DYNAMICS

Procedure for Analysis

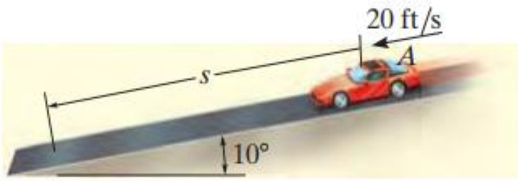
Work (Free-Body Diagram).

- Establish the inertial coordinate system and draw a free-body diagram of the particle in order to account for all the forces that do work on the particle as it moves along its path.

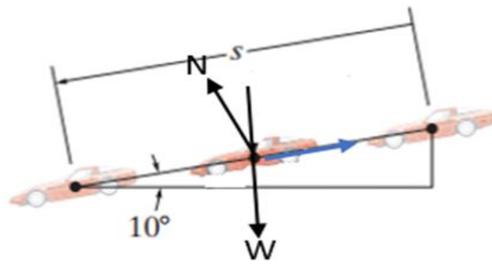
Principle of Work and Energy.

- Apply the principle of work and energy, $T_1 + U_{1\text{-}2} = T_2$.
- The kinetic energy at the initial and final points is always positive, since it involves the speed squared 1T = 1/2 mv_2^2.
- A force does work when it moves through a displacement in the direction of it.
- Work is positive when the force component is in the same sense of direction as its displacement, otherwise it is negative.
- Forces that are functions of displacement must be integrated toobtain the work. Graphically, the work is equal to the area under the force-displacement curve.
- The work of a weight is the product of the weight magnitude andthe vertical displacement, UW = {Wy. It is positive when theweight moves downwards.
- The work of a spring is of the form Us = 1 2 ks2, where k is thespring stiffness and s is the stretch or compression of the spring.

PROBLEM.35: The 3500-lb automobile shown in Fig. 2–10a travels down the 10° inclined road at a speed of 20 ft/s. If the driver jams on the brakes, causing his wheels to lock, determine how far S the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$



(a)



SOLUTION

This problem can be solved using the principle of work and energy, since it involves force, velocity, and displacement.

Work (Free-Body Diagram). As shown in Fig. 2-10b, the normal force $\mathbf{N}_A$ does no work since it never undergoes displacement along its line of action. The weight, 3500 lb, is displaced *s* sin 10° and does positive work. Why? The frictional force $\mathbf{F}_A$ does both external and internal work when it undergoes a displacement *s*. This work is negative since it is in the opposite sense of direction to the displacement. Applying the equation of equilibrium normal to the road, we have

$(+\uparrow)$ $\sum F_n = 0$; $N_A - mg\cos\theta = 0 \Rightarrow N_A - 3500\cos 10 = 0 \Rightarrow$

$N_A = 3446.8$ lb

Thus, $F_k = \mu_k N_A = 0.5\,(3446.8) = 1723.4$ lb

Principle of Work and Energy.

$U_W = wy$, $y = S \sin\theta$

$$T_1 + \sum U_{1\text{-}2} = T_2 \Rightarrow \frac{1}{2}\left(\frac{3500}{32.2}\right)(20)^2 + 3500\,(S \sin 10) - (1723.4)S = 0 \Rightarrow S = 19.5 \text{ ft}$$

NOTE: If this problem is solved by using the equation of motion, two steps are involved. First, from the free-body diagram, Fig. 2-10b, the equation of motion is applied along the incline. This yields

$(+\swarrow)$ $\sum F_s = ma_s$; $3500 \sin 10 - 1723.4 = (3500/32.2)\,a \Rightarrow a = -10.3$ m/s^2

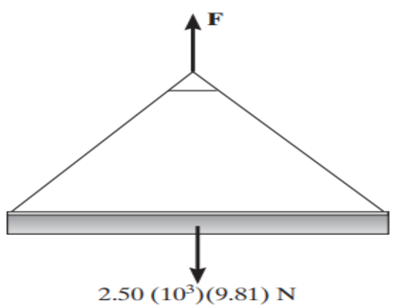
Then, since a is constant, we have

$(+\swarrow)$ $V^2 = V_o^2 + 2a\,(S - S_o)$

$(0)^2 = (20)^2 + 2(-10.3)(S - 0) \Rightarrow S = 19.5$ ft

PROBLEM.36: For a short time the crane in Fig. 2–11*a* lifts the 2.50-Mg beam with a force of $F = (28 + 3S^2)$ kN. Determine the speed of the beam when it has risen

S = 3 m. Also, how much time does it take to attain this height starting from rest?



SOLUTION

Work (Free-Body Diagram). As shown on the free-body diagram, Fig. 2–11b, the lifting force **F** does positive work, which must be determined by integration since this force is a variable. Also, the weight is constant and will do negative work since the displacement is upward.

Principles of Work and Energy.

$T_1 + \sum U_{1\text{-}2} = T_2$, $T_1 = \frac{1}{2}mv_1^2$, $T_2 = \frac{1}{2}mv_2^2$, $U_W = mgy$ $\quad (T_1 = 0)$

$\frac{1}{2}mv_1^2 + \int_{S_1}^{S_2} F.ds - mgy = \frac{1}{2}mv_2^2$

$0 + \int_0^S (28 + 3S^2)(10^3)\, dS - (2.50)(10^3)(9.81)S = \frac{1}{2}(2.50)(10^3)v_2^2$

$28(10^3)S + (10^3)S^3 - 24.525(10^3)S = 1.25(10^3)v^2$

When S = 3 m, ⇒ $V = [(111-73.575)/1.25]^{0.5}$ ⇒ V = 5.47 m/s

Or $v = (2.78s + 0.8S^3)^{1/2} = 5.47$ m/s

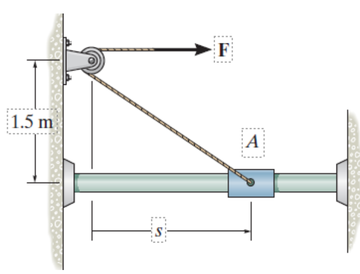
Kinematics. Since we were able to express the velocity as a function of displacement, the time can be determined using v = ds/dt. In this case.

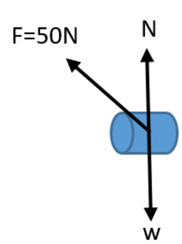
The integration can be performed numerically using a pocket calculator. The result

Hint $\quad v = (2.78s + 0.8S^3)^{1/2}$ ⇒ t = 1.79 s [Homework)

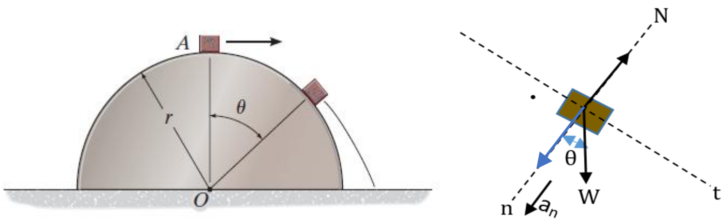
NOTE: The acceleration of the beam can be determined by integrating Eq. (1) using v dv = a ds, or more directly, by applying the equation of motion, ∑F = ma.

(Homework) The force of F = 50 N is applied to the cord when S = 2 m. If the 6-kg collar is orginally at rest, determine its velocity at S = 0. Neglect friction.





PROBLEM.37: A small box of mass *m* is given a speed v = $\frac{1}{2}\sqrt{gr}$ of at the top of the smooth half cylinder. Determine the angle θ at which the box leaves the cylinder.



SOLUTION

Principle of Work and Energy: By referring to the free-body diagram of the block, Fig. *a*, notice that **N** does no work, while **W** does positive work since it displaces downward though a distance of y = r-r cos θ

$$T_1 + \sum U_{1\text{-}2} = T_2 \Rightarrow \frac{1}{2}mv_1^2 + wy = \frac{1}{2}mv_2^2$$

$$\frac{1}{2} m \left(\frac{1}{2}\sqrt{gr}\right)^2 + mg(r - r\cos\theta) = \frac{1}{2} mv_2^2 \Rightarrow v_2^2 = gr\left(\frac{9}{4} - 2\cos\theta\right)$$

$$a_n = \frac{v^2}{r} = \frac{gr\left(\frac{9}{4} - 2\cos\theta\right)}{r} = g\left(\frac{9}{4} - 2\cos\theta\right)$$ By referring to Fig above

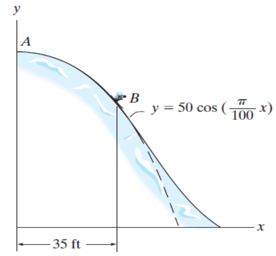
(+↙) $\sum F_n = ma_n$, $mg\cos\theta - N = mg\left(\frac{9}{4} - 2\cos\theta\right) \Rightarrow N = mg\left(3\cos\theta - \frac{9}{4}\right)$

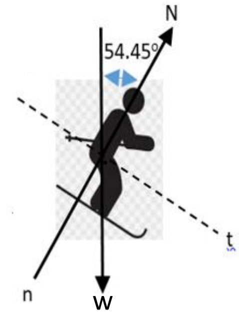
It is required that the block leave the track.Thus, . N = 0

$0 = mg\left(3\cos\theta - \frac{9}{4}\right) \Rightarrow 3\cos\theta = \frac{9}{4}$ then $\theta = \cos^{-1}\left(\frac{9}{12}\right) = 41.4^{o}$

DYNAMICS

PROBLEM.38: When the 150-lb skier is at point A he has a speed of 5 ft/s. Determine his speed when he reaches point B on the smooth slope. For this distance the slope follows the cosine curve shown. Also, what is the normal force on his skis at B and his rate of increase in speed? Neglect friction and air resistance.





SOLUTION

$y = 50\cos\left(\frac{\pi}{100}\right)x\Big]_{x=35} = 22.70$ ft

$\frac{dy}{dx} = \tan\theta = -50\left(\frac{\pi}{100}\right)\sin\left(\frac{\pi}{100}\right)x\Big]_{x=35} = -1.3996$

$\theta = \tan^{-1}(-1.3996) = -54.45°$

$\frac{d^2y}{dx^2} = -\left(\frac{\pi^2}{200}\right)\cos\left(\frac{\pi}{100}\right)x\Big]_{x=35} = -0.02240$

But we have $\rho = [1 + (\frac{dy}{dx})^2]^{3/2} / |\frac{d^2y}{dx^2}| \Rightarrow \rho = [1 + (-1.3996)^2]^{3/2} / |-0.02240| = 227.2$ ft

$T_1 + \sum U_{1\text{-}2} = T_2 \Rightarrow \frac{1}{2}\left(\frac{50}{32.2}\right)(5)^2 + 50\,(50 - 22.7) = \frac{1}{2}\left(\frac{50}{32.2}\right)v^2$

$19.4 + 1365 = 0.776\,v^2 \Rightarrow v = 42.22$ ft/s

(+↙) $\sum F_n = ma_n$, $N - mg\cos\theta = m\left(\frac{v^2}{\rho}\right) \Rightarrow N = 50.6$ lb

(+↘) $\sum F_t = ma_t$, $mg\sin\theta = ma_t \Rightarrow 150\sin(-54.45) = \left(\frac{150}{32.2}\right)a_t$

$a_t = 32.2\sin(-54.45) = -26.2$ ft/s^2

[Noted the tangential acceleration sign appears negative, why?]