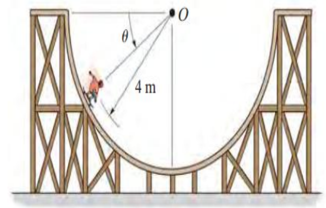
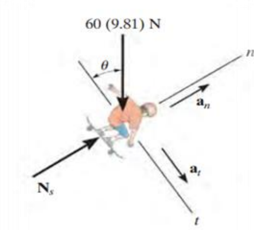
DYNAMICS

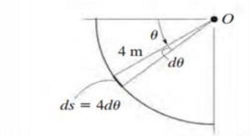
PROBLEM.21: The 60-kg skateboarder in Fig. 1–15*a* coasts down the circular track. If he starts from rest when θ = 0, determine the magnitude of the normal reaction the track exerts on him when θ=60. Neglect his size for the calculation.



(a)



(b)



SOLUTION

Free-Body Diagram.

The free-body diagram of the skateboarder when he is at an arbitrary position θ is shown in Fig. 1-15b. At θ = 60 there are three unknowns, Ns, a_t, and n (or v).

Equations of Motion.

(+↗) $\sum F_n = ma_n$; NS-mgsinθ = ma_n

N_S - [60(9.81)] sin θ = (60)a_n

(+↘) $\sum F_t = ma_t$; mg cosθ = ma_t ⇒[60 (9.81)] cosθ = 60 a_t,

a_t = 9.81 cosθ

Kinematics.

Since at is expressed in terms of θ, the equation v dv = a_t ds must be used to determine the speed of the skateboarder when θ = 60. Using the geometric relation S = θr, where dS = r dθ= (4)dθ, ⇒ ds = 4dθ

Fig.1-15c, and the initial condition

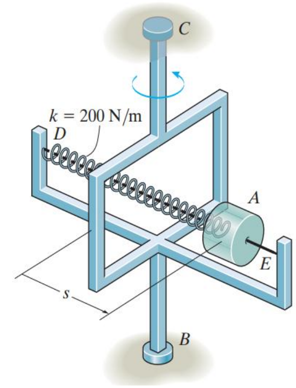
v_0 = 0, a_t= 9.81cos θ, θ= 0, we have, v dv = a_t ds

$\int_0^v v\,dv = \int_0^{60} 9.81(\cos\theta)(4d\theta) \Rightarrow \frac{1}{2}v^2\Big]_0^v = 39.24 \sin\theta\Big]_0^{60}$

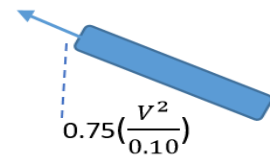
$\frac{1}{2}v^2 - 0 = 39.24(\sin 60 - \sin 0) \Rightarrow v^2 = 67.97\ m^2/s^2$

Substituting this result and θ = 60 into Eq. (1), yields N_S = 1529.23 N = 1.53 kN

DYNAMICS

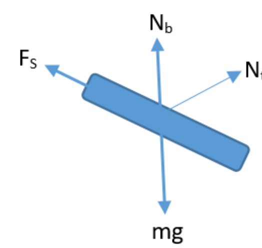


PROBLEM.22: The collar A, having a mass of 0.75 kg, is attached to a spring having a stiffness of k = 200 N/m. When rod BC rotates about the vertical axis, the collar slides outward along the smooth rod DE. If the spring is upstretched when S = 0, determine the constant speed of the collar in order that S = 100 mm. In addition, what is the normal force of the rod on the collar? Neglect the size of the collar



$\sum F_b = 0;\quad N_b - mg = 0 \Rightarrow N_b = mg = 0.75(9.81) = 7.36$ N

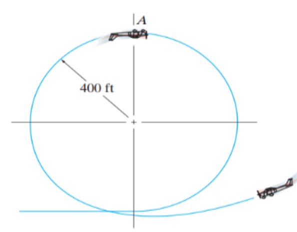
$\sum F_n = ma_n$; $F_{Sp} = m(\frac{V^2}{r}) \Rightarrow k.s = 200(0.1) = 0.75(\frac{V^2}{0.10})$



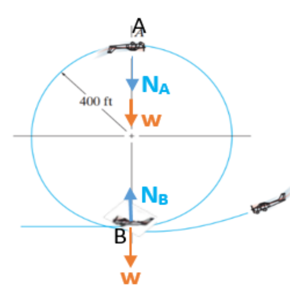
$\sum F_t = ma_t$; $N_t = 0$

v = 1.63 m/s

$N = \sqrt{(N_b)^2 + (N_t)^2} = \sqrt{(7.36)^2 + (0)^2} = 7.36$ N



PROBLEM.23: A pilot weight 150 lb and is traveling at a constant speed of 120 ft/s. Determine the normal force he exerts on the seat of the plane when he is upside down at A and down side at B. The loop has a radius of curvature of 400 ft.



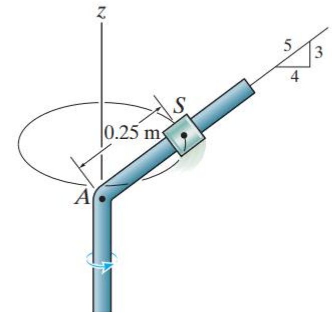
$(+\downarrow)\ \sum F_n = ma_n \Rightarrow N + mg = m\,(\frac{v^2}{r})$

$N = m[(\frac{v^2}{r}) - g] = (150/32.2)[\,(\frac{v^2}{400}) - 9.8]$

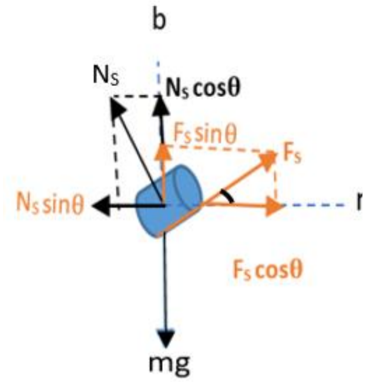
$= 4.66(\frac{120^2}{400})\text{-}150 = 17.76$ Ib

Homework find N_B

PROBLEM.24: The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is μs = 0.2. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod



SOLUTION



$r = S\cos\theta = 0.25(\frac{4}{5}) = 0.2$ m

$(\overset{\leftarrow}{+})$ $\sum F_n = ma_n$; $N_S \sin\theta - \mu_s N_S \cos\theta = m(\frac{v^2}{r})$

$N_S(\frac{3}{5}) - 0.2 N_S(\frac{4}{5}) = 2(\frac{v^2}{0.2})$

$0.6 N_S - 0.16 N_S = 10 v^2$ (1)

$(+\uparrow)$ $\sum F_b = 0$; $N_S\cos\theta + \mu_s N_S \sin\theta - mg = 0 \Rightarrow N_S\cos\theta + \mu_s N_S \sin\theta = mg$

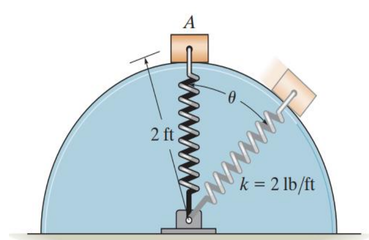
$N_S(\frac{4}{5}) + 0.2N_S(\frac{3}{5}) = 2(9.81)$

$0.8 N_S + 0.12 N_S = 19.62$ (2)

By solving the Eq 2, and substituting it in 1 , then we get

$N_S = 21.3$ N and $v = 0.969$ m/s

PROBLEM.25: The 2-lb block is released from rest at A and slides down along the smooth cylindrical surface. If the attached spring has a stiffness k = 2 lb/ft, determine its upstretched length so that it does not allow the block to leave the surface until θ = 60



Solution

(+↙) $\sum F_n = ma_n$; $F_{sp} + w\cos\theta = ma_n$

$F_{Sp} + 2\cos 60 = (\frac{2}{32.2})(\frac{v^2}{2})$ (1)

(+↘) $\sum F_t = ma_t$; $w\sin\theta = m\,a_t$

$2\sin\theta = (\frac{2}{32.2})\,a_t \Rightarrow a_t = 32.2\sin\theta$

$S = r\,\theta \Rightarrow dS = rd\theta = 2d\theta$ (2)

$v\,dv = a_t\,ds$; $\Rightarrow \int_0^v vdv = \int_0^\theta 32.2(\sin\theta)2d\theta \Rightarrow (\frac{1}{2}v^2) = 64.4(-\cos\theta - (-\cos 0))$

$\Rightarrow (\frac{1}{2}v^2) = 64.4(-\cos\theta + 1)$ (3)

When $\theta = 60° \Rightarrow v^2 = 64.4$

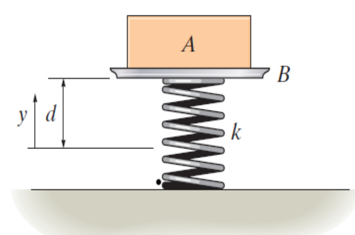
From Eq. (1)

$F_{SP} + 2\cos 60 = (\frac{2}{32.2})(\frac{v^2}{2}) \Rightarrow 2S + 2\cos 60 = (\frac{2}{32.2})(\frac{64.4}{2}) \Rightarrow S = 0.5$ ft

$S = \Delta l = l - l_0 \Rightarrow l_0 = l - S = 2 - 0.5 = 1.5$ ft

PROBLEM.26: The block A has a mass m_A and rests on the pan B, which has a mass m_B both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes upstretched.

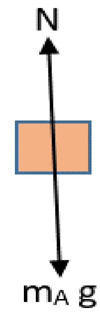
SOLUTION



For Equilibrium

(+↑) $\sum F_y = ma_y$; $F_{SP} - (m_A + m_B)g = m(0)$

$F_{SP} = (m_A + m_B)g$

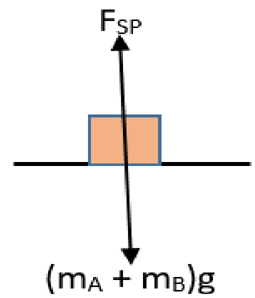


$\Rightarrow K.y_{eq} = (m_A + m_B)g$ 1

Block: A

$(+\uparrow)$ $\sum F_y = ma_y \Rightarrow N - m_A g = m_A a$

$a = [\frac{(N - m_A g}{m_A}]$ 2

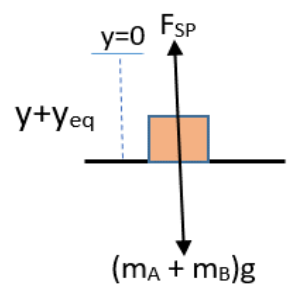


Block: A and pan

$(+\uparrow)$ $\sum F_y = ma_y$; $F_S - (m_A + m_B)g = (m_A + m_B)a$

$k(y_{eq} + y) - (m_A + m_B)g = (m_A + m_B)a$ 3

From Eqs 1, 2 and 3



$[(m_A + m_B)g + K y] - (m_A + m_B)g = (m_A + m_B)[\frac{(N - m_A g}{m_A}]$

Require

When block A is released then N=0

$k.y = -(m_A + m_B)g$

Since d is downward,

Distance $= -y \Rightarrow$ distance $= (\frac{m_A + m_B}{K})\, g$