DYNAMICS

Kinetics of a Particle: Work and Energy

CHAPTER OBJECTIVES

■ To develop the principle of work and energy and apply it to solve problems that involve force, velocity, and displacement.

■ To study problems that involve power and efficiency.

■ To introduce the concept of a conservative force and apply the theorem of conservation of energy to solve kinetic problems

2.1 The Work of a Force

In this chapter, we will analyze motion of a particle using the concepts of work and energy. The resulting equation will be useful for solving problems that involve force, velocity, and displacement. Before we do this, however, we must first define the work of a force. Specifically, a force **F** will do work on a particle only when the particle undergoes a displacement in the direction of the force. For example, if the force **F** in Fig. 2-1 causes the particle to move along the path s from position **r** to a new position **r**' the displacement is then d**r** = **ŕ** - **r**. The magnitude of d**r** is ds, the length of the differential segment along the path. If the angle between the tails of d**r** and **F** is θ, Fig. 2-1, then the work done by **F** is a scalar quantity, defined by

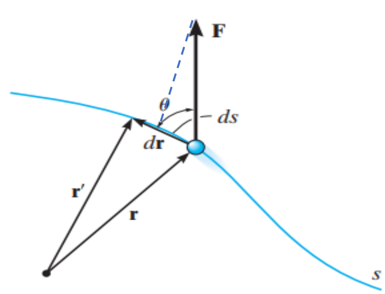


Fig. 2-1

$$dU_{1-2} = F\ ds \cos\theta$$

By definition of the dot product this equation can also be written as

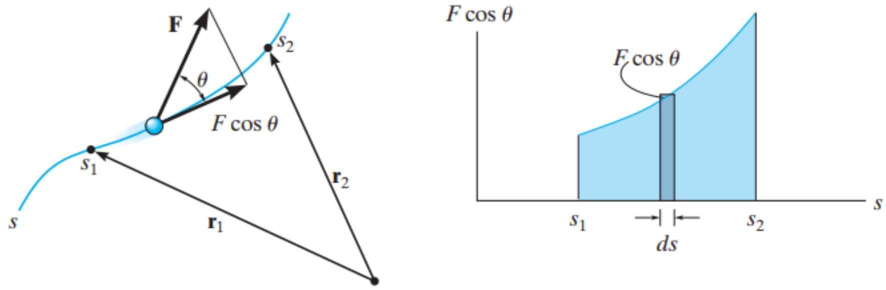
$$dU = \mathbf{F} d\mathbf{r}$$

This result may be interpreted in one of two ways: either as the product of F and the component of displacement ds cosθ in the direction of the force, or as the product of ds and the component of force, F cos θ, in the direction of displacement. Note that if $0 \leq \theta > 90^0$, then the force component and the displacement have the same sense so that the work is positive; whereas if $90 < \theta \leq 180$, these vectors will have opposite sense, and therefore the work is negative. Also, dU = 0 if the force is perpendicular to displacement, since cos 90 = 0, or if the force is applied at a fixed point, in which case the displacement is zero. The unit of work in SI units is the joule (J), which is the amount of work done by a one-newton force when it moves through a distance of one meter in the direction of the force (1J = 1 N.m). In the FPS system, work is measured in units of foot-pounds (ft.lb), which is the work done by a one-pound force acting through a distance of one foot in the direction of the force.

Work of a Variable Force. If the particle acted upon by the force **F** undergoes a finite displacement along its path from $\mathbf{r}_1$ to $\mathbf{r}_2$ or S_1 to S_2, Fig. 2–2a, the work of force **F** is determined by integration. Provided **F** and θ can be expressed as a function of position, then

$$U_{1-2} = \int_{r_1}^{r_2} F dr = F \int_{s_1}^{s_2} \cos\theta ds \qquad (2\text{-}1)$$

Sometimes, this relation may be obtained by using experimental data to plot a graph of F cos θ vs. S. Then the area under this graph bounded by S_1 and S_2 represents the total work, Fig. 1–2b



(a) (b)

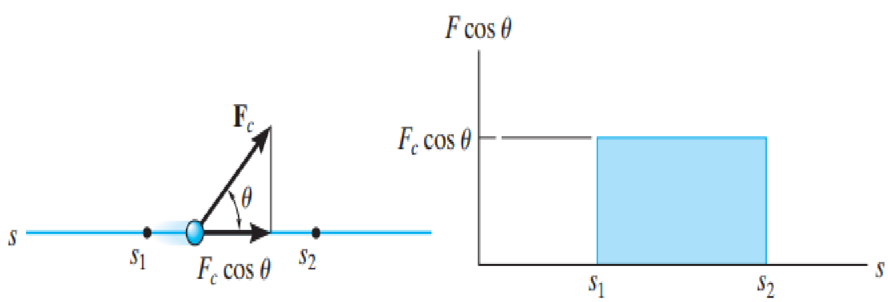
By convention, the units for the moment of a force or torque are written as lb . ft, to distinguish them from those used to signify work, ft . lb

Work of a Constant Force Moving Along a Straight Line.

If the force $\mathbf{F}_c$ has a constant magnitude and acts at a constant angle θ from its straight-line path, Fig. 2–3a, then the component of $\mathbf{F}_c$ in the direction of displacement is always Fc cos θ. The work done by $\mathbf{F}_c$ when the particle is displaced from s_1 to s_2 is determined from Eq. 2–1, in which case

$$U_{1-2} = F_C \cos\theta \int_{s_1}^{s_2} ds$$

$$\boxed{U_{1-2} = F_C \cos\theta \, (s_2 - s_1)} \qquad (2\text{-}2)$$



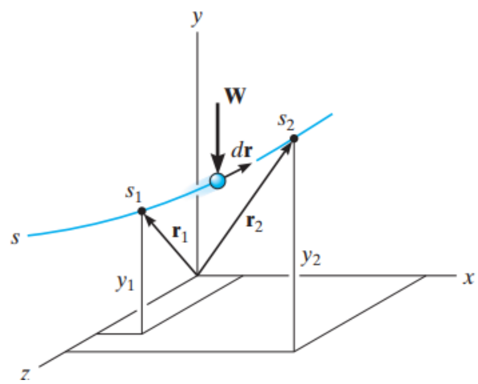
(a) (b)

Here the work of **F** represents the area of the rectangle in Fig. 2-3b.

Work of a Weight. Consider a particle of weight **W**, which moves up along the path S shown in Fig. 2-4 from position S1 to position S_2. At an intermediate point, the displacement d**r** = dx**i** + dy**j** + dz**k**. Since **W** = -W**j**, applying Eq. 2-1 we have

$$U_{1\text{-}2} = \int F\,dr = -\int_{S_1}^{S_2} (Wj)\,(dxi + dyj + dzk) = -\int_{y_1}^{y_2} W\,dy \Rightarrow U_{1\text{-}2} = -W\,(y_2 - y_1)$$

$$U_{1\text{-}2} = -W\Delta y \qquad (2\text{-}3)$$

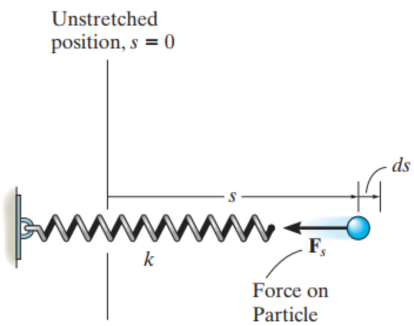


2-4

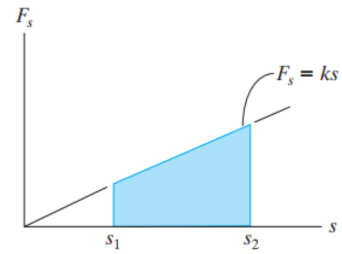
Thus, the work is independent of the path and is equal to the magnitude of the particle's weight times its vertical displacement. In the case shown in Fig. 3-4 the work is negative, since W is downward and Δy is upward. Note, however, that if the particle is displaced downward (- Δy), the work of the weight is positive. Why?

Work of a Spring Force. If an elastic spring is elongated a distance ds, Fig. 2–5a, then the work done by the force that acts on the attached particle is dU = F_{SP}ds = - ks ds. The work is negative since **F**s acts in the opposite sense to ds. If the particle displaces from S_1 to S_2, the work of $\mathbf{F_{SP}}$ is then.

$$U_{1\text{-}2} = F_{\mathbf{SP}}.\,dS = -\int_{S_1}^{S_2} (k.\,S).\,ds = -\frac{1}{2}K(S_2^2 - S_1^2) \qquad (2\text{-}4)$$



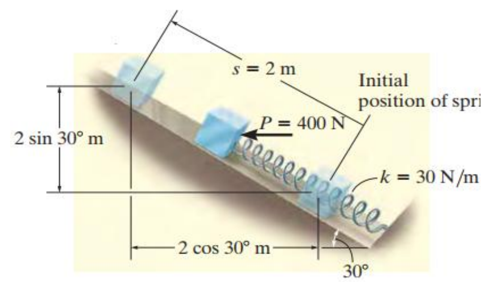
(a)



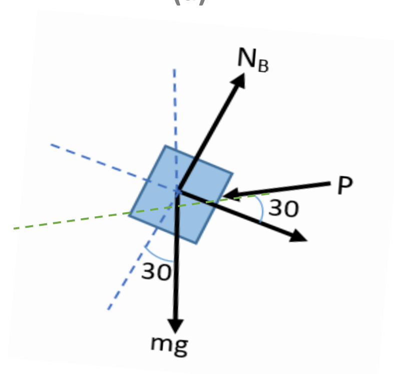
(b)

Fig 2-5

PROBLEM.34: The 10-kg block shown in Fig. below rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force P = 400 N pushes the block up the plane S = 2 m.



(a)



(b)

SOLUTION

First the free-body diagram of the block is drawn in order to account for all the forces that act on the block, Fig. 2–6b.

Horizontal Force P. Since this force is constant, the work is determined using Eq. 2-2. The result can be calculated as the force times the component of displacement in the direction of the force; i.e.

$U_P = P \times S\cos\theta = 400(2\cos 30) = 692.8$ J

or the displacement times the component of force in the direction of displacement,

Spring Force F_{SP}. In the initial position the spring is stretched S_1 = 0.5 m and in the final position it is stretched S_2 = 0.5 m + 2 m = 2.5 m. We require the work to be negative since the force and displacement are opposite to each other. The work of **F**s is thus

U_{SP} = = - $\frac{1}{2}$K (S_2^2-S_1^2) = - $\frac{1}{2}$ (30)[$(2.5)^2$ - $(0.5)^2$] = -90 Joule

Weight W. Since the weight acts in the opposite sense to its vertical displacement, the work is negative; i.e.,

U_W = - mgy =10(9.81)(2sin 30)= - (98.1) (2sin 30) = - 98.1 Joule

Note that it is also possible to consider the component of weight in the direction of displacement; i.e.

Normal Force N_B. This force does no work since it is always perpendicular to the displacement.

Total Work. The work of all the forces when the block is displaced 2 m is therefore

U_T =∑U= U_P+U_{SP}+U_W= 692.8 J - 90 J - 98.1 J = 505 Joule