$$\sum F = ma_G \qquad (1\text{-}6)$$

Hence, the sum of the external forces acting on the system of particles is equal to the total mass of the particles times the acceleration of its center of mass *G*. Since in reality all particles must have a finite size to possess mass, Eq.1-6 justifies application of the equation of motion to a body that is represented as a single particle.

Important Points

1. The equation of motion is based on experimental evidence and is valid only when applied within an inertial frame of reference.
2. The equation of motion states that the unbalanced force on a particle causes it to accelerate.
3. An inertial frame of reference does not rotate, rather its axes either translate with constant velocity or are at rest.
4. Mass is a property of matter that provides a quantitative measure of its resistance to a change in velocity. It is an absolute quantity and so it does not change from one location to another.
5. Weight is a force that is caused by the earth's gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth's surface.

1.4 Equations of Motion: Rectangular Coordinates

DYNAMICS

When a particle moves relative to an inertial x, y, z frame of reference, the forces acting on the particle, as well as its acceleration, can be expressed in terms of their i, j, k components, Fig. 1–5. Applying the equation of motion, we have

$$\sum F = ma; \sum F_x i + \sum F_y j + \sum F_z k = m(a_x i + a_y j + a_z k)$$

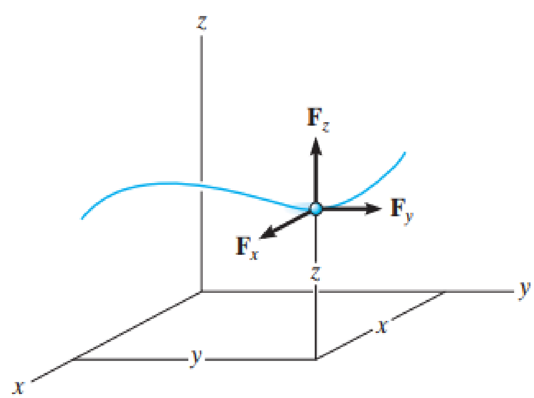


Fig. 1-5

Fig. 2-5 For this equation to be satisfied, the respective i, j, k components on the left side must equal the corresponding components on the right side. Consequently, we may write the following three scalar equations.

$$\sum F_x = ma_x, \quad \sum F_y = ma_y, \quad \sum F_z = ma_z \qquad (1\text{-}7)$$

In particular, if the particle is constrained to move only in the *x*–*y* plane, then the first two of these equations are using to specify the motion.

DYNAMICS

Free-Body Diagram (FBD)

1. The equations of motion are using to solve problems, which require a relationship between the forces acting on a particle, and the accelerated motion they cause Select the inertial coordinate system. Most often, rectangular or x, y, z coordinates are chosen to analyze problems for which the particle has rectilinear motion.
2. Once the coordinates are established, draw the particle's free body diagram. FBD drawing this diagram is very important since it provides a graphical representation that accounts for all the forces (F) which act on the particle, and thereby makes it possible to resolve these forces into their x, y, z components
3. The direction and sense of the particle's acceleration should also be established. If the sense is unknown, for mathematical convenience assume that the sense of each acceleration component acts in the same direction as its positive inertial coordinate axis
4. The acceleration may be represented as the m**a** vector on the kinetic diagram.
5. Identify the unknowns in the problem

Equations of Motion.

1. If the forces can be resolved directly from the free-body diagram, apply the equations of motion in their scalar component form.
2. If the geometry of the problem appears complicated, which often occurs in three dimensions, Cartesian vector analysis can be used for the solution.
3. Friction. If a moving particle contacts a rough surface, it may be necessary to use the frictional equation, which relates the frictional and normal forces F_f and N acting at the surface of contact by using the coefficient of kinetic friction, i.e., F_f =

$\mu_k N$. Remember that F_f always acts on the free-body diagram such that it opposes the motion of the particle relative to the surface it contacts. If the particle is on the verge of relative motion, then the coefficient of static friction should be use $F_S = \mu_S N$

4. If the particle is connected to an elastic spring having negligible mass, the force Fs can be related to the deformation of the spring by the equation $F_{Sp} = -kS$. Here k is the spring's stiffness measured as a force per length units, and S is the stretch or compression defined as the difference between the deformed length l and the undeformed length l_0, i.e., $S = \Delta l = l - l_0$. (Hook's Law)

Kinematics.

- If the velocity or position of the particle is to be found, it will be necessary to apply the necessary kinematic equations once the particle's acceleration is determined from $\sum F = m\mathbf{a}$.
- If acceleration is a function of time, use a = dv/dt and v = ds/dt which, when integrated, yield the particle's velocity and position, respectively.
- If acceleration is a function of displacement, integrate a ds=v dv to obtain the velocity as a function of position.
- If acceleration is constant, use $v = v_0 + a_c t$, $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$,
- $v^2 = v_0^2 + 2a_c(S - S_0)$ to determine the velocity or position of the particle.
- If the problem involves the dependent motion of several particles, use the

method outlined in secto relate their accelerations. In all cases, make sure the positive inertial coordinate directions used for writing the kinematic equations are the same as those used for writing the equations of motion; otherwise, simultaneous solution of the equations will result in errors.

- If the solution for an unknown vector component yields a negative scalar, it indicates that the component acts in the direction opposite to that which was assumed

Useful relation

$g = 32.174\ \frac{ft}{s^2}$	Acceleration due to gravity at the earth's surface.
$slug = 14.594\ kg$	English system unit of mass
$lbf = 4.448\ N$	English system unit of force
$\frac{lbf}{slug} = 1\ \frac{ft}{s^2}$	The lbf is the force required to accelerate one slug at 1 ft/sec squared.
$lb = 0.454\ kg$ $lbm = 0.454\ kg$	lb, or lbm is a common unit of mass.
$lbm \cdot g = 1\ lbf$ $lbm = 0.031\ slug$ $slug = 32.174\ lb$	lbm is the mass that will exert 1 lbf of force due to gravity at the earth's surface. (Scales typically measure force, not mass.)

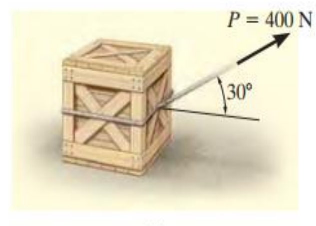
Illustrative Examples

PROBLEM.1: the 50-kg crate shown in Fig. 1–6a rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-

N towing force as shown, determine the velocity of the crate in 3 s starting from rest.

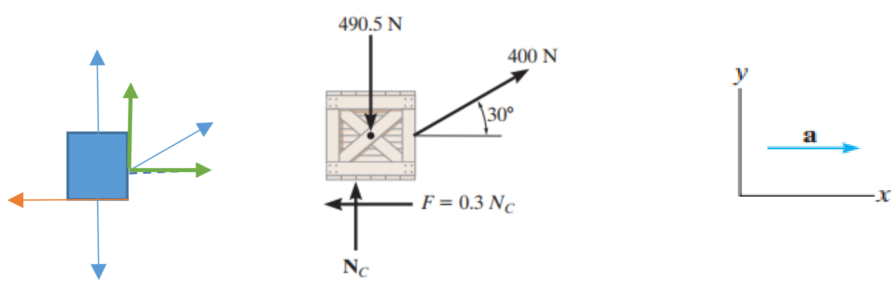
SOLUTION

Using the equations of motion, we can relate the crate's acceleration to the force causing the motion. The crate's velocity can then be determined using kinematics.



(a)

Free-Body Diagram. The weight of the crate is W=mg =50×9.81 =490.5 N, As shown in Fig. 1–6b, the frictional force has a magnitude F= $\mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration a is assumed to act horizontally, in the positive x direction. There are two unknowns, namely N_C and a.



(b)

$(\overset{\rightarrow}{+})$ $\sum F_X = ma$, $P\cos\theta - F_{\mathbf{k}} = ma \rightarrow 400\cos(30) - 0.3N = 50a$ (1)

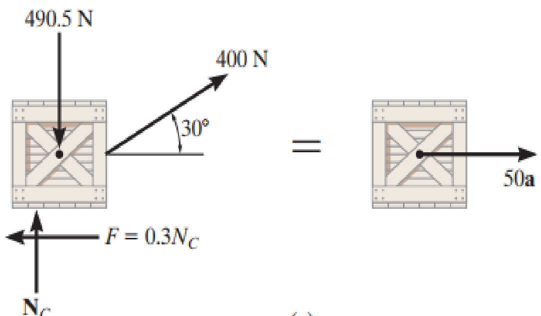
$(+\uparrow)$ $\sum F_Y = 0$; $N_C - 490.5 + 400\sin 30 = 0$

Solving Eq. 2 for N_C, substituting the result into Eq. 1, and solving.

$N_C = 290.5$ N, $a = 5.185\ m/s^2$

Kinematics. Notice that the acceleration is *constant*, since the applied force P is constant. Since initial velocity is zero, the velocity of the crate in 3s is

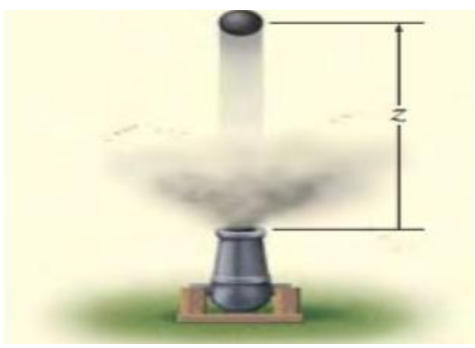
(+↑) $V= V_0 +at = 5.185(3) = 15.6$ m/s



(c)

NOTE: we can also use the alternative procedure of drawing the crate's FBD and kinetic diagrams, Fig. 1-6c, prior to applying the equations of motion.

PROBLEM.**2:** A 10-kg projectile is fired vertically upward from the ground, with an initial velocity of 50 m/s, Fig. 1-7a. Determine the maximum height to which it will travel if (a) atmospheric resistance is neglected; and (b) atmospheric resistance is measured as $F_D=(0.01v^2)$ N, where v is the speed of the projectile at any instant, measured in m/s.



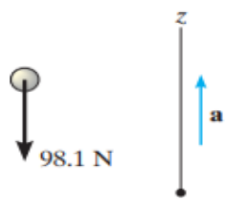
a

SOLUTION

In both cases the known force on the projectile can be related to its acceleration using the equation of motion. Kinematics can then be used to relate the projectile's acceleration to its position.

Part (a) Free-Body Diagram.

As shown in Fig. 1–7b, the projectile's weight is $W = mg = 10\times9.81 = 98.1$ N. We will assume the unknown acceleration **a** acts upward in the positive z direction.



Equation of Motion.

$(+\uparrow)\ \sum F_z = ma_z \Rightarrow -mg = ma = -98.1 = 10\,a,\ a = -9.81\ m/s^2$

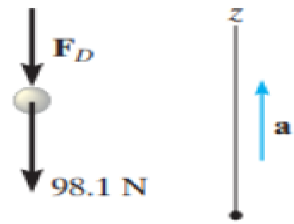
The result indicates that the projectile, like every object having free flight motion near the earth's surface, is subjected to a downward acceleration of 9.81 m/s^2.

Kinematics. Initially, $z_0 = 0$ and $v_0 = 50$ m/s, and at the maximum height z= h, v =0. Since the acceleration is constant, then

$\left(\overset{\rightarrow}{+}\right)\quad v^2 = v_0^2 + 2a(z - z_0);\ 0 = (50)^2 + 2(-9.81)(z-0) \rightarrow 2500 = 19.62\ Z$

$$Z = 127.4\ m,\ \therefore h = Z = 127.2\ m$$

Part (b) Free-Body Diagram. Since the force $F_D = (0.01v^2)$ N tends to retard the upward motion of the projectile, it acts downward as shown on the free-body diagram, Fig. 1-7c



c

Equation of Motion.

(↑+) $F_z = ma_z; \Rightarrow -F_D - mg = ma = -0.01v^2 - 98.1 = 10\,a \rightarrow a = -(0.001v^2 + 9.81)$

The result indicates that the projectile, like every object having free flight motion near the earth, is subjected to a constant downward acceleration of 9.81 m/s^2

Kinematics. Here the acceleration is not constant since FD depends on the velocity. Since a=f(v), we can relate a to position using

(+↑) $a\,dz = v\,dv \rightarrow -(0.001v^2 + 9.81)dz = v\,dv, \quad dz = -\,vdv/(0.001v^2 + 9.81)$

Separating the variables and integrating, realizing that initially $z_0 = 0$, $v_0 = 50$ m/s (positive upward), and at z = h, v = 0, we have

$$z = \int_{z_o}^{z} dz = -\int_{50}^{0} \frac{v}{(0.001v^2 + 9.81)} dv = \int_{0}^{50} \frac{v}{(0.001v^2 + 9.81)} dv = \frac{50}{\sqrt{0.01}} \ln\,(0.1v^2 + 981]_{0}^{50}$$

500 ln[250- 981]- 500 ln[981]=113.5 m, h=z=113.5 m