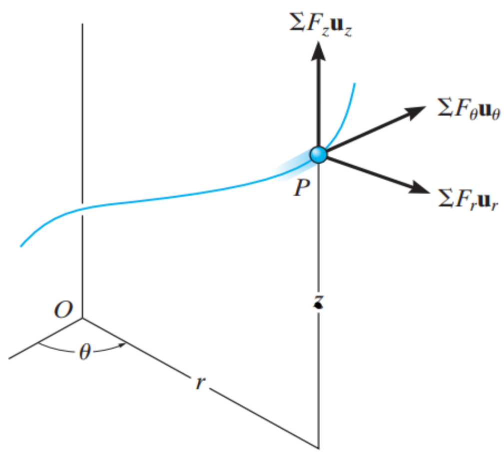
# DYNAMICS

## .1.6 Equations of Motion: Cylindrical Coordinates

When all the forces acting on a particle are resolved into cylindrical components, i.e., along the unit-vector directions **u**r, **u**θ, **u**z, Fig. 1–16, the equation of motion can be expressed as:



Inertial coordinate system

$$\sum F = ma$$

$$\sum F_r u_r + \sum F_\theta u_\theta + \sum F_z u_z = ma_r u_r + ma_\theta u_\theta + ma_z u_z$$

To satisfy this equation, we require

$$\left.\boxed{\begin{array}{l} \sum F_r u_r = ma_r u_r \\ \sum F_\theta u_\theta = ma_\theta u_\theta \\ \sum F_z u_z = ma_z u_z \end{array}}\right\} \qquad (1\text{-}9)$$

If the particle is constrained to move only in the r-θ plane, then only the first two of Eq. 1–9 are using to specify the motion.

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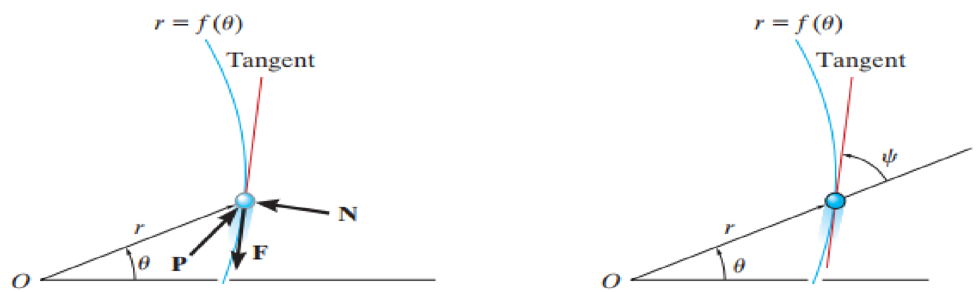
**Tangential and Normal Forces.** The most straight forward type of problem involving cylindrical coordinates requires the determination of the resultant force components $\sum F_r$, $\sum F_\theta$, $\sum F_z$ which, cause a particle to move with a known acceleration. If, however, the particle's accelerated motion is not completely specified at the given instant, then some information regarding the directions or magnitudes of the forces acting on the particle must be known or calculated in order to solve Eqs. 1-9 For example, the force **P** causes the particle in Fig. 1–17a to move along a path $r = f(\theta)$. The normal force **N** that the path exerts on the particle is always perpendicular to the tangent of the path, whereas the frictional force **F** always acts along the tangent in the opposite direction of motion. The directions of **N** and **F** can be specified relative to the radial coordinate by using the angle $\psi$ (psi), Fig.1-17b, which is defined between the extended radial line and the tangent to the curve



(a) (b)

This angle can be obtained by noting that when the particle is displaced a distance dS along the path, Fig. 1–17c, the component of displacement in the radial direction is dr and the component of displacement in the transverse direction is $r\,d\theta$.

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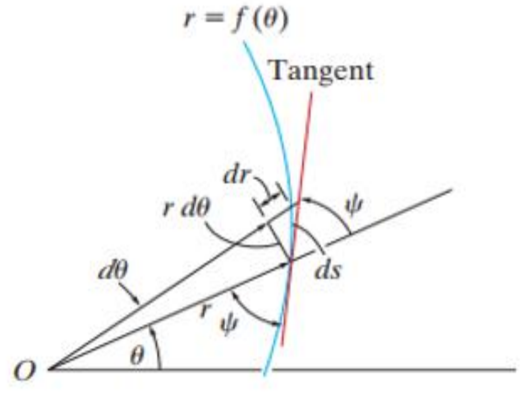


c

Since these two components are mutually perpendicular, the angle c can be determined from tan $\psi$ = r dθ/dr, or

$$\boxed{\tan\psi = \frac{r}{dr/d\theta}} \qquad (1\text{-}10)$$

If $\psi$ is calculated as a positive quantity, it is measured from the extended radial line to the tangent in a counterclockwise sense or in the positive direction of θ. If it is negative, it is measured in the opposite direction to positive θ. For example, consider the cardioid r = a(1 + cos θ), shown in Fig. 1–18. Because dr/dθ = -a sin θ, then when θ = 30, tan $\psi$ = a(1 + cos 30)/(-a sin 30) = -3.732, or $\psi$ = -75, measured clockwise, opposite to +θ as shown in the figure 1-18.

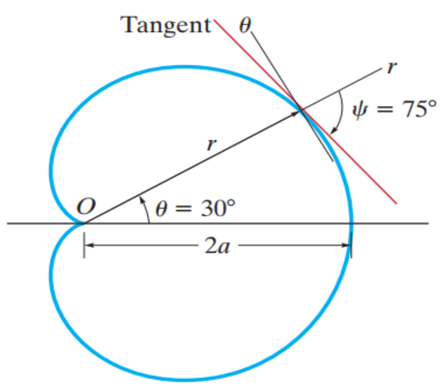


Fig. 1-18

## Procedure for Analysis

Cylindrical or polar coordinates are a suitable choice for the analysis of a problem for which data regarding the angular motion of the radial line r are given, or in cases where the path can be conveniently expressed in terms of these coordinates. Once these coordinates have been established, the equations of motion can then be applied in order to relate the forces acting on the particle to its acceleration components. The method for doing this has been outlined in the procedure for analysis given in the following is a summary of this procedure.

### Free-Body Diagram.

- Establish the r, $\theta$, z inertial coordinate system and draw the particle's free-body diagram.
- Assume that $\mathbf{a}_r$, $\mathbf{a}_\theta$, $\mathbf{a}_z$ act in the positive directions of r, $\theta$, z if they are unknown.
- Identify all the unknowns in the problem.

### Equations of Motion.

- Apply the equations of motion, Eq. 1–9.

### Kinematics.

- Use the methods to determine r and the time derivatives $\dot{r}, \ddot{r}, \dot{\theta}, \ddot{\theta}, \ddot{z}$ and then evaluate the acceleration components $a_r = \ddot{r} - r\dot{\theta}^2$, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, $a_z = \ddot{z}$
- If any of the acceleration components is computed as a negative quantity, it indicates that it acts in its negative coordinate direction.
- When taking the time derivatives of r = f($\theta$), it is very important to use the chain rule of calculus, which is discussed in Appendix C

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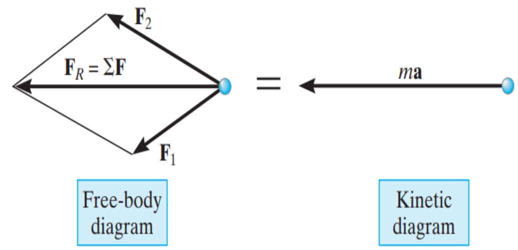
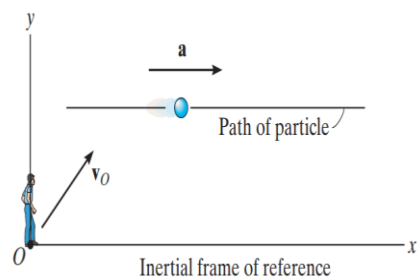
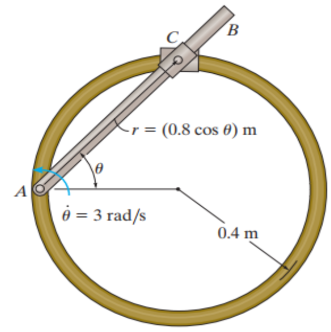
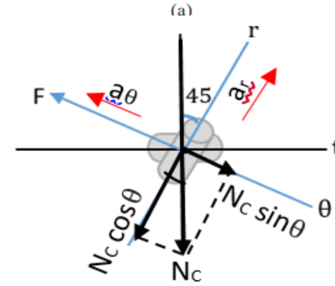
| CHAPTER REVIEW | |
|---|---|
| **Kinetics**<br>Kinetics is the study of the relation between forces and the acceleration they cause. This relation is based on Newton's second law of motion, expressed mathe -matically as $\sum F = ma$. Before applying the equation of motion, it is important to first draw the particle's free-body diagram in order to account for all of the forces that act on the particle. Graphically, this diagram is equal to the kinetic diagram, which shows the result of the forces, that is, the ma vector |  |
| **Inertial Coordinate Systems**<br>When applying the equation of motion, it isimportant to measure the acceleration from an inertial coordinate system. This system has axes that do not rotate but are either fixed or translate with a constant velocity. Various types of inertial coordinate systems can be used to apply $\sum F = ma$ , in component form. |  |
| **Force components**<br>Rectangular x, y, z axes are used to describe the motion along each of the axes. | $\sum F_x = ma_x$, $\sum F_y = ma_y$, $\sum F_z = ma_z$ |
| **Normal, tangential, and binormal axes n, t, b,**<br>are often used when the path is known. Recall that an is always directed in the +n direction. It indicates the change in the velocity direction. also recall that a t is tangent to the path. It indicates the change in the velocity magnitude. | $\sum F_t = ma_t$, $\sum F_n = ma_n$, $\sum F_b = 0$<br>t = dv/dt or at = v dv/ds<br>$\rho = [1 + (\frac{dy}{dx})^2]^{3/2} / \lvert \frac{d^2y}{dx^2} \rvert$<br>$a_n = (\frac{v^2}{\rho})$ |
| **Cylindrical coordinates**<br>are useful when angular motion of the radial line r is specified or when the path can conveniently be described with these coordinates | $\sum F_r = m(\ddot{r} - r\dot{\theta}^2)$<br>$\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$<br>$\sum F_z = m\ddot{z}$ , tan $\psi = \frac{r}{dr/d\theta}$ |
| **Central-Force Motion**<br>When a single force acts upon a particle, such as during the free-flight trajectory of a satellite in a gravitational field, then the motion is referred to as central-force motion. The orbit depends upon the eccentricity e; and as a result, the trajectory can be either circular, parabolic, elliptical, or hyperbolic. | |

PROBLEM.27: The smooth 0.5-kg double-collar in Fig. 1-19a can freely slide on arm AB and the circular guide rod. If the arm rotates with a constant angular velocity of $\dot{\theta}$ = 3 rad/s, determine the force the arm exerts on the collar at the instant $\theta$ = 45. Motion is in the horizontal plane.




(a)




SOLUTION

Free-Body Diagram. The normal reaction $N_C$ of the circular guide rod and the force F of arm AB act on the collar in the plane of motion, Fig. 1-19b. Note that F acts perpendicular to the axis of arm AB, that is, in the direction of the u axis, while $N_C$ acts perpendicular to the tangent of the circular path at $\theta = 45^0$. The four unknowns are $N_C$, F, $a_r$, $a_\theta$.

Equations of Motion.

(+↗) $\sum F_r = ma_r$; $-N_C \cos\theta = m\, a_r$

$-N_C \cos 45 = 0.5\, a_r$ 1

(+↖) $\sum F_\theta = ma_\theta$; $F - N_C \sin\theta = ma_\theta$

$F - N_C \sin 45 = 0.5a_\theta$ 2

Kinematics. Using the chain rule (see Appendix C), the first and second time derivatives of r when $\theta$ = 45, $\dot{\theta}$= 3 rad/s, are

$r= 0.8 \cos\theta = 0.8 \cos 45 = 0.5657$ m

$\dot{r} = -0.8 \sin\theta.\dot{\theta} = -0.8 \sin 45(3) = -1.6971$ m/s

$\ddot{r} = -0.8 \sin\theta.\ddot{\theta} - \cos\theta.\dot{\theta}^2 = -0.8\sin 45(0) -0.8 \cos 45(3^2) = -5.091\ m/s^2$

But we have

$a_r = \ddot{r} - r\,\dot{\theta}^2 = -5.091 - (0.5657)(3)^2 = -10.18\ m/s^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta} = (0.5657)(0) + 2(-1.6971)(3) = -10.18\ m/s^2$

Substituting these results into Eqs. (1) and (2) and solving, we get

$N_C = 7.20$ N, $F \approx 0$

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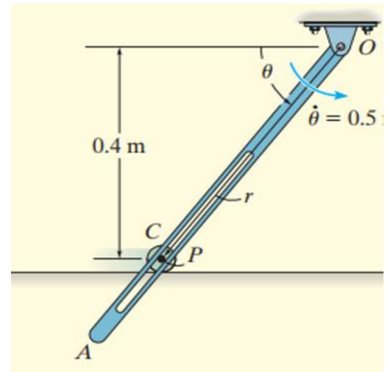
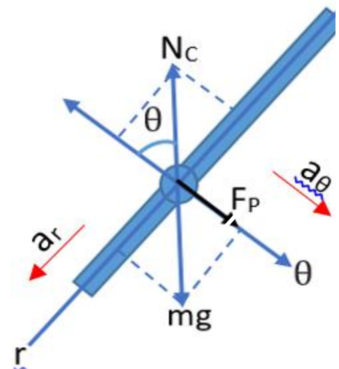
PROPLEM.28: The smooth 2-kg cylinder C in Fig. 1-20a has a pin P through its center which passes through the slot in arm OA. If the arm is forced to rotate in the vertical plane at a constant rate $\dot{\theta}$= 0.5 rad/s, determine the force that the arm exerts on the peg at the instant θ = 60.



(a)

SOLUTION

**Free-Body Diagram.** The free-body diagram for the cylinder is shown in Fig. 1-20b. The force on the peg, $\mathbf{F}_P$, acts perpendicular to the slot in the arm. As usual, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are assumed to act in the directions of positive r and θ, respectively. Identify the four unknowns.




**Equations of Motion.** Using the data in Fig. 1–20b, we have

(+↙) $\sum F_r = ma_r$;  $mg \sin\theta - N_C \sin\theta = ma_r \Rightarrow$

$$19.6\,(0.86) - N_C(0.86) = 2a_r \qquad (1)$$

(+↘) $\sum F_\theta = ma_\theta$; $mg \cos\theta - N_C \cos\theta = ma_\theta \Rightarrow$

$$19.6(0.5) - N_C(0.5) = 2a_\theta \qquad (2)$$

**Kinematics.** From Fig. 1–20a, r can be related to θ by the equation

$r = 0.4 \csc\theta \Rightarrow$ Since $d(\csc\theta) = -(\csc\theta \cot\theta)d\theta$ and $d(\cot\theta) = -(\csc^2\theta)\, d\theta$, then r and the necessary time derivatives become

$r = 0.4 \csc\theta \Rightarrow$ $r = 0.4616$ m and $\dot{r} = -0.4 \csc\theta \cot\theta.\, \dot{\theta} \Rightarrow \dot{r} = -1.333$ m/s

$\ddot{r} = -0.4[(\csc\theta \cot\theta)(\ddot{\theta}) + \dot{\theta}(\csc\theta.(-\csc^2\theta)\, \dot{\theta} + \cot\theta(-\csc\theta.\cot\theta)\dot{\theta}]$

$\ddot{r} = 0.4 \csc\theta.\, \dot{\theta}^2[\csc^2\theta + .\cot^2\theta.)] = 0.4(1.154)(0.5)^2[(1.154)^2 + (0.577)^2] = 0.192$ m/s$^2$ ,

$a_r = \ddot{r} - r\dot{\theta}^2 = 0.192 - 0.4616(0.5)^2 = 0.77 m/s^2$, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(-1.333)(0.5) = -1.333 m/s^2$

Substituting these results into Eqs. 1 and 2 with $\theta = 60$ and solving yields

$N_C = 19.4$ N, $F_P = -0.356$ N