Carson's Rule for FM Bandwidth.

An approximation for the bandwidth of an FM signal is given by BW = 2(Maximum frequency deviation + highest modulating frequency)

$$B_{FM} = 2(m_f + 1) f_m$$

$$= 2(\Delta f + f_m)$$

The same rule applies to PM bandwidth,

$$B_{PM} = 2(\Delta f + f_m)$$

For PM the $\boldsymbol{\Delta f} = \frac{\mathbf{1}}{\mathbf{2\pi}} \boldsymbol{K_p} \left| \frac{d\, m(t)}{dt} \right|_{\max}$

$\Delta f = \Delta\omega / 2\pi$

When $\Delta f << f_m$, modulation index < *1,* the scheme is called Narrow Band (NBFM, NBPM).

Then the bandwidth is $$B_{NBFM} = B_{NBPM} = 2(f_m)$$

- Therefore, no matter how small we make the deviation around f_c , the bandwidth of the modulated signal does not get smaller than $2f_m$ or $2B_m$.

Example: If the modulating frequency is 1 kHz and the maximum deviation is 10 kHz, what is the modulation index and the required bandwidth for an FM signal?

Solution

$f_m = 1\text{kHz}, \Delta f = 10\text{kHz}$

$$\mu_f = \frac{\Delta f}{f_m} = \frac{10\ kHz}{1kHz} = 10$$

$$B_{FM} = 2(m_f + 1) f_m$$
$$= 22kHz$$

Example : Consider the angle modulated (PM) signal **φ (t) = 10 cos ($2\pi\ 10^8$t + 3 sin $2\pi\ 10^3$t).** Find the bandwidth of the PM signal when the message frequency is (1) doubled, and (2) is halved.

Solution :φ_{PM} (t) = $A_c\ Cos\ (\theta(t))$= 10 cos ($2\pi\ 10^8$t + 3 sin $2\pi\ 10^3$t)

= $A_c\ Cos\ (2\pi f_c\, t + K_p\, m(t))$

We have $f_m = 1\ kHz$ and $K_p = 3$

$m(t)$=sin $2\pi\ 10^3$t

$\frac{d\,m(t)}{dt} = 2\pi\ 10^3 \cos(2\pi\ 10^3 t)$

$\left|\frac{d\,m(t)}{dt}\right|_{max} = 2\pi\ 103$

$\Delta\omega$ for PM = $K_p \left|\frac{d\,m(t)}{dt}\right|_{max}$=6π kHz : $\Delta f = \Delta\omega/2\pi = 3\ kHz$

$$B_{PM} = 2(\Delta f + f_m) = 2(3+1) = 8kHz$$

When the message frequency is doubled

$\Delta\omega$ for PM = $K_p \left|\frac{d\,m(t)}{dt}\right|_{max}$=12 π kHz

$$B_{PM} = 2(\Delta f + f_m) = 2(6+2) = 16\,kHz$$

When the message frequency is halved $\Delta f = 1.5\ kHz$

$$B_{PM} = 2(\Delta f + f_m) = 2(1.5+0.5) = 4\,kHz$$

Deviation Ratio (DR)is defined as the worst-case modulation index and is equal to the maximum peak frequency deviation divided by the maximum modulating-signal frequency.

$$DR = \frac{\triangle f_{max}}{f_{m(max)}}$$

The worst-case modulation index produces the widest output frequency spectrum.

Example : Determine the deviation ratio and the bandwidth for an FM broadcast-band transmitter with a maximum frequency deviation of 75kHz and a maximum modulating-signal frequency of 15kHz.

Solution:

$$DR = \frac{\triangle f_{max}}{f_{m(max)}} = \frac{75KHz}{15KHz} = 5$$

$$B_{FM} = 2\left(\triangle f + f_m\right) = 2(75+15) = 180KHz$$

Some mobile phone companies use FM with a very low modulation index, i.e. $\mu_f < 1$. This is known as **narrowband FM**. Mobile phone companies use this because it offers many of the advantages of FM, with the minimum bandwidth requirement.

Angle Modulation versus Amplitude Modulation

Advantages

- Wideband FM gives significant *improvement in the SNR* at the output of the RX which proportional to the square of modulation index.
- Angle modulation is *resistant to propagation-induced* selective fading since amplitude variations are unimportant and are removed at the receiver using a limiting circuit.
- Angle modulation is very *effective in rejecting interference*. (minimizes the effect of noise).
- Angle modulation allows the use of *more efficient* transmitter power in information.
- Angle modulation is capable of *handing a greater dynamic range* of modulating signal without distortion than AM.

Disadvantages

- Angle modulation requires a transmission bandwidth *much larger* than the message signal bandwidth.
- Angle modulation requires *more complex* and expensive circuits than AM.

Questions?

Thank you for
Listening & Attending